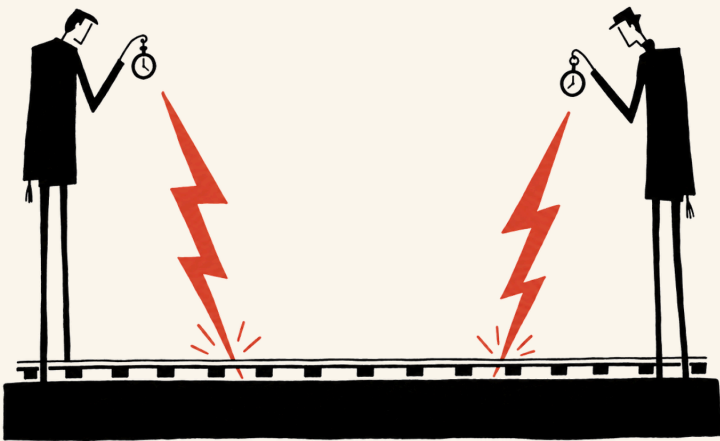


Relativity: The Special and General Theory



Albert Einstein · 1917

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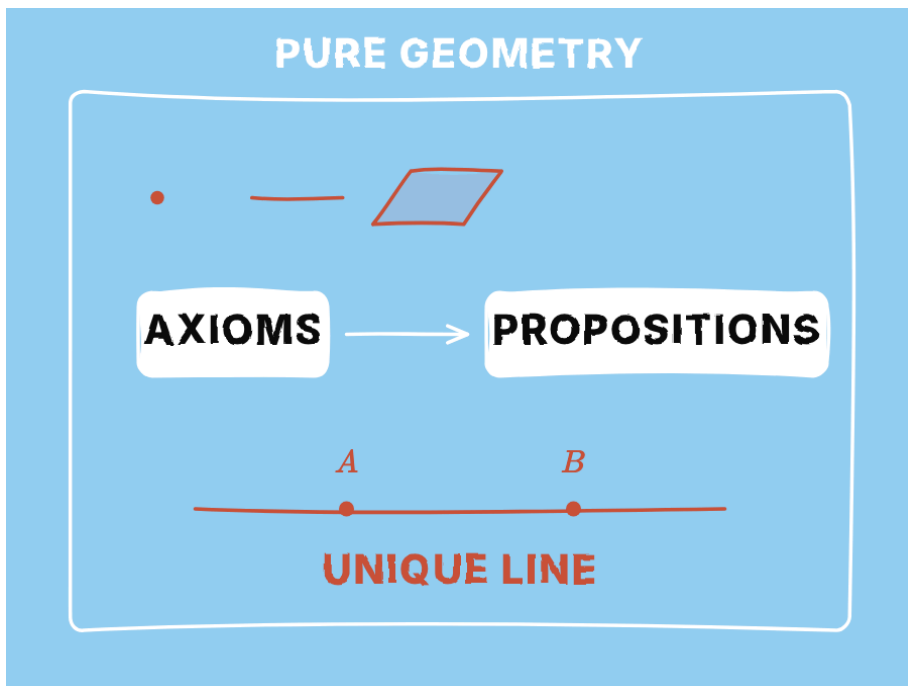
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Physical Meaning of Geometrical Propositions

Surely you too, dear reader, made the acquaintance as a boy or girl of the proud edifice of Euclid's geometry, and you remember perhaps with more respect than love the proud structure up whose high staircases you were chased for countless hours by conscientious teachers. Surely, by virtue of this past of yours, you would punish with contempt anyone who declared even the most remote little proposition of this science to be untrue. But this feeling of proud certainty would perhaps leave you at once if someone asked you: "What, then, do you mean by the assertion that these propositions are true?" Let us linger a little over this question.



Inside Pure Geometry

Geometry starts out from certain basic concepts, such as plane, point, straight line, with which we are able to associate more or less definite ideas, and from certain simple propositions, the axioms, which, on the strength of those ideas, we are inclined to accept as "true". All the remaining propositions are then traced back to those axioms, that is, proved, by means of a logical method whose justification we feel compelled to acknowledge. A proposition is then correct, or "true", when it has been derived from the axioms in the recognised manner. The question of the "truth" of the individual geometrical propositions thus leads back to the question of the "truth" of the axioms. But it has long been known that this last question is not only unanswerable by the methods of geometry, but is in itself without any meaning at all. One cannot ask whether it is true that only one straight line passes

through two points. One can only say that Euclidean geometry deals with structures it calls "straight lines", and to which it ascribes the property of being uniquely determined by two of their points. The concept "true" does not fit the statements of pure geometry, because by the word "true" we are in the habit of designating, in the last resort, agreement with a "real" object; geometry, however, is not concerned with the relation of its concepts to the objects of experience, but only with the logical connection of these concepts among themselves.

That we nevertheless feel drawn to call the propositions of geometry "true" is easily explained. To the geometrical concepts there correspond, more or less exactly, objects in nature, and these are without doubt the sole cause of the origin of those concepts. Geometry may keep aloof from this in order to give its edifice the greatest possible logical unity; but the habit of seeing in a segment, for example, two marked places on a practically rigid body is deeply rooted in our habits of thought. We are further accustomed to regard three places as lying on a straight line when, by a suitable choice of the point of observation, we can make their apparent positions coincide in one-eyed vision.

If now, following our habit of thought, we add to the propositions of Euclidean geometry the single proposition that to two points of a practically rigid body there always corresponds the same distance, the same segment, whatever changes of position we may make with the body, then the propositions of Euclidean geometry become propositions about the possible relative placement of practically rigid bodies. The geometry thus completed is then to be treated as a branch of physics. Now we may rightly ask after the "truth" of geometrical propositions so interpreted, for we may ask whether those propositions hold for the real things which we have assigned to the

geometrical concepts. Somewhat inexactly, then, we can say that by the "truth" of a geometrical proposition in this sense we understand its holding good in a construction with compass and straightedge.

The conviction of the "truth" of geometrical propositions in this sense rests, of course, exclusively on rather imperfect experience. We shall assume that truth of the geometrical propositions for the present, in order to see afterwards, in the last part of our considerations, in the general theory of relativity, that and in what way that truth has its limits.

Questions

1. Within geometry itself, when is a proposition called true?
 - a. When it agrees with measurements made on real bodies
 - b. When it has been derived from the axioms in the recognised manner
 - c. When every careful reader finds it obvious
2. Why does the author say the concept "true" does not fit the statements of pure geometry?
 - a. Because its axioms have never been proved by anyone
 - b. Because "true" designates agreement with a real object, and pure geometry deals only with the logical connection of its concepts
 - c. Because the ideas we associate with point and straight line are too vague
 - d. Because geometrical constructions are never exact

3. What single proposition, added to Euclidean geometry, turns it into a branch of physics?
 - a. That to two points of a practically rigid body there always corresponds the same distance, whatever changes of position the body undergoes
 - b. That three places lie on a straight line when they appear to coincide in one-eyed vision
 - c. That a construction with compass and straightedge is always possible
 - d. That the objects of nature are the sole cause of geometrical concepts
4. On what does the conviction of the truth of geometrical propositions, in the physical sense, rest?
 - a. On the logical unity of the edifice of geometry
 - b. On the fact that the axioms feel obvious
 - c. On rather imperfect experience, assumed for the present

Answers

1. **b)** A proposition is correct, or true, when it has been derived from the axioms in the recognised manner; the question of its truth thus leads back to the truth of the axioms.
2. **b)** By the word true we designate, in the last resort, agreement with a real object; geometry is concerned only with the logical connection of its concepts among themselves, not with their relation to the objects of experience.
3. **a)** With that one added proposition, the propositions of Euclidean geometry become propositions about the possible relative placement of

practically rigid bodies, and geometry so completed is to be treated as a branch of physics.

4. c) The conviction rests exclusively on rather imperfect experience; the author assumes that truth for the present, to show later, in the general theory of relativity, where it has its limits.

Practice

1. Explain in your own words why the question "Is it true that only one straight line passes through two points?" cannot be asked inside pure geometry, while the question whether a drawn triangle obeys Pythagoras can be.

Hint: Ask of each question whether it is about concepts or about real objects.

Answer: Pure geometry deals with structures it calls straight lines and gives them, by definition, the property of being fixed by two of their points; asking whether that is true would ask for agreement with a real object, which pure geometry does not concern itself with. The drawn triangle is a real thing assigned to the geometrical concepts, so asking whether the proposition holds for it is a question about agreement with reality, which has a meaning.

2. The author says we feel drawn to call geometrical propositions true, and that this is easily explained. Give the explanation, with the two habits of thought he mentions.

Hint: Where did the concepts of geometry come from in the first place?

Answer: The geometrical concepts correspond, more or less exactly, to objects in nature, which are the sole cause of their origin. Two habits show this: we see in a segment two marked places on a practically rigid body, and we regard three places as lying on a straight line when, from a

suitable point of observation, their apparent positions coincide in one-eyed vision.

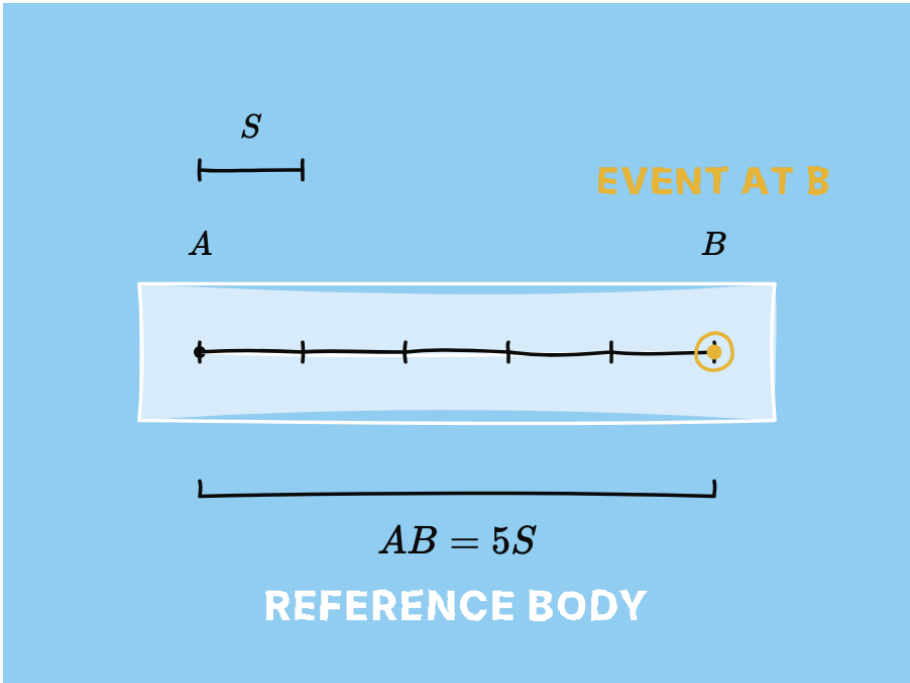
3. In what sense, somewhat inexactly, does the author say we may understand the truth of a geometrical proposition once geometry is treated as a branch of physics, and why does he call that conviction provisional?

Hint: Think of what one can do with compass and straightedge.

Answer: Its truth means that it holds good in a construction with compass and straightedge, that is, for the real things assigned to the geometrical concepts. The conviction rests only on rather imperfect experience, so it is assumed for the present; the general theory of relativity, at the end of the book, will show that and in what way it has limits.

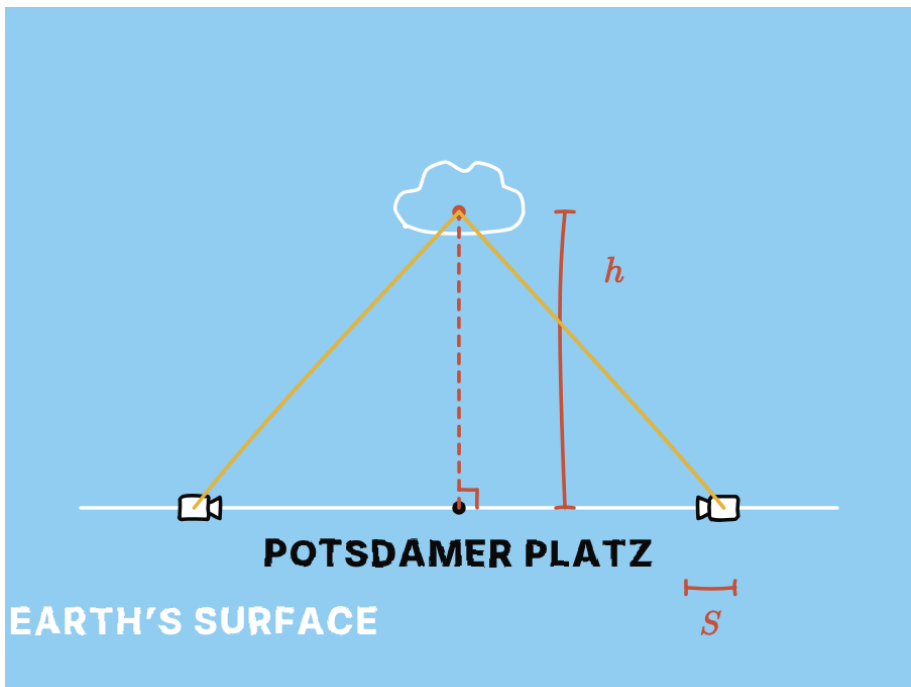
The Coordinate System

On the basis of the physical interpretation of distance just indicated, we are also in a position to establish the distance between two points of a rigid body by means of measurements. For this we need a segment to be used once and for all, a little rod S , which serves as the unit of measure. If now A and B are two points of a rigid body, the straight line joining them can be constructed according to the laws of geometry; then, on this joining line, the segment S can be laid off from A as many times as it takes to reach B . The number of repetitions of this laying off is the measure of the segment AB . All measuring of lengths rests on this.



Measuring on a Reference Body

Every spatial description of the place of an event or of an object rests on one's naming the point of a rigid body, the reference body, with which that event coincides. This holds not only for scientific description but also for daily life. If I analyse the statement of place "in Berlin, on Potsdamer Platz", it means the following. The surface of the Earth is the rigid body to which the statement of place refers; on it, "Potsdamer Platz in Berlin" is a marked point, furnished with a name, with which the event spatially coincides.



From Named Place to Height

This primitive way of stating a place knows only places on the surface of rigid bodies, and it is tied to the existence of distinguishable points of that surface. Let us see how the human mind frees itself from these two restrictions without the nature of the statement of place undergoing any change. If, for instance, a cloud is floating above Potsdamer Platz, its place, referred to the surface of the Earth, can be fixed by erecting on the square a pole, standing vertically, that reaches up to the cloud. The length of the pole, measured with the unit of measure, together with the statement of the place of the foot of the pole, is then a complete statement of place. From this example we see by what road a refinement of the concept of place has come about.

First, the rigid body to which the statement of place refers is extended in such a way that the object to be located is reached by the completed rigid body. Second, a number is used to characterise the place, instead of named marks; here, the length of the pole measured with the unit of measure. Third, one speaks of the height of the cloud even when a pole reaching the cloud has not been erected at all. In our case one determines, from optical photographs of the cloud taken from various places on the ground, and taking into account the properties of the propagation of light, how long the pole would have to be made in order to reach the cloud.

From this consideration one sees that it will be an advantage for the description of places if, by the use of measured numbers, we succeed in making ourselves independent of the existence of named marks on the rigid body to which the statement of place refers. Measuring physics achieves this by the use of the Cartesian coordinate system.

This consists of three mutually perpendicular rigid plane walls, joined together into one rigid body. The place of any event with respect to the coordinate system is described, in essence, by stating the lengths of the three perpendiculars, or coordinates (x, y, z) , which can be dropped from the event onto those three plane walls. The lengths of these three perpendiculars can be determined by a sequence of manipulations with rigid rods, manipulations which are prescribed by the laws and methods of Euclidean geometry.

In applications, the rigid walls forming the coordinate system are mostly not realised; nor are the coordinates actually determined by constructions with rigid rods, but indirectly. The physical meaning of statements of place must, however, always be sought in accordance

with the above considerations, if the results of physics and astronomy are not to dissolve into obscurity.

The following therefore results: every spatial description of events makes use of a rigid body to which the events are to be spatially referred. That reference presupposes that the laws of Euclidean geometry hold for "segments", where the "segment" is physically represented by two marks on a rigid body.

Questions

1. What is the measure of the segment AB, according to the passage?
 - a. The number of times the unit rod S must be laid off along the straight line from A to B
 - b. The time a signal needs to travel from A to B
 - c. The number of named marks lying between A and B
2. What does the statement of place "in Berlin, on Potsdamer Platz" mean when it is analysed?
 - a. That the event is at a certain distance from the centre of the Earth
 - b. That the event spatially coincides with a marked, named point of a rigid body, the surface of the Earth
 - c. That the event has three coordinates referred to perpendicular walls
 - d. That a pole could be erected on the square to reach the event

3. The height of a cloud is found from photographs taken from several places on the ground. Which refinement of the concept of place does this illustrate?
 - a. Extending the rigid body until it reaches the object
 - b. Using a number instead of named marks
 - c. Speaking of a height even though no pole reaching the cloud has been erected
 - d. Treating the cloud as a point of the reference body
4. Why must the physical meaning of statements of place always be sought in rigid rods and rigid bodies, even though coordinates are usually determined indirectly?
 - a. Because indirect determinations are never exact enough
 - b. Because the three rigid walls are always actually built
 - c. Because otherwise the results of physics and astronomy dissolve into obscurity

Answers

1. **a)** On the straight line joining A and B the segment S is laid off from A as many times as it takes to reach B; the number of repetitions is the measure of the segment, and all measuring of lengths rests on this.
2. **b)** The surface of the Earth is the rigid body to which the statement refers; on it Potsdamer Platz in Berlin is a marked point furnished with a name, with which the event spatially coincides.
3. **c)** One speaks of the height of the cloud even when no pole reaching it exists; from the photographs, allowing for how light propagates, one determines how long the pole would have to be.
4. **c)** The walls are mostly not realised and the coordinates are found indirectly, yet their physical meaning must be sought in accordance with the

rod constructions, or the results of physics and astronomy dissolve into obscurity.

Practice

1. Describe in your own words what the Cartesian coordinate system consists of and what the three coordinates of an event are, and say how, in principle, their values are determined.

Hint: Start from three rigid plane walls and ask what can be dropped from the event onto each.

Answer: It consists of three mutually perpendicular rigid plane walls joined into one rigid body. The place of an event is described by the lengths of the three perpendiculars dropped from the event onto the walls, the coordinates x , y , z . These lengths are determined, in principle, by a sequence of manipulations with rigid rods prescribed by the laws and methods of Euclidean geometry.

2. The primitive way of stating a place is tied to two restrictions. Name them, and explain how the pole erected on Potsdamer Platz frees us from each without changing the nature of a statement of place.

Hint: What can "Potsdamer Platz in Berlin" locate, and what can it not?

Answer: It knows only places on the surface of rigid bodies, and it needs distinguishable, named points on that surface. The pole extends the rigid body upward so that the cloud, off the surface, is reached; and the measured length of the pole, a number, replaces a named mark. The statement still names a point of a rigid body with which the cloud coincides, so its nature is unchanged.

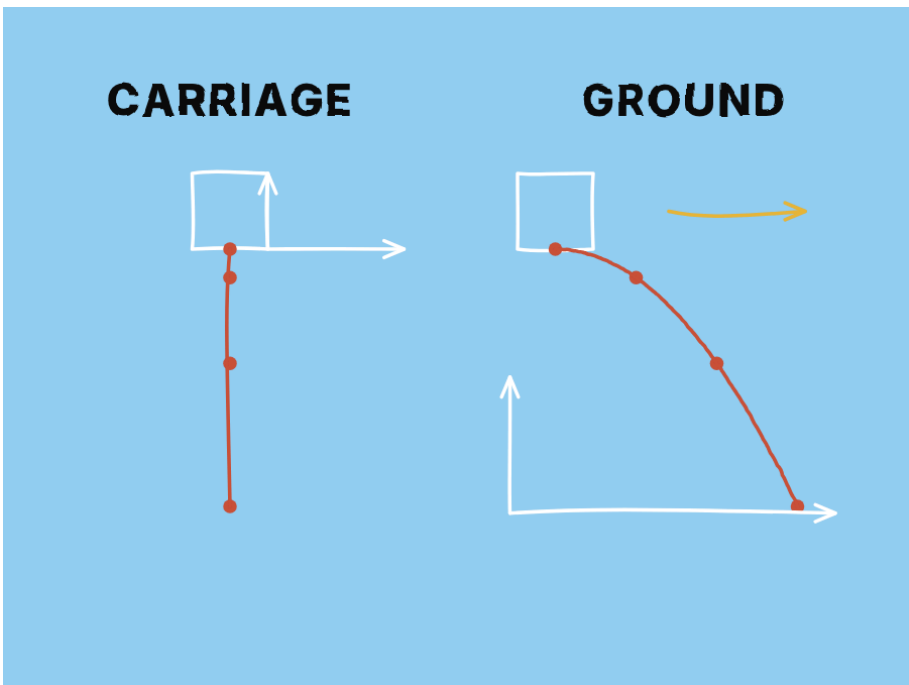
3. State the conclusion the passage reaches about every spatial description of events, and say what that description presupposes about segments.

Hint: Two sentences at the end of the passage carry the whole result.

Answer: Every spatial description of events makes use of a rigid body to which the events are spatially referred. That reference presupposes that the laws of Euclidean geometry hold for segments, where a segment is physically represented by two marks on a rigid body.

Space and Time in Classical Mechanics

If, without serious misgivings and detailed explanations, I formulate the task of mechanics thus: "Mechanics has to describe how bodies change their place in space with time", I take upon my conscience some mortal sins against the holy spirit of clarity; these sins shall first be uncovered.



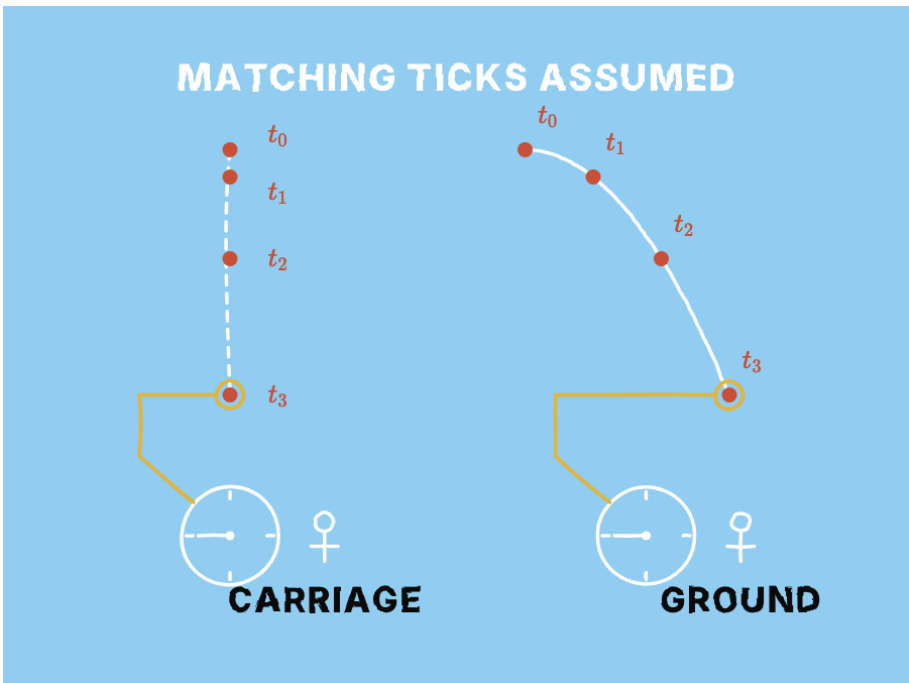
One Fall, Two Paths

It is unclear what is to be understood here by "place" and "space". I stand at the window of a railway carriage travelling uniformly and let a stone fall onto the embankment, without giving it any push. Then I

see, apart from the influence of air resistance, the stone fall straight down. A pedestrian who watches the misdeed from the footpath observes that the stone falls to the Earth in a parabolic arc. I now ask: do the "places" the stone passes through lie "in reality" on a straight line or on a parabola? What, moreover, does motion "in space" mean here? After the considerations of the previous section the answer is self-evident. First of all we set aside entirely the obscure word "space", by which, if we are honest, we can think of nothing whatever; in its place we put "motion with respect to a practically rigid reference body". The places with respect to the reference body, the railway carriage or the ground, have already been defined in detail in the previous section. If, instead of "reference body", we introduce the concept "coordinate system", which is useful for the mathematical description, we can say: the stone describes a straight line with respect to a coordinate system rigidly attached to the carriage, and a parabola with respect to a coordinate system rigidly attached to the ground. One sees clearly from this example that there is no such thing as a path in itself, but only a path with respect to a particular reference body.

A complete description of the motion, however, comes about only when one states how the body changes its place with time; that is, for every point of the path it must be stated at what time the body is there. These statements must be completed by such a definition of time that, by virtue of that definition, these time values can be regarded as quantities observable in principle, as results of measurements. Standing on the ground of classical mechanics, we meet this requirement for our example in the following way. We imagine two clocks of exactly the same construction; the man at the window of the railway carriage holds one, the man on the footpath the

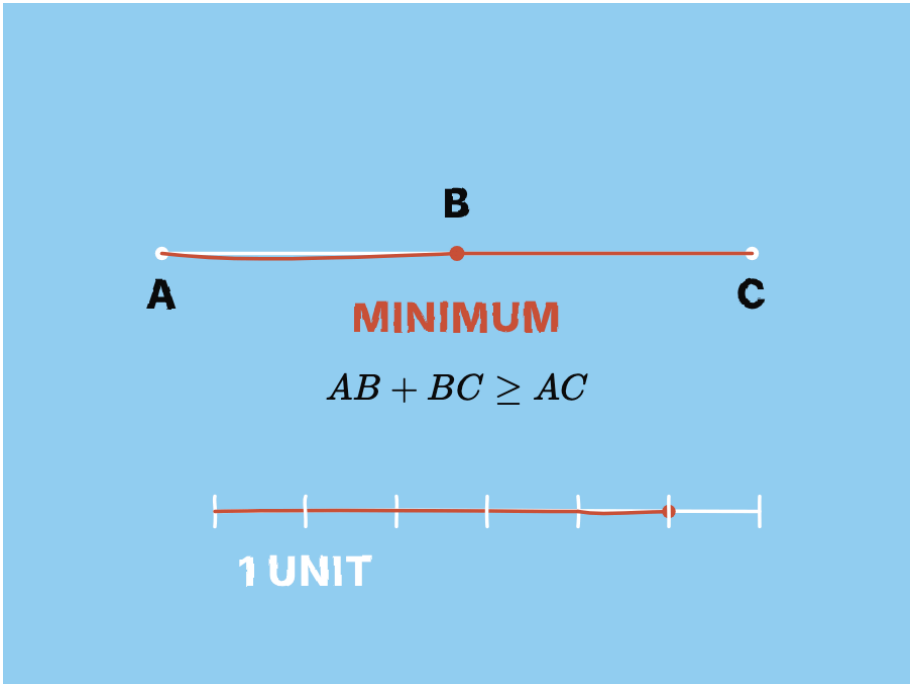
other. Each of them determines at which place of his reference body the stone is at the moment the clock he holds in his hand ticks. In doing so we refrain from going into the inaccuracy introduced by the finite speed of propagation of light. Of this, and of a second difficulty that prevails here, there will be a detailed account later.



Positions Paired With Times

Notes.

With this, a natural object has also been assigned to the straight line. Three points A, B, C of a rigid body lie on a straight line when, the points A and C being given, the point B is so chosen that the sum of the distances AB and BC is as small as possible. This incomplete indication may suffice in this context.



Straightness and Divided Rods

Here it is assumed, to be sure, that the measurement comes out even, that is, yields a whole number. One frees oneself from this difficulty by the use of divided measuring rods, whose introduction demands no new method in principle.

A further investigation of what "spatial coincidence" means here is not necessary, for this concept is clear in so far as, in any individual real case, differences of opinion as to whether it applies or not are hardly likely to arise.

Only the general theory of relativity, treated in the second part of this little book, makes a refinement and alteration of these views necessary.

Questions

1. The stone dropped from the carriage window falls straight down for the man at the window and in a parabolic arc for the pedestrian. Which path do the places of the stone lie on "in reality"?
 - a. On the parabola, because the ground is at rest
 - b. On the straight line, because the stone was given no push
 - c. Neither: there is only a path with respect to a particular reference body
2. Why does the author set aside the word "space" and speak instead of motion with respect to a practically rigid reference body?
 - a. Because, if we are honest, we can think of nothing whatever by "space", whereas places with respect to a reference body were defined in the previous section
 - b. Because space is curved and cannot be measured with rigid rods
 - c. Because the carriage and the ground move through space at different speeds
 - d. Because reference bodies are easier to picture than empty space
3. What is still missing from a description of the motion once the path with respect to a reference body is known?
 - a. The mass of the stone and the force acting on it

- b. For every point of the path, the time at which the body is there, defined as a quantity observable in principle
 - c. The speed of the carriage with respect to the ground
- 4. How, standing on the ground of classical mechanics, do the two observers obtain the times for their descriptions?
 - a. They read a single clock placed on the embankment
 - b. Each holds one of two clocks of the same construction and notes where the stone is on his reference body when his clock ticks
 - c. They calculate the time from the height the stone has fallen
 - d. They measure how long the light from the stone takes to reach them

Answers

- 1. **c)** There is no such thing as a path in itself; the stone describes a straight line with respect to a coordinate system attached to the carriage and a parabola with respect to one attached to the ground.
- 2. **a)** The obscure word space is one by which we can think of nothing whatever; places with respect to a reference body, the carriage or the ground, were already defined in detail.
- 3. **b)** A complete description comes about only when one states how the body changes its place with time, and those time values must, by their definition, count as results of measurements.
- 4. **b)** Two clocks of exactly the same construction, one at the carriage window and one on the footpath; each observer determines at which place of his reference body the stone is at the moment his own clock ticks.

Practice

1. Describe the stone's path with respect to a coordinate system attached to the carriage and with respect to one attached to the ground, and explain why the question which path is the real one has no answer.

Hint: Name the reference body before describing either path.

Answer: With respect to the carriage the stone falls straight down; with respect to the ground it falls in a parabolic arc. There is no path in itself, only a path with respect to a particular reference body, so a question that names no reference body asks nothing.

2. The notes say that three points A, B, C of a rigid body lie on a straight line when B is so chosen that the sum of the distances AB and BC is as small as possible. Explain in your own words how this assigns a natural object to the straight line, and which earlier idea it rests on.

Hint: Think about what you would need in order to compare the sum of two distances.

Answer: Instead of defining straightness by a picture, it defines it by measurement: among the points of the rigid body, those that make the sum of AB and BC least lie on the straight line through A and C. It rests on the measurement of distance by laying off the unit rod, so a straight line becomes something found with rods on a real body.

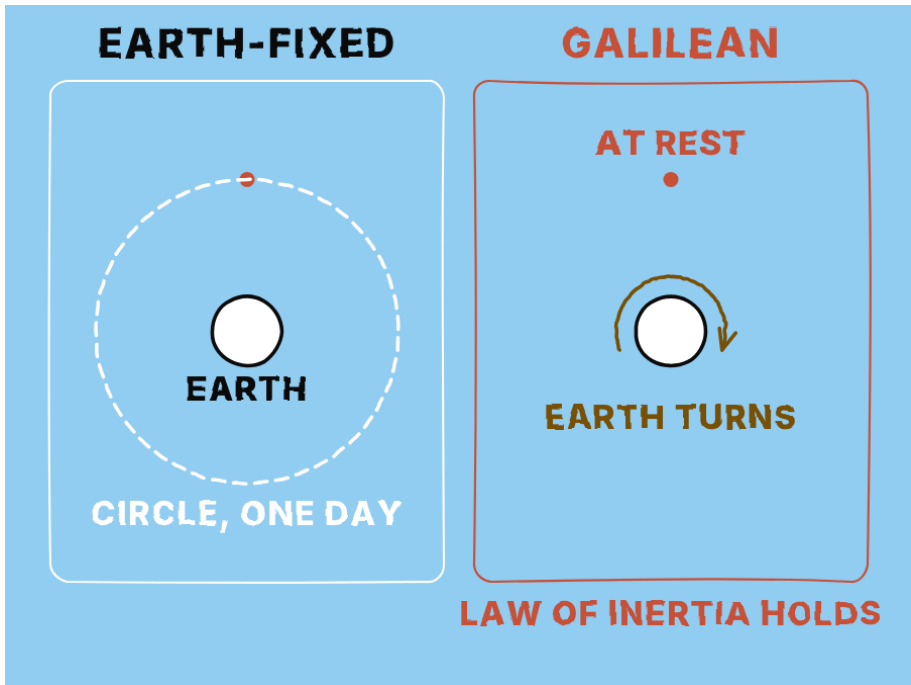
3. What inaccuracy does the author say he refrains from going into when a clock tick is paired with the stone's place, and what does he promise about it?

Hint: Ask how the pedestrian knows where the stone is at the moment of a tick.

Answer: The inaccuracy introduced by the finite speed of propagation of light: the observer sees the stone by light that took time to arrive. He sets it aside here, together with a second difficulty, and promises a detailed account of both later.

The Galilean Coordinate System

As is well known, the fundamental law of the mechanics of Galileo and Newton, known as the law of inertia, runs: a body sufficiently far removed from other bodies persists in a state of rest or of uniform motion in a straight line. This proposition says something not only about the motion of bodies, but also about the reference bodies or coordinate systems admissible in mechanics, which may be used in mechanical description. Bodies to which the law of inertia can certainly be applied with great approximation are the visible fixed stars. If now we use a coordinate system rigidly connected with the Earth, then relative to it every fixed star describes, in the course of one astronomical day, a circle of enormous radius, in contradiction with the wording of the law of inertia. If, then, we hold to this law, we may refer motions only to coordinate systems relative to which the fixed stars perform no circular motions. A coordinate system whose state of motion is such that the law of inertia holds relative to it we call a "Galilean coordinate system". Only for a Galilean coordinate system do the laws of the mechanics of Galileo and Newton claim validity.



Circles Or Rest

Questions

1. With respect to a coordinate system rigidly connected with the Earth, what does a fixed star do in one astronomical day?
 - a. It stays at rest, as the law of inertia requires
 - b. It describes a circle of enormous radius, against the wording of the law of inertia
 - c. It moves uniformly in a straight line
2. What is a Galilean coordinate system?

- a. A coordinate system rigidly connected with the Earth
 - b. A coordinate system whose state of motion is such that the law of inertia holds relative to it
 - c. Any coordinate system built from three walls at right angles
 - d. A coordinate system moving along with the fixed stars
3. For which coordinate systems do the laws of the mechanics of Galileo and Newton claim validity?
- a. For every coordinate system
 - b. For Earth-fixed coordinate systems only
 - c. For Galilean coordinate systems only

Answers

- 1. b)** Relative to a system rigidly connected with the Earth every fixed star describes, in one astronomical day, a circle of enormous radius, in contradiction with the wording of the law of inertia.
- 2. b)** A coordinate system whose state of motion is such that the law of inertia holds relative to it is called a Galilean coordinate system; the fixed stars perform no circular motion relative to it.
- 3. c)** Only for a Galilean coordinate system do the laws of the mechanics of Galileo and Newton claim validity.

Practice

- 1.** Explain in your own words why a coordinate system rigidly attached to the Earth is not a Galilean coordinate system, using the fixed stars.

Hint: Ask what path a fixed star describes with respect to the ground in one day, and what the law of inertia says it should do.

Answer: A fixed star is far from all other bodies, so by the law of inertia it should be at rest or move uniformly in a straight line. With respect to a system attached to the Earth it describes a circle of enormous radius once a day instead. The law does not hold with respect to that system, so the system is not Galilean.

2. Einstein keeps the law of inertia and gives up some reference bodies. State the rule this leads to and name the kind of coordinate system that results.

Hint: What must the fixed stars not do with respect to an allowed system?

Answer: Motions may be referred only to coordinate systems with respect to which the fixed stars do not circle. A coordinate system in which the law of inertia holds is a Galilean coordinate system, and the mechanics of Galileo and Newton claims validity only for such systems.

3. A friend says the law of inertia must be false, since the stars plainly circle every night. Answer them with the ideas of this part.

Hint: Separate the statement about bodies from the statement about reference bodies.

Answer: The circles are the stars' paths with respect to the turning Earth, and a path exists only with respect to a reference body. The law claims validity only for Galilean coordinate systems, in which the fixed stars do not circle; the Earth-fixed system is not one, so the night sky contradicts the choice of reference body, not the law.