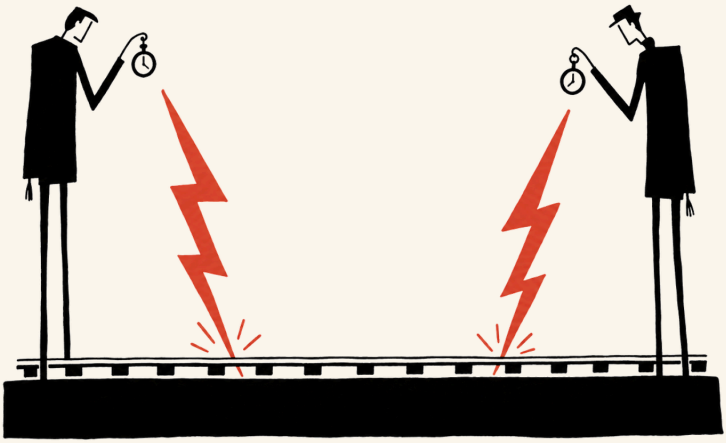
Relativity: The Special and General Theory



Albert Einstein · 1917

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Relativity: The Special and General Theory

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From the original: Über die spezielle und die allgemeine Relativitätstheorie
(Gemeinverständlich)

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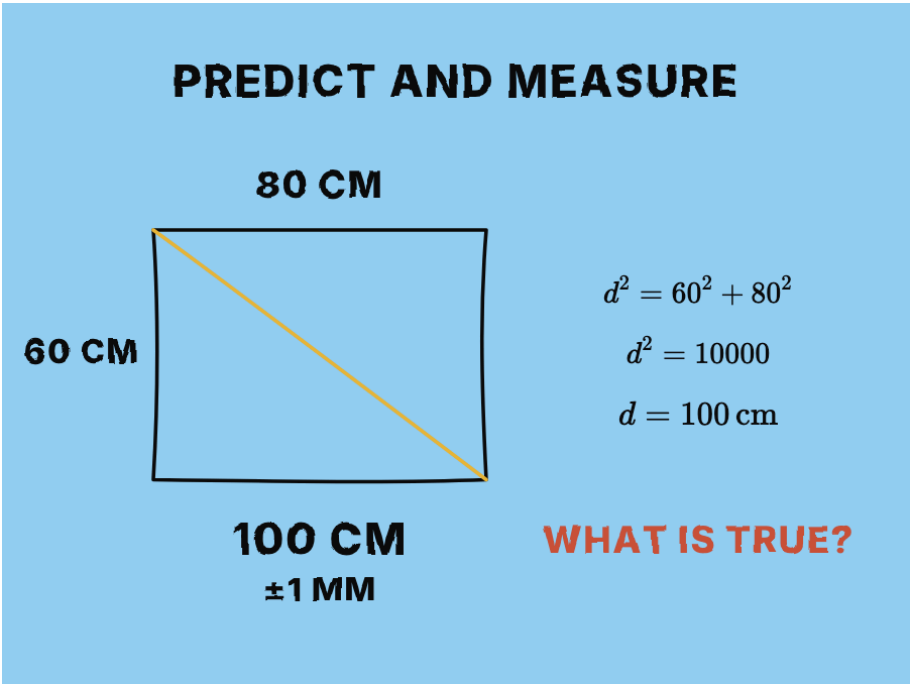
When Is a Theorem True?

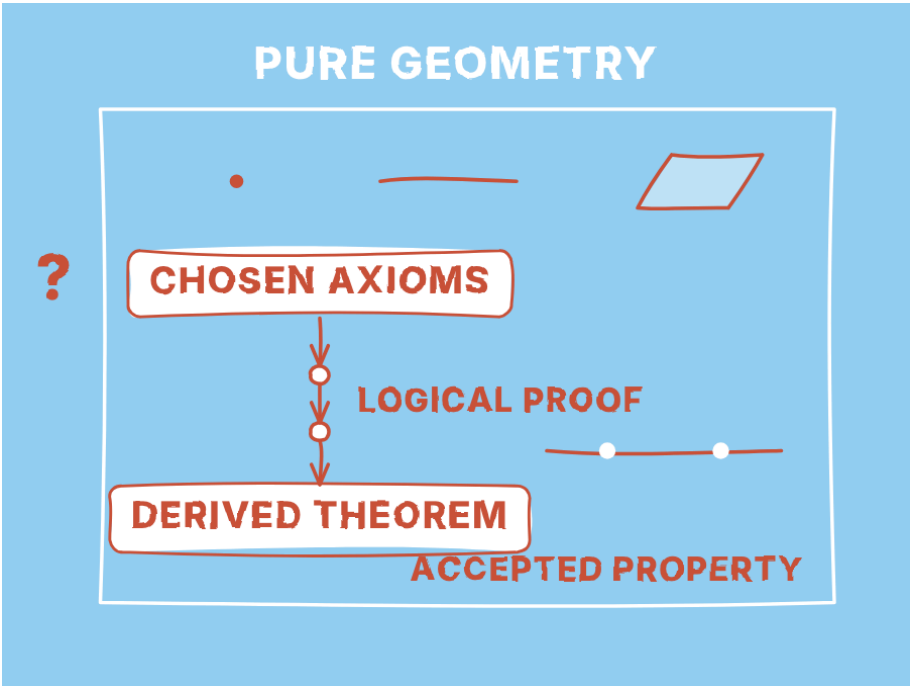
Take a tape measure to a rectangular table, the kitchen table will do. Measure the short side and the long side, and write the two numbers down. Now stretch the tape from one corner to the opposite corner, along the diagonal, but before you read the number, predict it with the one theorem everybody remembers from school. Suppose the sides came out at 60 centimetres and 80 centimetres. Sixty squared is 3600, eighty squared is 6400, the sum is 10 000, and the square root of 10 000 is 100. You look at the tape, and there it is, 100 centimetres, give or take a millimetre. What exactly have you just confirmed?



Prediction Meets Measurement

Most people would say they have checked that Pythagoras was right, that the theorem is true. Albert Einstein opens his little book by stopping us precisely here and asking, with some mischief, what we even mean when we call such a statement true. He reminds us that most of us met the proud edifice of Euclid's geometry as children, chased up its high staircases by conscientious teachers, and that we remember it with more respect than affection. Out of loyalty to that schooling we would scorn anyone who doubted the humblest of its theorems. And yet, he says, our proud certainty may evaporate the moment someone asks what "true" is supposed to mean.

Here is how geometry is built. It starts from a few basic notions, such as plane, point and straight line, which we picture more or less clearly, and from a few simple statements about them, the axioms, which we are willing to accept because the pictures make them feel obvious. Every other statement is then derived from the axioms by logical steps whose validity we feel obliged to grant; that derivation is what we call a proof. So within geometry a theorem is correct, or "true", when it has been properly derived from the axioms, and nothing more. The whole question of truth is thereby pushed back onto the axioms themselves. And there it dies, because, as Einstein says, the question whether an axiom is true cannot be answered by geometry, and worse, it has no meaning at all.



Inside Pure Geometry

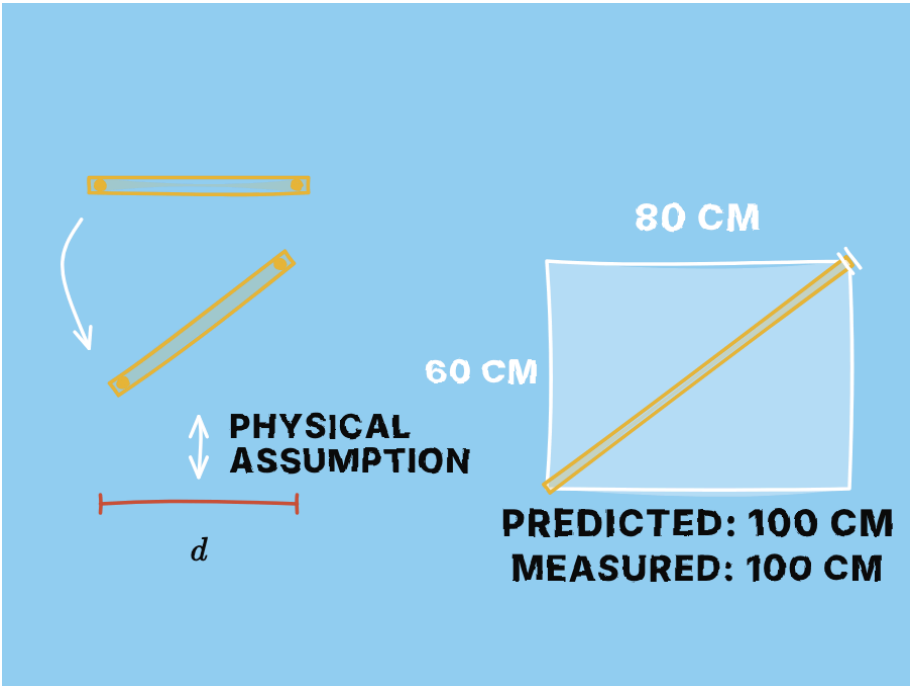
Try it on the simplest axiom. Is it true that through two points only one straight line passes? You cannot ask that. All you can say is that Euclidean geometry deals with certain objects it chooses to call straight lines, and that it grants them the property of being fixed by two of their points. The word "true" does not fit the statements of pure geometry, because when we say "true" we always mean, in the end, agreement with some real thing, and pure geometry is not about real things. It is only about the logical connections among its own concepts.

Why, then, does it feel so natural to call the theorems true? Because the concepts of geometry correspond, more or less exactly, to objects in nature, and those objects are beyond doubt the reason the concepts

arose in the first place. Geometry may keep its distance from nature in order to make its structure logically watertight, but the habit of seeing a segment as two marks on a practically rigid body sits deep in our thinking. We also habitually take three places to lie on a straight line when, looking with one eye from a suitably chosen spot, we can make their apparent positions coincide. That is a physical test, done with an eye and a body, not a logical one.

> **Try it yourself**
>
> Stand three pencils upright in lumps of modelling clay on a table, roughly in a row, and look at them with one eye closed. Move your head slowly from side to side. If they are properly in line there is one viewpoint from which the nearest pencil hides the other two, and the three shapes merge into one narrow silhouette. You have just used your eye as a straightness test; keep this picture, because the whole section rests on it.

Now Einstein makes the decisive move, and it is a single added sentence. To the theorems of Euclid we add one more statement: two points of a practically rigid body always correspond to the same distance, the same segment, whatever changes of position we put the body through. Nothing in pure geometry says that; it is a claim about steel rulers and wooden rods and the behaviour of real matter. Once it is added, the theorems of Euclidean geometry turn into statements about the possible relative arrangement of practically rigid bodies. The geometry so completed is to be treated as a branch of physics.

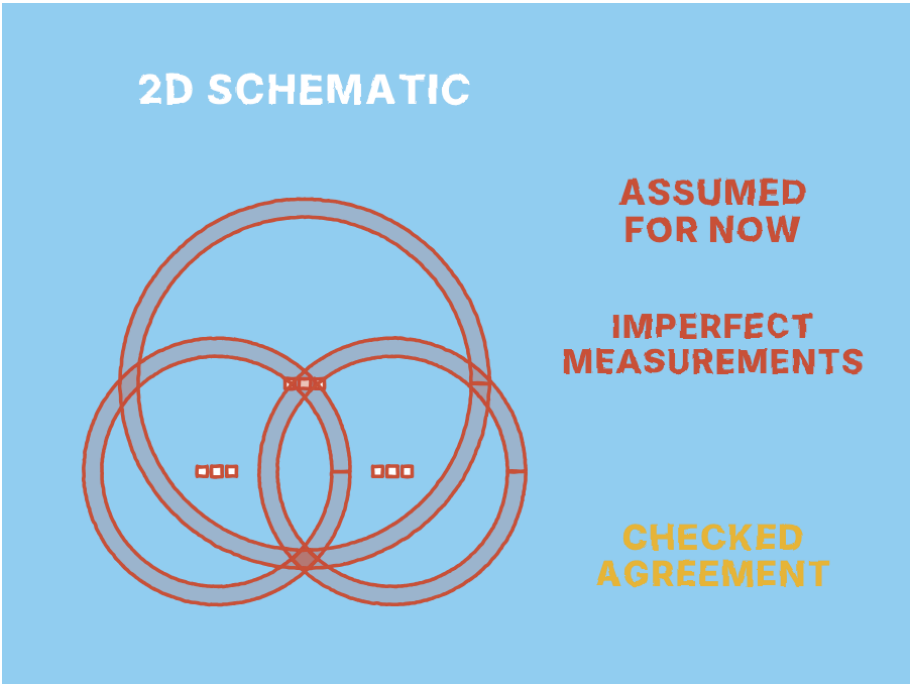


The Added Physical Assumption

And now the question of truth is allowed back in, because now it has something to bite on. We can ask whether those theorems hold for the real things we have assigned to the geometrical concepts, the marks, the edges, the rods. A little loosely, Einstein says, we may take the "truth" of a geometrical theorem in this sense to mean that it holds in a construction with compass and straightedge. That is exactly what you did at the kitchen table. The theorem about the diagonal was true in the physical sense because the tape, a practically rigid body, agreed with the prediction.

The same move is made, on a grand scale, every time your phone finds its position. A satellite navigation receiver measures its distance to several satellites and then applies plain geometry, the same

geometry of segments and points, to turn those distances into a place on the map. Nobody could prove in advance that this would work; it works because the applied geometry agrees with experience, with where the phone actually turns out to be. Geometry earns its physical truth the way any physical statement does, by being checked against the world.



Distances Locate a Receiver

Einstein is careful to add that the conviction that geometrical theorems are true in this sense rests on nothing more than rather imperfect experience. A tape measure is never perfectly rigid, an eye is never perfectly straight. For the moment, he says, we will assume that truth, because without it we cannot describe anything, and in the last part of the book, when we come to the general theory of relativity, we

shall see that it has limits, and where they lie. That promise is the thread to hold on to: the geometry you trusted at the table is being accepted on probation.

> **Remember**
>
> A theorem of pure geometry is "true" only in the sense that it follows from the axioms, and whether the axioms are true is a question without meaning inside geometry. The theorems feel true because their concepts grew out of real objects, and we are used to seeing a segment as two marks on a rigid body. Add one physical statement, that two points of a practically rigid body keep the same distance however the body is moved, and geometry becomes a branch of physics about the arrangement of such bodies. Then a theorem is true in the physical sense when a construction with compass and straightedge, or a tape measure on a table, or a satellite receiver, agrees with it. That agreement rests on imperfect experience and is assumed for now; its limits appear in the general theory of relativity.

Questions

1. Inside pure geometry, what does it mean to say that a theorem is true?
   a. It agrees with measurements made on real objects
   b. It has been properly derived from the axioms, and nothing more

c. Most people who think about it carefully find it obvious

2. Why does Einstein say the question whether an axiom is true has no meaning inside geometry?
   a. Because the axioms are too simple to be doubted
   b. Because an axiom can only be proved from other axioms
   c. Because pure geometry deals only with the logical connections among its own concepts, not with real things
   d. Because nobody has yet found a way to test them

3. What single addition turns Euclid's geometry into a branch of physics?
   a. The statement that two points of a practically rigid body keep the same distance however the body is moved
   b. The statement that the axioms are true because experience confirms them
   c. The statement that a straight line is the shortest path between two points
   d. The statement that light always travels in straight lines

4. After that addition, when is a geometrical theorem true in the physical sense?
   a. When it can be derived from the axioms in the accepted way
   b. When it holds in a construction with real bodies, such as compass and straightedge
   c. When it has been accepted by conscientious teachers for a long time

Answers

1. b) Within geometry a theorem is correct, or true, only when it has been derived from the axioms by accepted logical steps; agreement with real things is not what pure geometry is about.

2. c) The word true always means, in the end, agreement with some real thing, and pure geometry is not about real things; it is only about how its concepts are connected logically.

3. a) Once we add that two points of a practically rigid body always correspond to the same segment whatever we do with the body, the theorems become statements about the possible arrangement of such bodies, and that is physics.

4. b) Einstein says, a little loosely, that the truth of a geometrical theorem in this sense means that it holds in a construction with compass and straightedge, which is a check against real things.

Practice

1. A friend says: Pythagoras's theorem is true, I checked it with a tape measure on my table. In what sense of the word true is your friend right, and in what sense have they not checked anything at all?

> **Hint:** Separate the theorem as a statement derived from axioms from the theorem as a statement about a tape measure and a tabletop.
>
> **Answer:** Your friend is right in the physical sense: with the added statement that a tape measure is a practically rigid body whose marks keep their distance, the theorem becomes a claim about real segments, and the table agreed with it. In the sense of pure geometry they have checked nothing, because there the theorem is true only by being derived from the axioms, and no measurement can bear on that.

2. Why can the question whether only one straight line passes through two points not even be asked, according to Einstein, while the

question whether a measured diagonal matches the calculated one can be?

Hint: Ask what each question is about: concepts, or real things.

Answer: The first is a statement of pure geometry about objects it chooses to call straight lines, defined by having that property; asking whether it is true would mean asking for agreement with some real thing, and pure geometry is not about real things. The second is about marks on a rigid tape and a tabletop, real things assigned to the concepts, so agreement or disagreement with them is a meaningful question.

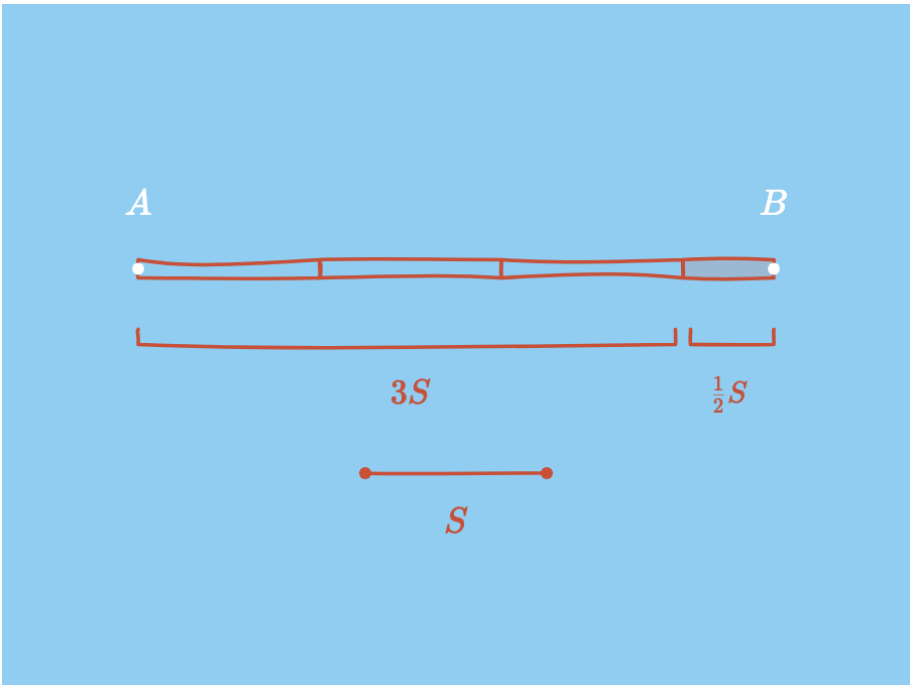
3. Einstein says our conviction that geometry holds for real bodies rests on rather imperfect experience, and that we accept it for now. Give two reasons the experience is imperfect, using the examples of this part, and say what Einstein promises about it.

Hint: Think about what a tape measure and a sighting eye really are.

Answer: A tape measure is never perfectly rigid, so two marks on it do not keep exactly the same distance; and an eye lining up three pencils is a rough test of straightness, not an exact one. So agreement between geometry and real bodies is only approximate. Einstein promises to assume that agreement for now and to show, in the last part of the book on the general theory of relativity, that it has limits and where they lie.

Giving a Place Three Numbers

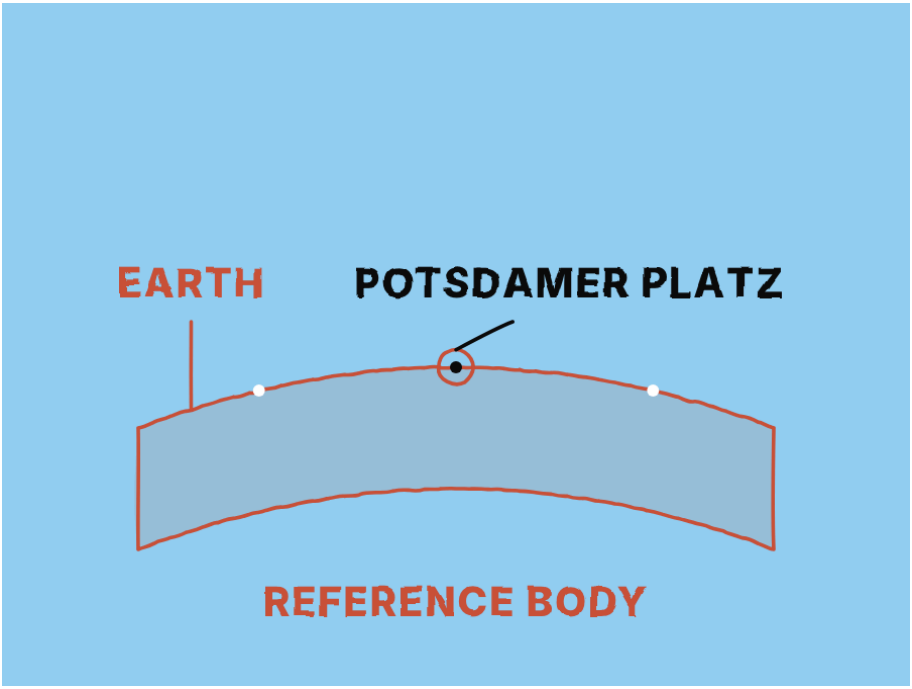
Take a wooden ruler, a short one, and measure the width of a doorway with it. You lay the ruler against the frame, mark where it ends with a fingertip, lay it down again from the mark, and count: one, two, three ruler-lengths and a bit. Nobody taught you to do this; it is what measuring is. Einstein asks us to look at this everyday action closely, because every statement about where something is, in physics and in daily life, rests on it.



Length Is a Repeated Unit

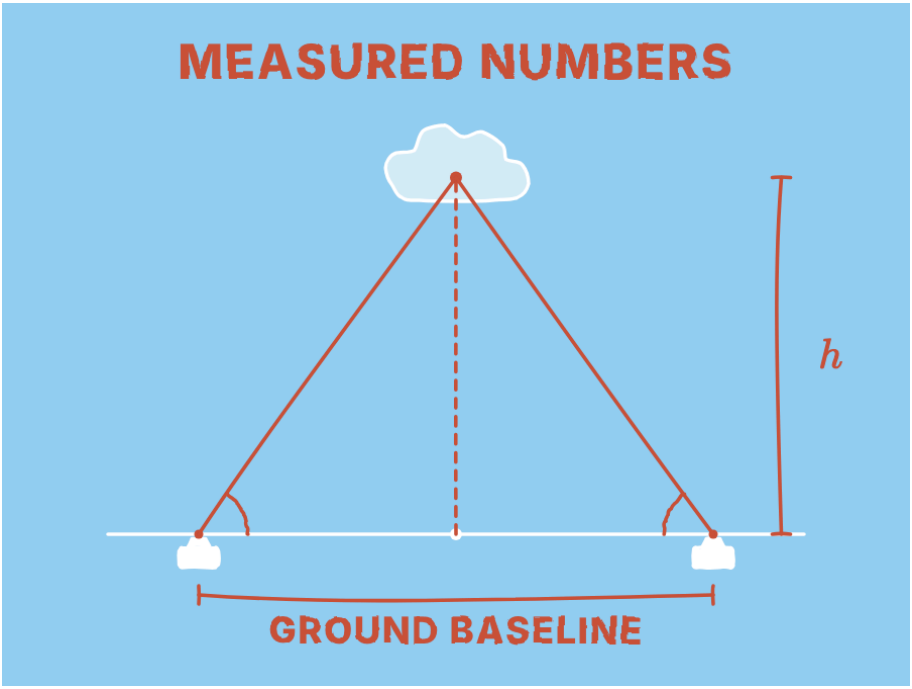
Last time we agreed that two marks on a practically rigid body are what a segment really is. Now take one such segment and promise to use it once and for all: a little rod, call it S, the unit of measure. If A and B are two points of a rigid body, the straight line joining them can be constructed by the rules of geometry, and along that line you lay the rod S down from A as many times as it takes to reach B. The number of times you had to lay it down is the measure of the segment AB. All measuring of lengths, every tape and every laser rangefinder, comes down to this counting of a repeated unit.

Now to the question of where a thing is. Every spatial description of the place of an event or an object, Einstein says, consists in naming the point of a rigid body with which the event coincides. That rigid body is the reference body. This is true not only in science; it is how everyday speech works. Say "in Berlin, on Potsdamer Platz". The surface of the Earth is the rigid body to which the statement refers; on it, "Potsdamer Platz in Berlin" is a marked point with a name, and the event is at that point.



A Place on a Reference Body

But this way of giving a place is primitive in two ways. It knows only places on the surface of rigid bodies, and it depends on there being distinguishable, nameable points on that surface. Einstein shows how the human mind frees itself from both restrictions without changing what a place statement is. Suppose a cloud is floating above Potsdamer Platz. Its place, relative to the Earth's surface, can be fixed by erecting a pole vertically on the square, tall enough to reach the cloud. The length of the pole, measured with the unit rod, together with the place of the foot of the pole, is a complete statement of where the cloud is.

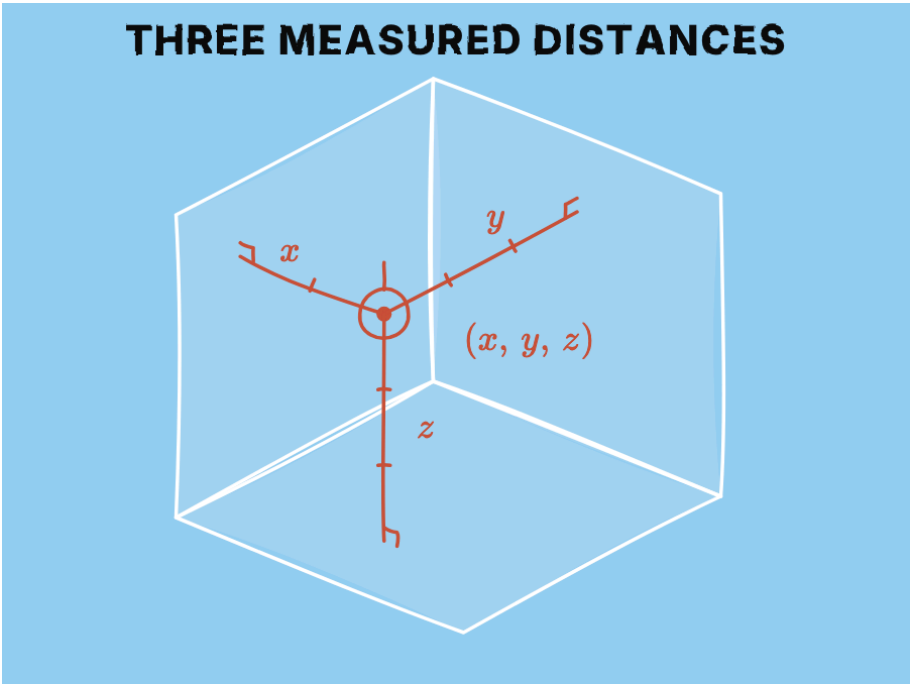


Measuring an Unbuilt Pole

Look at what has happened in that example, because three refinements of the idea of place are hidden in it. First, the rigid body of reference has been extended, with the pole, until it reaches the thing to be located. Second, a number, the measured length of the pole, has replaced a named landmark in characterising the place. Third, we speak of the height of the cloud even when no pole reaching it has been built at all. In that case one takes photographs of the cloud from several points on the ground and, knowing how light travels, works out how long the pole would have to be. The pole is imagined, but its length is measured all the same.

From this Einstein draws the lesson: for describing places it pays to free ourselves, by using measured numbers, from the need for named

landmarks on the reference body. Measuring physics does this with the Cartesian coordinate system. Picture three rigid flat walls, each perpendicular to the other two, joined into one rigid body, like the two walls and the floor meeting in the corner of a room. The place of any event with respect to this system is described, essentially, by the lengths of the three perpendiculars that can be dropped from the event onto the three walls. Those three lengths are the coordinates, written (x, y, z). Each of them can be found by a sequence of manipulations with rigid rods, carried out by the laws and methods of Euclidean geometry.



Three Distances Fix a Place

Try it in your own room. Choose the corner where two walls meet the floor. A lamp hanging in the room is so far from one wall, so far from

the other wall, and so high above the floor: three distances, three rods laid down and counted, and the lamp's place is fixed without any landmark at all. Move the lamp, and only the three numbers change; the walls stay what they were.

In practice, Einstein admits, the three rigid walls are hardly ever built, and the coordinates are hardly ever found by constructions with rigid rods; they are found indirectly. A satellite receiver in your pocket never lays a rod against anything; it measures how long the signals from several satellites took to arrive and computes its three coordinates from those times, much as the cloud's height was computed from photographs. Yet the physical meaning of every such statement of place must always be sought in the way we have just described, in rods laid against a rigid body. Otherwise, he warns, the results of physics and astronomy dissolve into vagueness.

So here is where the section arrives. Every spatial description of events makes use of a rigid body to which the events are referred. That reference presupposes that the laws of Euclidean geometry hold for segments, where a segment is represented physically by two marks on a rigid body. The assumption we accepted on probation last time is now carrying the whole weight of the idea of place.

Remember

Length is measured by laying a chosen unit rod along the straight line between two points and counting the repetitions; every measurement of length comes down to this. To say where something is means to name the point of a rigid reference body that it coincides with, as "Potsdamer Platz in Berlin" names a point on the Earth's surface. The idea of place is refined by extending the reference body, by replacing named landmarks with measured numbers, and by speaking of a length even when the rod is only imagined. The Cartesian coordinate system makes this systematic: three mutually perpendicular rigid walls, and the three perpendicular distances from an event to them are its coordinates. Whether found with rods or computed from signals, a place statement means this, and it assumes that Euclidean geometry holds for segments on rigid bodies.

Questions

1. According to the lecture, what does the measure of a segment AB actually consist of?
   a. The number of times the unit rod S must be laid along the line from A to B
   b. The distance a signal needs to travel from A to B
   c. The number of named marks that lie between A and B on the rigid body

2. What does a statement like "in Berlin, on Potsdamer Platz" mean, once it is analysed the way the lecture does?
   a. That the event lies at a certain distance from the centre of the Earth
   b. That the event has three coordinates measured from the walls of a room
   c. That the event coincides with a marked, named point of a rigid body, the surface of the Earth
   d. That the event can be reached by a pole erected on the square

3. A cloud floats above the square. Which refinement of the idea of place is at work when its height is computed from photographs taken at several points on the ground?
   a. The reference body is extended by a real pole until it reaches the cloud
   b. A number stands in for a named landmark on the Earth's surface
   c. A length is spoken of even though the rod that would measure it was never built
   d. The cloud is treated as a point of the rigid reference body

4. Why does Einstein insist that the physical meaning of a place statement must always be sought in rods laid against a rigid body, even when coordinates are found indirectly?
   a. Because indirect methods such as signals are never accurate enough
   b. Because otherwise the results of physics and astronomy lose their definite meaning

c. Because the three rigid walls of the coordinate system are always built in practice

Answers

1. a) Along the straight line joining A and B you lay the rod S down from A as many times as it takes to reach B; the number of repetitions is the measure of the segment, and all measuring of lengths comes down to this.

2. c) Every spatial description names the point of a rigid reference body with which the event coincides; here the Earth's surface is the reference body and Potsdamer Platz is a named point on it.

3. c) We speak of the height of the cloud even when no pole reaching it exists; from the photographs and the way light travels one works out how long the pole would have to be, so the pole is imagined but its length is measured all the same.

4. b) The walls are hardly ever built and the coordinates are found indirectly, yet their meaning must be the one given by rods and a rigid body, or the results of physics and astronomy dissolve into vagueness.

Practice

1. A lamp hangs in a room. Using the corner where two walls meet the floor as the coordinate system, explain in your own words what its three coordinates are and how, in principle, each one is measured.

Hint: Think of the three flat walls as the rigid body and ask what a perpendicular dropped from the lamp onto each wall is.

Answer: The two walls and the floor are three mutually perpendicular rigid planes joined into one rigid body. From the lamp drop a perpendicular onto each of them; the lengths of these three perpendiculars are the coordinates x, y, z. Each length is found, in

principle, by laying the unit rod along the perpendicular and counting the repetitions, a construction governed by Euclidean geometry.

2. Einstein says the primitive way of giving a place is bound by two restrictions. Name them, and say for each how the cloud example frees us from it without changing what a place statement is.

Hint: Start from what "Potsdamer Platz in Berlin" can and cannot locate.

Answer: The primitive way knows only places on the surface of a rigid body, and it needs distinguishable, named points on that surface. The pole extends the rigid body upward, so a place off the surface can be given; and the measured length of the pole, a number, replaces a named landmark. In both cases the statement still names a point of a rigid body that the cloud coincides with, so its nature is unchanged.

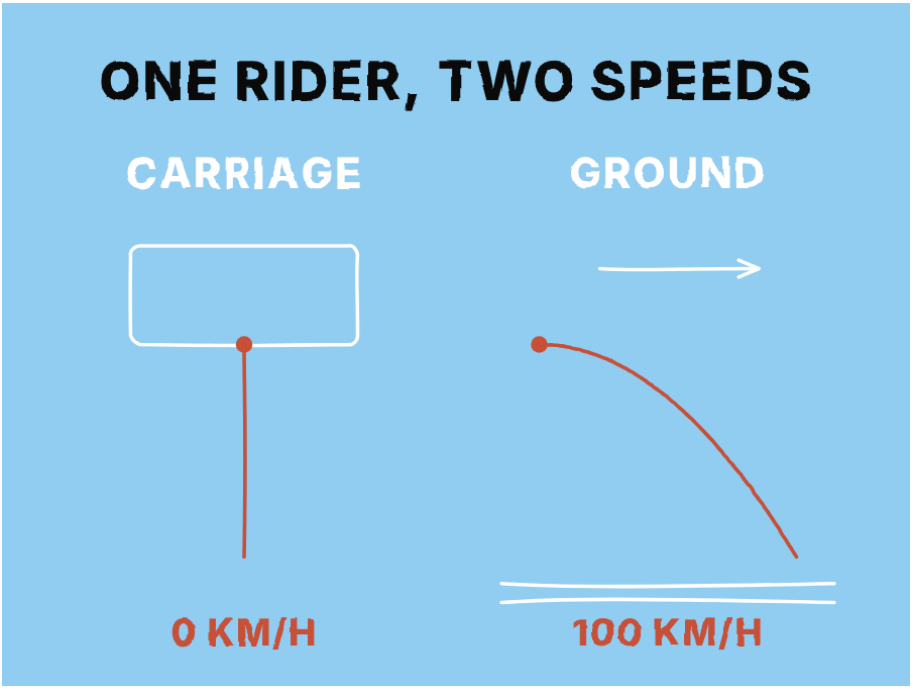
3. A satellite receiver computes its position from signal travel times and never touches a rod. In what sense is its answer still a place statement of the kind the lecture describes, and what assumption about geometry does it rest on?

Hint: Compare it with the cloud whose height is computed from photographs.

Answer: The receiver finds its coordinates indirectly, as the cloud's height was found from photographs, but their physical meaning is still the lengths of three perpendiculars to a rigid reference body, lengths a rod could in principle measure. Like every spatial description, it presupposes that Euclidean geometry holds for segments, a segment being two marks on a rigid body.

The Stone That Fell Two Ways

Sit by the window of a train that is running smoothly along a straight track, at a steady speed, with no jolts. Hold a stone out of the window and simply let it go, without giving it any push. Watch it. What you see, if we ignore the air, is a stone dropping straight down, as if the train were standing still; it leaves your hand and falls vertically to the embankment below. Now put yourself in the shoes of a walker on the path beside the track, who sees the same stone leave the same hand. The walker sees it fall in a curve, a parabola, sweeping forward with the train while it drops, and landing well ahead of the spot where it was released. Two honest witnesses, one stone, and two different paths.



One Stone, Two Paths

Einstein opens this section with a confession. If he says, without further thought, that mechanics has to describe how bodies change their place in space with time, he commits, in his own words, several mortal sins against the holy spirit of clarity. The sins hide in two small words, "place" and "space", and the falling stone is his way of exposing them. Ask the question that seems to demand an answer: do the places the stone passes through "really" lie on a straight line or on a parabola? And what does it even mean to say that the stone moves "in space"?

The answer, Einstein says, is obvious after what we did last time, and here is why. The word "space", if we are honest, is a dark word to which we can attach no definite idea at all, so we set it aside entirely.

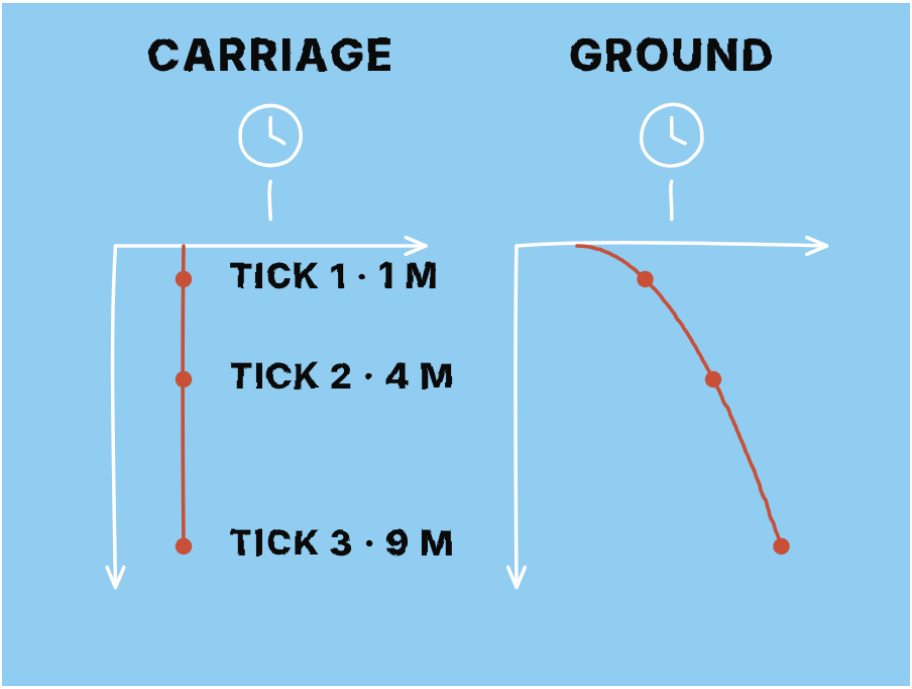
In its place we put "motion with respect to a practically rigid reference body". And places with respect to a reference body are exactly what the previous section defined, by coincidence with a point of the body, or by three measured perpendiculars. For the stone there are two reference bodies on offer: the railway carriage and the ground. With respect to the carriage, the stone passes through a sequence of places that lie one below another, on a straight line. With respect to the ground, it passes through places that lie on a parabola.

Since the coordinate system, the three rigid walls, is the mathematically useful form of a reference body, Einstein puts it this way: the stone describes a straight line with respect to a coordinate system rigidly attached to the carriage, and a parabola with respect to a coordinate system rigidly attached to the ground. Neither description is the real one and the other an illusion. Say it twice, because the whole of relativity grows from this seed: there is no such thing as a path in itself. There is only a path with respect to a particular reference body. Drop the reference body from the sentence and the sentence says nothing.

> **Try it yourself**
>
> Next time you are a passenger in a car on a straight, smooth road, let a coin fall from your hand into your lap. It drops straight down, and it lands in your lap, not behind you. Then picture yourself standing on the pavement watching the coin through the window: it travels forward with the car the whole time it falls, tracing a shallow arc. Both of you are right; you have simply chosen different rigid bodies, the car or the road, to refer the coin's places to.

A satellite receiver lives by this rule. It reports your motion with respect to the Earth's surface, the reference body you care about, and a passenger reading it on a train sees a speed of a hundred kilometres an hour while sitting perfectly still with respect to the carriage. Neither number is wrong; each belongs to its own reference body, and the receiver has quietly chosen one.

But a path, even with its reference body named, is not yet a description of motion. A complete description must say how the body changes its place with time; for every point of the path we must state at what time the body is there. And those times, Einstein insists, must be defined in such a way that they count as quantities observable in principle, results of measurements, not numbers assumed out of the air. Standing on the ground of classical mechanics, he meets the requirement like this. Imagine two clocks, made exactly alike; the person at the carriage window holds one, the walker on the path holds the other. Each of them notes at which point of their own reference body the stone is at the moment their own clock ticks. Click, the stone is here; click, it is there; and a list of places paired with ticks is a complete description of the motion, with respect to that reference body and that clock.

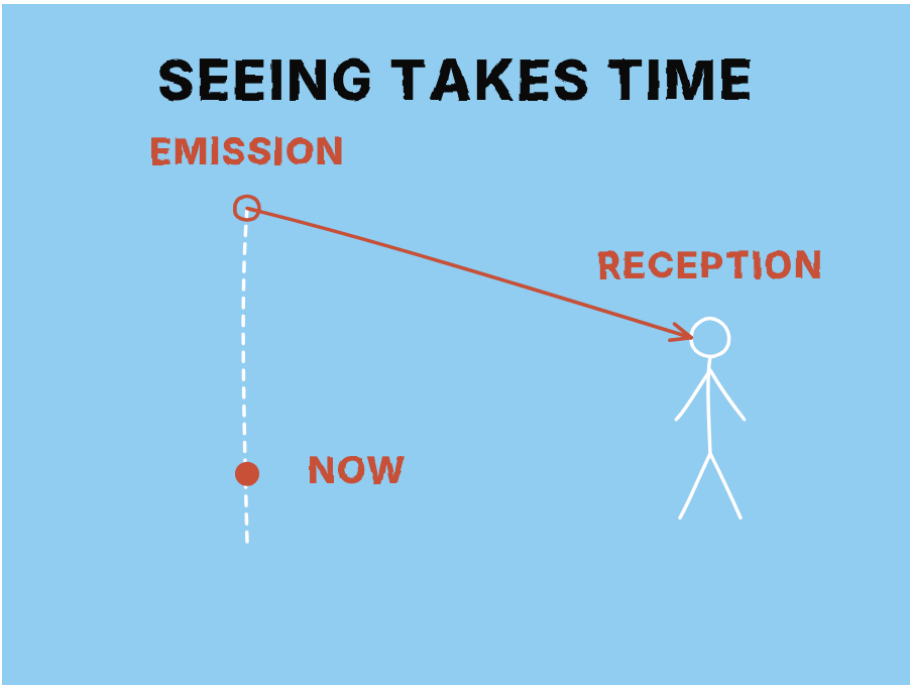


Places Paired With Ticks

Picture the two lists side by side. The passenger's list reads: tick one, one metre below the hand; tick two, four metres below; tick three, nine metres below, always on the same vertical line. The walker's list has the same heights, but each place also sits a little farther along the track than the one before, so joined up they make the parabola. Same stone, same ticks, two reference bodies, two lists.

Einstein closes the section with two quiet warnings, and he leaves them for later. In pairing a tick with a place we have ignored the inaccuracy that creeps in because light travels at a finite speed: the walker sees the stone where it was a moment ago, not where it is. A second difficulty is lurking here as well, and he promises to deal with both at length. They are not small; the satellite clocks that feed your

receiver, moving at thousands of metres a second high above the ground, would drift by kilometres a day if those difficulties were ignored, and the rest of the book is the story of taking them seriously. So these first three sections pause where Einstein's own argument pauses: geometry made physical by rigid bodies, places given by rods against a reference body, and motion that exists only with respect to a reference body and a clock. Everything that follows in the book rests on these three floors.



Seeing Takes Time

Remember

"Motion in space" means nothing until we replace the dark word "space" by a practically rigid reference body, or its mathematical form, a coordinate system. The stone dropped from the train falls on a straight line with respect to the carriage and on a parabola with respect to the ground, and neither path is more real than the other. There is no path in itself, only a path with respect to a particular reference body. A complete description of motion also needs the time at each point of the path, and times must be observable quantities: here, the ticks of a clock held by the observer, paired with the stone's places on their own reference body. Two difficulties were set aside, the finite speed of light and a second one not yet named, and the rest of the book takes them up.

Questions

1. The stone dropped from the train window falls straight down for the passenger and along a parabola for the walker. Which path is the real one?
   a. The parabola, because the ground is at rest and the train is moving
   b. The straight line, because the stone was released without any push
   c. Neither: a path exists only with respect to a chosen reference body

2. Why does Einstein set aside the word "space" and speak instead of motion with respect to a practically rigid reference body?
   a. Because we can attach no definite idea to "space", whereas places relative to a rigid body were already defined
   b. Because space is curved and cannot be measured with rigid rods
   c. Because rigid bodies are easier to draw than empty space
   d. Because the train and the ground move through space at different speeds

3. A path with its reference body named is still not a complete description of the stone's motion. What is missing?
   a. The mass of the stone and the force of gravity
   b. The time at which the stone is at each point of the path, given as a measurable quantity
   c. The speed of the train with respect to the ground

4. In classical mechanics, how do the passenger and the walker each obtain the times for their description of the falling stone?
   a. They agree on a single master clock placed on the embankment
   b. They compute the time from the height the stone has fallen
   c. Each holds one of two identical clocks and notes where the stone is on their own reference body at each tick
   d. Each measures how long the light from the stone takes to reach them

Answers

1. c) There is no path in itself; the stone describes a straight line with respect to a coordinate system attached to the carriage and a parabola with respect to one attached to the ground, and neither description is an illusion.

2. a) The word space is a dark word to which, if we are honest, we can attach no definite idea; places with respect to a reference body, by contrast, are exactly what the previous section defined.

3. b) A complete description must say how the body changes its place with time, so for every point of the path we must state when the body is there, and those times must be observable quantities, results of measurements.

4. c) Two clocks made exactly alike, one at the carriage window and one on the path; each observer notes at which point of their own reference body the stone is at the moment their own clock ticks.

Practice

1. A passenger on a smoothly moving train drops a stone from the window. Describe, in your own words, the stone's path with respect to the carriage and with respect to the ground, and explain why the question "which path is real?" has no answer.

> **Hint:** Begin by naming the reference body before you describe either path.
>
> **Answer:** With respect to a coordinate system rigidly attached to the carriage the stone falls on a straight vertical line; with respect to one attached to the ground it moves forward with the train while it drops and traces a parabola. There is no path in itself, only a path with respect to a particular reference body, so asking which path is real is asking a question with no reference body in it, and such a question says nothing.

2. Suppose the passenger and the walker each write down a list of the stone's places at the ticks of their own clocks. Say what the two lists

have in common and how they differ, and explain why such a list, and not the path alone, counts as a complete description of the motion.

Hint: Think about what each tick adds to a point on the path.

Answer: Both lists pair the same ticks with the stone's place; the passenger's places lie one below another on a vertical line, the walker's lie on a parabola, each a little farther along the track. The path alone says where the stone passed but not when; the list gives the time at every point, and because the times come from a clock the observer holds, they are observable quantities, which is what a complete description requires.

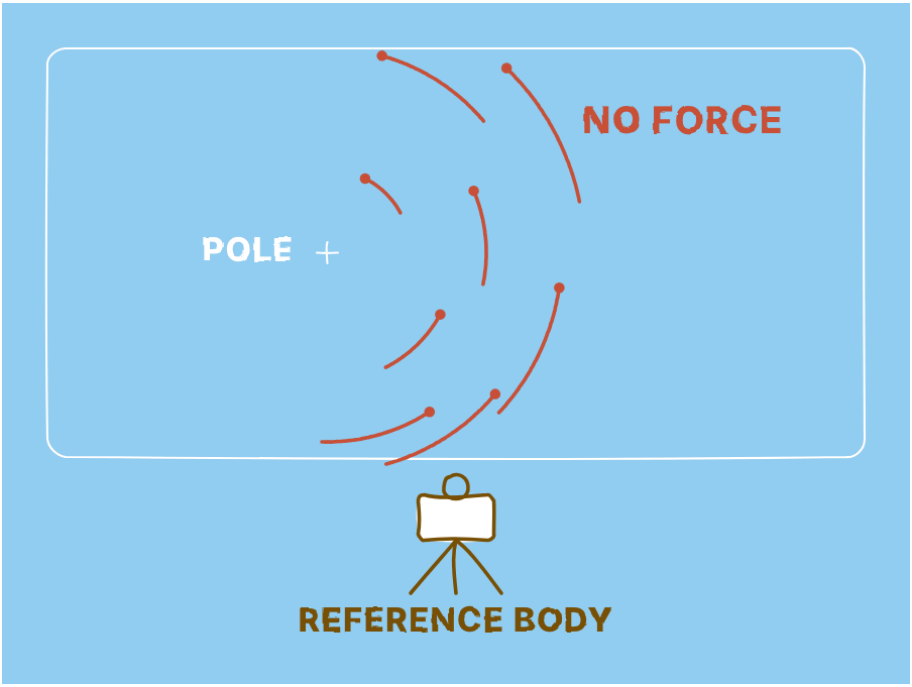
3. Einstein admits that pairing a clock tick with the stone's place ignores an inaccuracy. What is it, and why does he allow himself to ignore it here?

Hint: Ask how the walker learns where the stone is at the instant of a tick.

Answer: The walker sees the stone by light that took a finite time to arrive, so at the tick the stone is seen where it was a moment ago, not where it is. Einstein sets this aside because he is standing on the ground of classical mechanics for now, and he promises to deal with it, and with a second difficulty, at length later in the book.

Which Reference Bodies Are Allowed?

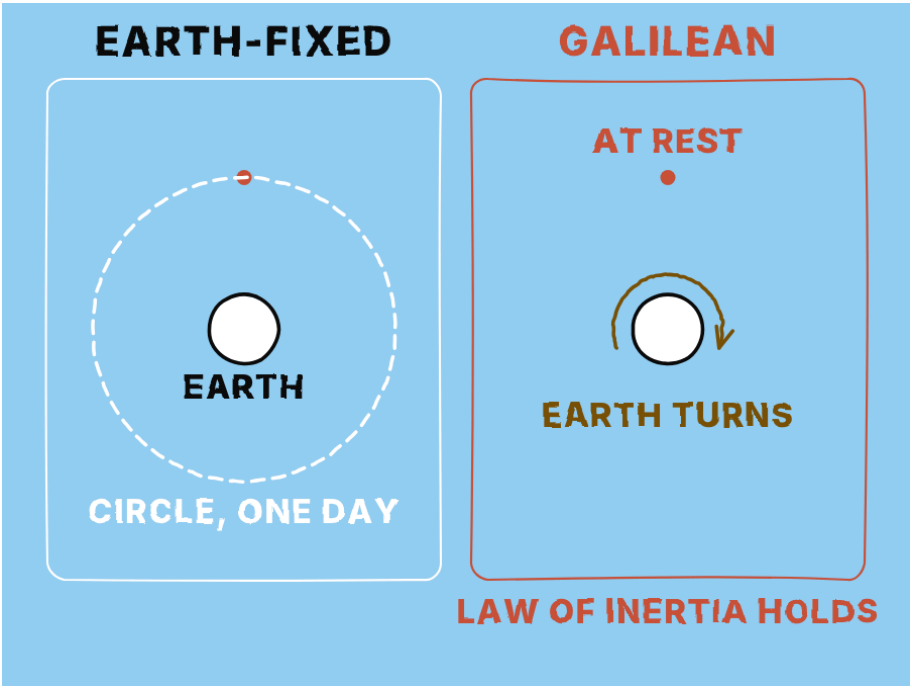
Set a camera on a tripod on a clear night, point it at the sky, and leave the shutter open for an hour. When you look at the picture, the stars are not dots but arcs, all of them pieces of circles drawn around one point in the sky. Nothing was pushing those stars; they are so far from everything that no pull worth mentioning reaches them. Yet with respect to the camera, the tripod and the ground they stand on, every one of them travelled along a curve. Keep that photograph in mind, because Einstein uses exactly this fact to decide which reference bodies physics may use at all.



Star Trails Over the Camera

He begins with the law of inertia, the first law of the mechanics of Galileo and Newton: a body far enough removed from all other bodies stays at rest or keeps moving uniformly in a straight line. Said that way, it sounds like a statement about bodies alone. Einstein points out that it is just as much a statement about reference bodies, the coordinate systems that mechanics is allowed to use when it describes motion. After the stone from the train, we know why: a path is only ever a path with respect to a reference body, so "moves in a straight line" is empty until the body is named.

Now put the two together. The visible fixed stars are bodies to which the law of inertia should apply with great accuracy; they are as far removed from everything as any body we can see. Take a coordinate system rigidly attached to the Earth, the three walls of the earlier part, fixed to the ground. With respect to that system every fixed star describes, in the course of one astronomical day, a circle of enormous radius. That is your photograph, and it contradicts the wording of the law of inertia, which promised a straight line or rest.



Earth-Fixed Versus Galilean

So one of the two has to give. If we hold on to the law, Einstein says, we may refer motions only to coordinate systems with respect to which the fixed stars do not move in circles. A coordinate system whose state of motion is such that the law of inertia holds with respect to it, he names a Galilean coordinate system. Only for a Galilean coordinate system do the laws of the mechanics of Galileo and Newton claim to be valid. The Earth, turning once a day, is not quite one; a system that keeps its directions fixed on the distant stars is.

This is not a historical curiosity. The satellites that feed your phone's receiver are tracked and predicted in exactly such a system, one whose axes point at the fixed stars and do not turn with the Earth, because only there do the equations of motion take their simple form. The

receiver then converts the result into a place on the turning Earth for you. The first choice every navigation engineer makes is Einstein's choice in this section: pick a Galilean coordinate system first, and describe motion afterwards.

Here the first four sections of the book come to rest. Geometry became physics through rigid bodies; places became three numbers on a reference body; motion became a path with respect to such a body and a clock; and now the reference bodies themselves have been sorted, with the Galilean ones admitted and the rest, for the moment, set aside. That "for the moment" is the hinge on which the rest of the book turns.

> **Remember**
>
> The law of inertia says a body far from all others stays at rest or moves uniformly in a straight line, and this is a statement about reference bodies as much as about bodies. With respect to a coordinate system fixed to the Earth, the fixed stars move in enormous circles once a day, which contradicts the law as worded; a long-exposure photograph shows it. To keep the law, motions may be referred only to coordinate systems in which the fixed stars do not circle: Galilean coordinate systems, defined as those in which the law of inertia holds. The laws of the mechanics of Galileo and Newton claim validity only for Galilean coordinate systems, and satellite navigation computes orbits in such a system before translating to the turning Earth.

Questions

1. A camera left open for an hour shows the stars as arcs of circles. What does the lecture draw from this?
   a. That the stars are being pushed along curved paths by some force
   b. That with respect to a coordinate system fixed to the Earth the fixed stars move in circles, against the wording of the law of inertia
   c. That the law of inertia does not hold for stars

2. Why does Einstein say the law of inertia is a statement about reference bodies as much as about bodies?
   a. Because reference bodies are themselves far from all other bodies
   b. Because a straight line or rest is only a straight line or rest with respect to a named reference body
   c. Because the law was first found by observing the fixed stars
   d. Because only rigid bodies can serve as reference bodies

3. What is a Galilean coordinate system?
   a. A coordinate system rigidly attached to the Earth
   b. A coordinate system whose state of motion is such that the law of inertia holds with respect to it
   c. Any coordinate system built from three mutually perpendicular walls
   d. A coordinate system that moves with the fixed stars at their speed

4. Why are the satellites behind a navigation receiver tracked in a system whose axes point at the fixed stars?
    a. Because the receiver cannot work with Earth-fixed coordinates
    b. Because only in a Galilean coordinate system do the laws of mechanics take their simple form
    c. Because the stars are needed to measure the satellites' distances

Answers

1. b) The fixed stars are bodies to which the law of inertia should apply with great accuracy, yet with respect to a system fixed to the Earth each describes a circle of enormous radius in one day, which contradicts the law as worded.

2. b) A path is only ever a path with respect to a reference body, so the law's talk of moving in a straight line is empty until the coordinate system is named.

3. b) Einstein names a coordinate system Galilean when its state of motion is such that the law of inertia holds with respect to it; the fixed stars do not circle in such a system.

4. b) The laws of the mechanics of Galileo and Newton claim validity only for a Galilean coordinate system, so the orbits are computed in one and converted to the turning Earth afterwards.

Practice

1. Explain in your own words why a coordinate system rigidly attached to the Earth is not a Galilean coordinate system, using the fixed stars.

Hint: Ask what path a fixed star describes with respect to the ground in one day, and what the law of inertia says it should do.

Answer: A fixed star is far from all other bodies, so by the law of inertia it should be at rest or move uniformly in a straight line. With respect to a system attached to the Earth it describes a circle of enormous radius once a day instead. The law does not hold with respect to that system, so the system is not Galilean.

2. Einstein keeps the law of inertia and gives up some reference bodies. State what rule this leads to for describing motion, and name the kind of coordinate system that results.

Hint: What must the fixed stars not do with respect to an allowed system?

Answer: Motions may be referred only to coordinate systems with respect to which the fixed stars do not move in circles. A coordinate system in whose state of motion the law of inertia holds is a Galilean coordinate system, and the laws of the mechanics of Galileo and Newton claim validity only for such systems.

3. A friend says the law of inertia is simply false, since the stars plainly move in circles every night. Answer them with the ideas of this part.

Hint: Separate the statement about bodies from the statement about reference bodies.

Answer: The circles are the stars' paths with respect to the turning Earth, and a path exists only with respect to a reference body. The law of inertia claims validity only for Galilean coordinate systems, in which the fixed stars do not circle; the Earth-fixed system is not one of them, so the night sky contradicts the choice of reference body, not the law.