

The Mathematical Principles of Natural Philosophy



Isaac Newton · 1687

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From the original: The Mathematical Principles of Natural Philosophy, translated by
Andrew Motte (first American edition, 1846)

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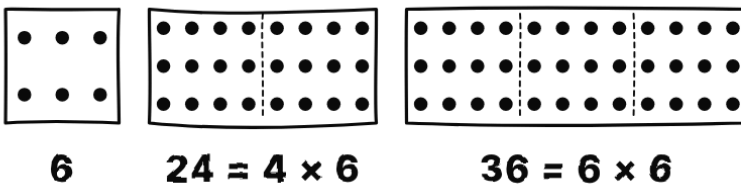
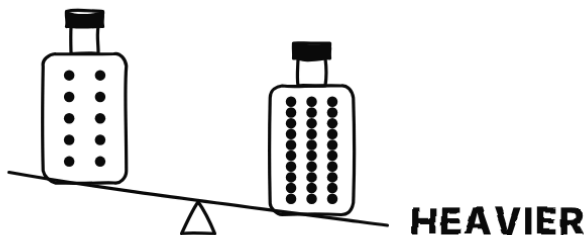
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Matter, Motion and the Pull Towards a Centre

Take two identical plastic bottles. Leave one holding air at ordinary pressure with its cap on; pump the other full of air with a bicycle pump until it is hard. They take up the same room, yet the hard one holds more air, and a sensitive kitchen scale even shows it a little heavier. So the size of a thing does not tell how much stuff is in it. That is the very first question Newton answers in his great book, before any law of motion is stated.

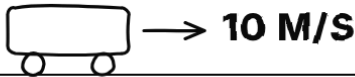


His answer is that the quantity of matter is measured by how tightly the matter is packed and how much room it takes, the two multiplied together. Packing and room multiply because each counts on its own: think of a crowd, where twice as many people on each square metre, over twice as many square metres, makes four times the crowd. Newton's own example is air: air twice as dense, filling twice the space, is four times as much air, and the same dense air in three times the space is six times as much. Snow pressed into ice, or powder squeezed into a cake, changes size and packing, but not the amount of matter. This quantity is what he calls, from then on, body or mass.

Newton leaves aside, on purpose, any fine medium that might flow between the parts of bodies; he counts only the matter of the body itself. Then he makes a claim that is far from obvious: the mass of a body is proportional to its weight. He found it by very careful experiments with pendulums, which he promises to describe later: pendulums of the same length swing in the same time whatever their bobs are made of, and that can only happen if weight grows exactly in step with mass. It is why the scale in your bathroom, which really feels a pull, can honestly show your mass in kilograms.

The second definition builds on the first: the quantity of motion is the mass and the speed multiplied together; today it is called momentum. The reason is that the motion of a whole body is just the motion of its parts added up, so twice the matter moving at the same speed is twice the motion. Newton's numbers: a body with twice the matter at the same speed has twice the motion, and if it also goes twice as fast, it has four times the motion. A small car of one thousand kilograms at ten metres per second carries ten thousand units of motion; a van of two thousand kilograms at twenty metres per second carries forty thousand, four times as many, and needs far more road to stop.

1000 KG



10 000

2000 KG

← INERTIA



40 000

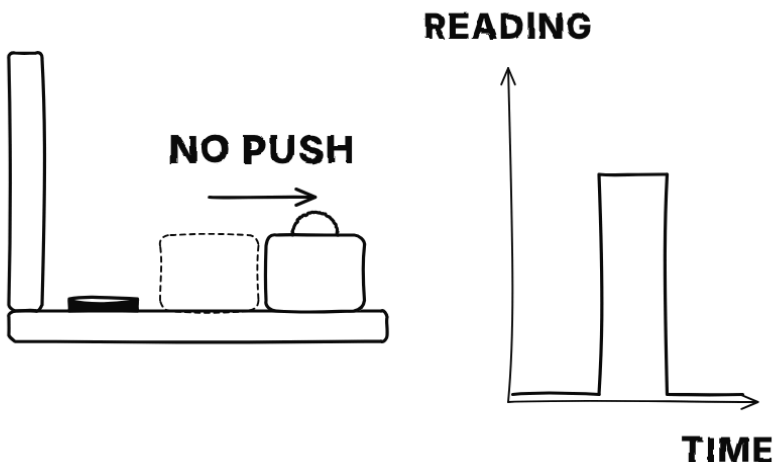
Mass Times Speed

The third definition names something everyone feels: the force born into matter itself, which Newton calls in Latin the *vis insita*. It is a power of resisting, by which every body, as far as it can, keeps doing what it is doing, staying at rest or moving straight ahead at a steady speed. This power is proportional to the amount of matter, and it is nothing but the sluggishness of matter seen from another side; Newton's better name for it is the force of inactivity, *vis inertiae*, from which we get the word inertia. A body shows it only when something tries to change its state: from the side of the body being pushed, it looks like resistance; from the side of a moving body pushing into another, it looks like impulse. And Newton adds a warning that will matter later: rest and motion, as people usually think of them, are told

apart only relative to other things, and bodies that look at rest are not always truly at rest.

Try it yourself

Lay your phone flat on the passenger seat of a car and open any free sensor app that shows the accelerometer. While the car rolls steadily along a straight road, the reading along the direction of travel stays near zero. When the driver brakes, the reading jumps, and a loose bag on the seat slides forward. Nothing pushed the bag forward: it simply kept going while the car was made to stop, and that is inertia.

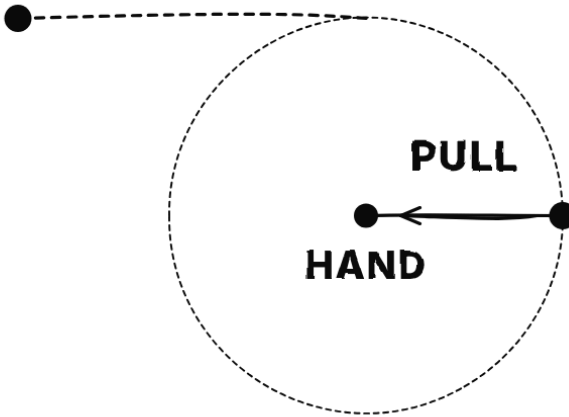


The Bag Keeps Going

The fourth definition: an impressed force is an action on a body that changes its state of rest or of steady straight motion. It exists only while the action lasts; nothing of it stays in the body afterwards, because the body keeps its new state by inertia alone. Such forces come from blows, from pressure, and from centripetal force. The brakes press only while the pedal is down; once the car has slowed to its new speed, it keeps that speed with nothing pressing at all.

The fifth definition brings in the force the whole book is about: a centripetal force, one that draws a body towards a point as towards a centre. Gravity is one, pulling bodies towards the middle of the earth; magnetism is another, pulling iron towards a lodestone; and so is whatever force bends the planets out of straight lines into curved orbits. Newton's picture is a stone whirled in a sling: it strains to fly away and stretches the sling, the more the faster it goes, and the moment it is let go it flies off along a straight line. The pull of the sling, always aimed back at the hand at the centre, is the centripetal force, the force that seeks the centre, and every body moving round any orbit needs one, or it would leave in a straight line.

STRAIGHT ON



The Pull Toward the Centre

Then comes the most famous picture of all: a cannon on a mountain top, firing a lead ball level with the ground. Without gravity and without air, the ball would go straight on forever; gravity bends its path down, less the faster it is fired. Newton gives numbers: if the ball lands two miles away, then with the air removed, twice the speed carries it four miles and ten times the speed twenty miles. The reason is that the ball takes the same time to fall the same height whatever its speed, so the distance it covers before landing grows exactly with the speed. Keep making it faster and the path flattens, until the ball lands a quarter of the way round the earth, or goes right round before it falls, or never comes down at all and travels on into the sky. That last case is real: every GPS satellite your phone listens to is Newton's

cannonball, moving so fast sideways that it keeps falling around the earth and never reaches it.

The moon is held in exactly this way. If the pull towards the earth were too weak, the moon would drift away in a straight line; if it were too strong, the moon would be drawn down out of its orbit. The pull has to be just right for the speed the moon has, and Newton names the two problems his book will solve: from a given orbit and speed, find the force; and from a given force, place and speed, find the curved path. Engineers placing a satellite in orbit solve exactly these two problems before every launch.

Remember

The amount of matter is packing times room, so air twice as dense in twice the space is four times as much, and that amount, which Newton calls mass, is proportional to weight. The amount of motion is mass times speed, so twice the mass at twice the speed is four times the motion. Inertia is a body's tendency to keep doing what it is doing, and an impressed force exists only while it acts. A centripetal force pulls towards a centre and turns straight motion into an orbit, as the sling holds the stone and gravity holds the moon and every satellite. Finding the force from the orbit, and the orbit from the force, is the task Newton sets for the rest of the book.

Questions

1. Air is made three times as dense and fills twice the room. How much more air is there?
 - a. Five times as much
 - b. Six times as much
 - c. Three times as much
 - d. Twice as much

2. What do pendulums of the same length swinging in the same time, whatever their bobs are made of, tell Newton?
 - a. That weight grows exactly in step with mass
 - b. That all materials have the same density
 - c. That pendulums are not affected by gravity
 - d. That heavier bodies fall faster

3. Once the brakes have slowed the car and the pedal is released, what keeps the car at its new speed?
 - a. Some of the braking force stored in the car
 - b. The engine taking over from the brakes
 - c. A pull towards the centre of the earth
 - d. Its inertia alone

4. Why does a cannonball fired twice as fast, with the air removed, land twice as far away?
 - a. Gravity pulls a faster ball more weakly
 - b. A faster ball has more inertia and falls more slowly
 - c. It takes the same time to fall the same height, so it travels twice as far in that time
 - d. A faster ball rises before it falls

Answers

1. **b)** Packing and room multiply, each counting on its own, so three times the packing over twice the room is six times the matter.
2. **a)** Pendulums of the same length swing in the same time whatever their bobs are made of, which can only happen if weight grows exactly in step with mass.
3. **d)** An impressed force exists only while it acts; nothing of it stays in the body, which keeps its new state by inertia alone.
4. **c)** The ball takes the same time to fall the same height whatever its speed, so the distance it covers before landing grows exactly with the speed.

Worked examples

Air twice as dense Air twice as dense fills twice the space. In a second case the same denser air fills three times the space. How many times the original amount of matter is each?

The amount of matter is how tightly it is packed times how much room it fills.

Twice the packing in twice the room: two times two, four times as much.

Twice the packing in three times the room: two times three, six times as much.

Answer: four times and six times

The cannonball's range A lead ball fired level from a mountain top lands two miles away. With the air removed, how far does it go at twice and at ten times the speed?

Without air, the ball falls the same height in the same time whatever its speed.

So the distance it covers before landing grows exactly with the speed.

Twice the speed gives twice two miles, four miles; ten times the speed gives ten times two miles, twenty miles.

Answer: four miles and twenty miles

Practice

1. One body has twice the matter of another and moves twice as fast. How many times the amount of motion does it carry?

Hint: The amount of motion is mass times speed: multiply the two factors.

Answer: Four times: twice the matter doubles the motion, and twice the speed doubles it again.

2. A car of one thousand kilograms moves at ten metres per second; a van of two thousand kilograms moves at twenty metres per second. How many times the car's amount of motion does the van carry?

Hint: Work out mass times speed for each.

Answer: Four times: the car carries ten thousand units, the van forty thousand.

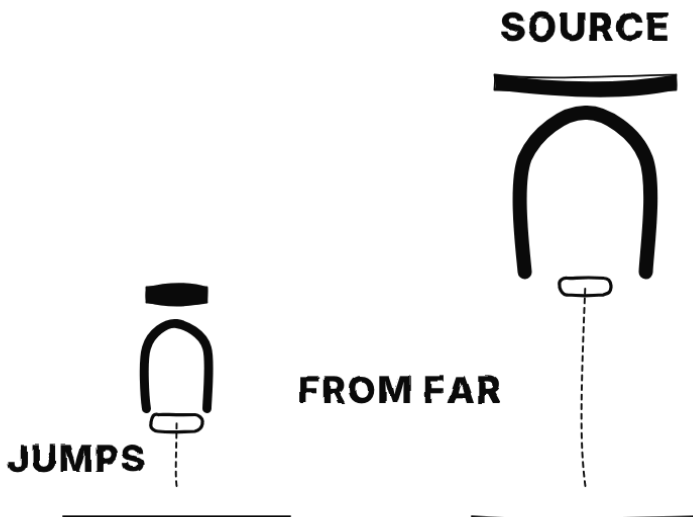
3. With the air removed, the ball that lands two miles away is fired three times as fast. How far does it go?

Hint: Its fall still takes the same time, so the distance grows with the speed.

Answer: Six miles: three times the speed carries it three times the two miles.

One Pull, Three Measures, and Time Itself

Hold a small fridge magnet above a paper clip and lower it slowly until the clip jumps up and sticks. Now do the same with a big, strong magnet: the clip jumps from much farther away. And with either magnet, the closer it is, the more sharply the clip leaps. So how strong a pull is has more than one honest answer, and Newton keeps three of them apart.

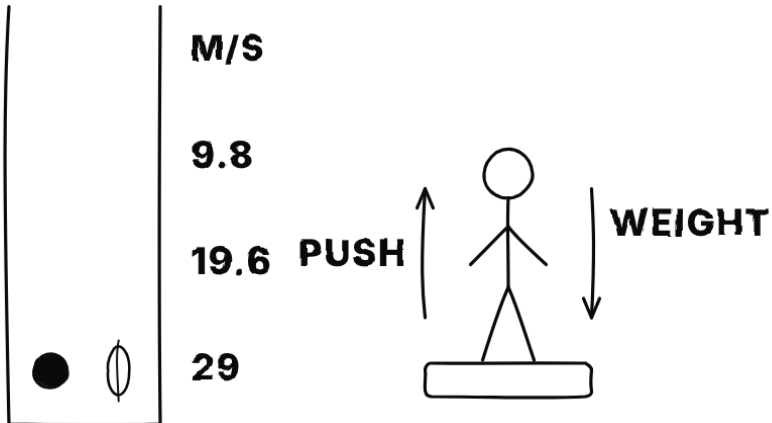


Small Magnet, Big Magnet

The first measure looks at the source. Newton calls it the absolute quantity of a centripetal force: how powerful the cause is that sends the pull out from the centre into the space around it. A big strong loadstone simply has more of it than a small weak one, whatever is placed near it. The earth's own magnetism, which turns your phone's compass, is an enormous source, though spread so wide that the needle hardly feels it.

The second measure looks at a place. The accelerative quantity is how much speed the pull adds to a body there in a given time, what we now call acceleration. The same loadstone speeds things up more close by than far away; gravity is a little stronger in a valley than on a very high mountain, and weaker still far from the earth. Yet at the same distance from the earth it speeds up every falling body alike, heavy or light, big or small, once the air is allowed for. A calculation in words: a stone dropped near the ground gains about nine point eight metres per second of speed every second, so after three seconds it falls at about twenty-nine metres per second, and a feather in a tube with the air pumped out keeps exact pace with it. Your phone's accelerometer, lying still on a table, shows that same nine point eight whether the phone is bare or inside a heavy case.

VACUUM



Speed Gained Each Second

The third measure looks at the whole body. The motive quantity is the amount of motion the pull produces in the body in a given time, and Newton says plainly that this is the body's weight, its whole tendency towards the centre. It is always measured by the equal and opposite push that is just enough to stop the body from falling. That is exactly what your bathroom scale does: it pushes up just hard enough to stop you sinking, and shows how hard that is. A bigger body has more weight, and the same body weighs more near the earth and less far away.

Newton gives each measure a home. Weight belongs to the body, as the sum of the pulls on all its parts; acceleration belongs to the place, as a power spread from the centre to every spot around it; and the

absolute force belongs to the centre, as whatever cause produces the pull, a loadstone, the earth, or something not yet known. He insists that he is giving only a mathematical idea of these forces, not saying what physically causes them.



 **HERE: 1**

PULL $\frac{1}{2}$  A QUARTER

PULL $\frac{1}{2}$  A SIXTH


GPS

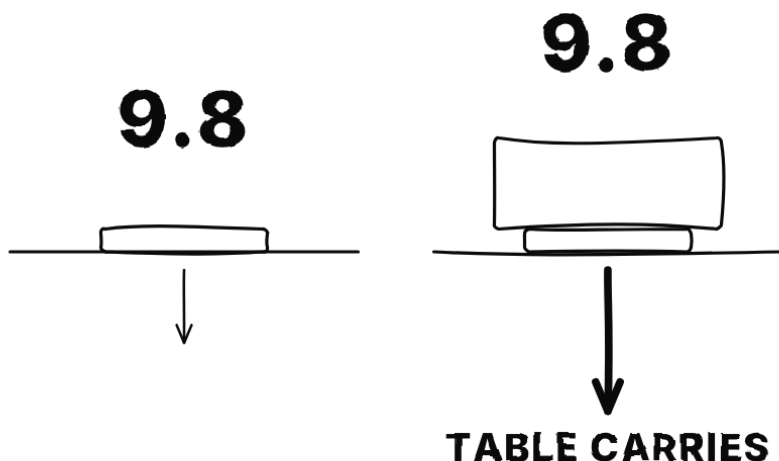
Half the Pull, Smaller Bodies

Then he links weight and acceleration the way motion is linked to speed. Motion is speed multiplied by the amount of matter; in the same way, weight is acceleration multiplied by the amount of matter, because the pull on the whole body is the pull on every little part, added up. So near the earth, where the acceleration is the same for everything, weight is simply in proportion to the amount of matter. His own calculation goes up high: in a region where gravity's acceleration is only half, a body half as large weighs a quarter of what

the big body weighs here, four times less, and a body a third as large weighs a sixth, six times less. At the height of the GPS satellites, far above the air, gravity's acceleration is only a small fraction of what it is at the ground, so a satellite there weighs much less than on the launch pad, though its amount of matter has not changed at all.

Try it yourself

Lay your phone flat on a table and open a free sensor app that shows the accelerometer. Note the reading pointing up from the table, close to nine point eight. Put a heavy book on top of the phone and look again: the reading does not change. The acceleration belongs to the place, not to what lies there; the book changed what the table carries, not the speed-up gravity gives at that spot.

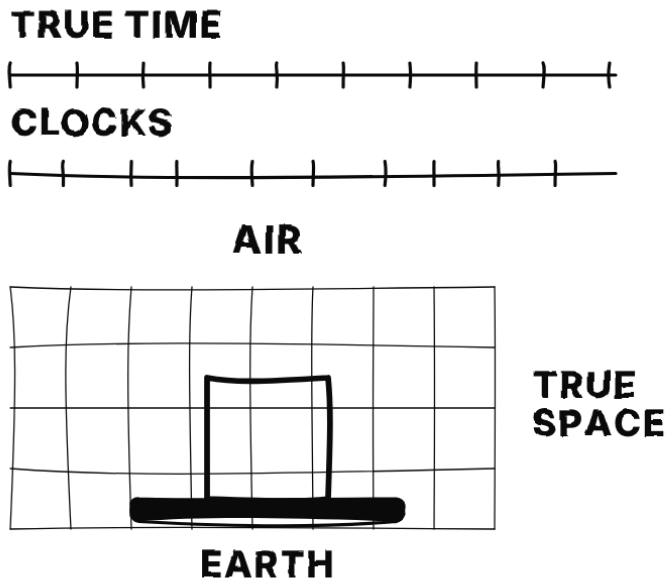


The Book Does Not Change It

Newton adds a warning about his words: he uses attraction, impulse and tendency towards a centre interchangeably, and never claims to say what these forces really are, nor that a centre, a mere mathematical point, truly pulls. Then the definitions end and a long explanatory note, the scholium, begins. Time, space, place and motion, he says, need no definitions, since everyone knows them; but people think of them only through the things they see, and this breeds prejudices. To clear them away he separates each into absolute and relative, true and apparent, mathematical and common.

Time comes first. True, absolute time flows evenly of itself, without regard to anything outside, and its other name is duration. Relative, common time is any visible measure of that duration made by some

motion, accurate or not, such as an hour, a day, a month or a year. Your phone's clock is relative time in exactly this sense: a measure made by counting the vibrations of a tiny crystal, checked against the satellites' clocks, and never the flow of time itself.



True and Common Time and Space

Space comes next. Absolute space, of its own nature and without regard to anything outside, stays always the same and never moves. Relative space is any movable measure of it that our senses fix by its position among bodies, such as the space of a cave, of the air, or of the heavens, measured from the earth. The two have the same shape and size, but they need not stay the same stretch of space: if the earth moves, the air that stays put around us is, absolutely speaking, passing through one part of space after another, forever changing.

Remember

A pull towards a centre has three measures: the power of its source, how fast it speeds things up at a place, and the weight it gives a whole body. Weight is acceleration times amount of matter, just as motion is speed times amount of matter, so where gravity's acceleration is halved, a body half or a third as large weighs a quarter or a sixth. At the same height gravity speeds up everything alike, which a phone's accelerometer shows with or without a book on top. True time flows evenly by itself, while hours, days and our phone clocks are only motions used to measure it. True space never moves, while the space we measure from the earth may itself be moving through it.

Questions

1. Putting a heavy book on a phone does not change its accelerometer reading. What does that show?
 - a. The acceleration belongs to the place, not to what lies there
 - b. Heavy things are not pulled by gravity
 - c. The phone measures the weight of the book
 - d. Gravity is weaker under heavy objects
2. Why is weight simply in proportion to the amount of matter near the earth?
 - a. Because all bodies there are the same size
 - b. Because the acceleration there is the same for everything

- c. Because weight does not depend on the pull at all
 - d. Because heavier bodies speed up faster
3. According to Newton's scholium, what is a day or an hour?
- a. True time itself
 - b. A visible measure of duration made by some motion
 - c. A part of absolute space
 - d. The duration of a single motion of the stars
4. What is true of absolute space, as opposed to the space we measure from the earth?
- a. It moves along with the earth
 - b. It is a different shape from relative space
 - c. It stays the same and never moves
 - d. It exists only near the stars

Answers

1. **a)** The acceleration belongs to the place, not to what lies there; the book changed what the table carries, not the speed-up gravity gives at that spot.
2. **b)** Weight is acceleration times the amount of matter, and near the earth the acceleration is the same for everything, so weight follows the amount of matter alone.
3. **b)** Relative, common time is any visible measure of duration made by some motion, such as an hour, a day, a month or a year.
4. **c)** Absolute space, of its own nature and without regard to anything outside, stays always the same and never moves.

Worked examples

Weight where gravity is halved In a region where gravity's acceleration is half of what it is here, how much do bodies half and a third as large weigh, compared with the larger body here?

Weight is acceleration multiplied by the amount of matter.

Half the acceleration times half the matter is a quarter: four times less.

Half the acceleration times a third of the matter is a sixth: six times less.

Answer: four times less and six times less

Practice

1. A stone near the ground gains about nine point eight metres per second of speed each second. Ignoring the air, how fast does it fall after two seconds?

Hint: Acceleration is the speed added each second; multiply by the number of seconds.

Answer: About nineteen point six metres per second: nine point eight added twice.

2. Where gravity's acceleration is halved, a body is a quarter as large as one here. How many times less does it weigh?

Hint: Multiply the share of the acceleration by the share of the matter.

Answer: Eight times less: a half times a quarter is an eighth.

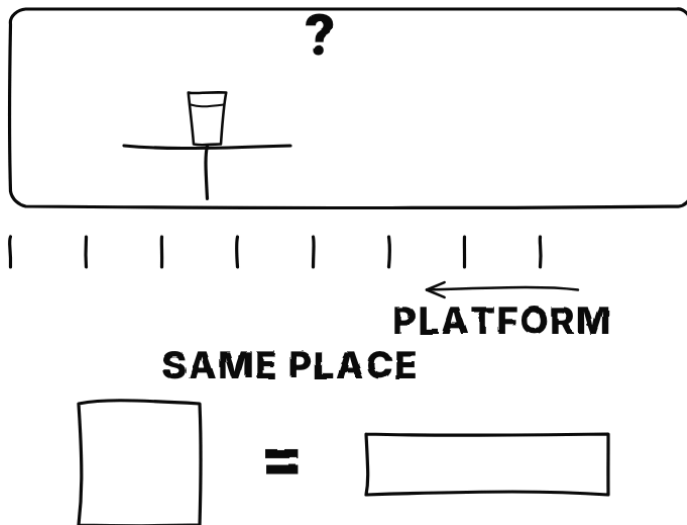
3. Explain in your own words why your phone's clock shows relative time, not true time.

Hint: Ask what the clock actually counts.

Answer: The clock counts a motion, the vibrations of a tiny crystal, and uses that count as a measure of how long things last. True time flows evenly by itself; any clock is only a motion used to measure it, accurate or not.

Places, a Sailor, and Why Rest Cannot Be Seen

Fill a glass of water to the brim and set it on the table of a train waiting at a platform. When the train glides off smoothly, the water barely trembles, and from your seat the glass, the table and you all seem perfectly still. Yet the platform outside is sliding away. Are you at rest or moving, and how could anyone tell for certain?

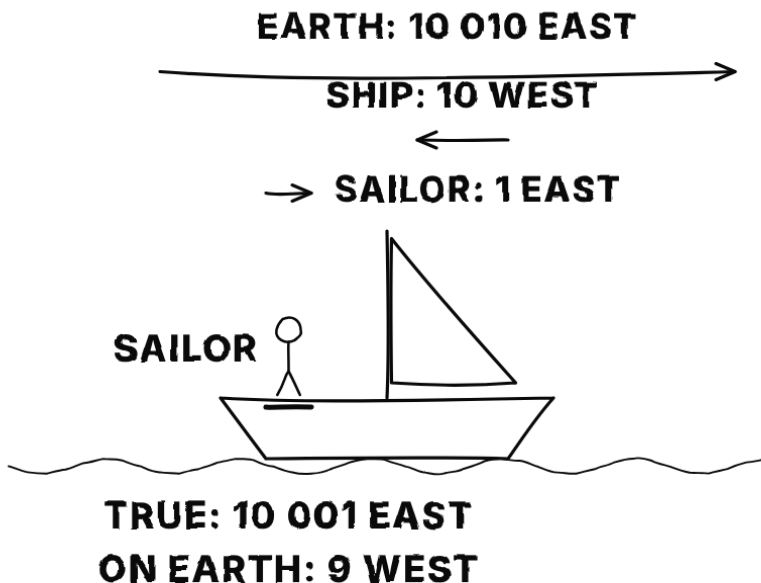


Still on a Moving Train

Newton starts from place. A body's place is the part of space it fills, true or relative as the space is; not its position, and not its outer skin.

Two solid blocks of equal size always fill equal places, even when their shapes, and so their surfaces, are different. And since the motion of a whole is the motions of all its parts together, the place of a whole is the places of all its parts together, inside the body.

Motion then splits in two. True motion is a body's passage from one place in true space to another; relative motion is its passage from one place to another among the things around it. On a ship under sail, a passenger sitting in a cabin keeps the same part of the ship, and so is at rest relative to the ship. True rest would mean staying in the same part of unmoving space, through which the whole ship is carried. So if the earth were truly still, the sitting passenger would truly move with the ship's speed; and if the earth moves too, the passenger's true motion is the earth's motion, the ship's motion over the earth, and any walking on deck, all put together.



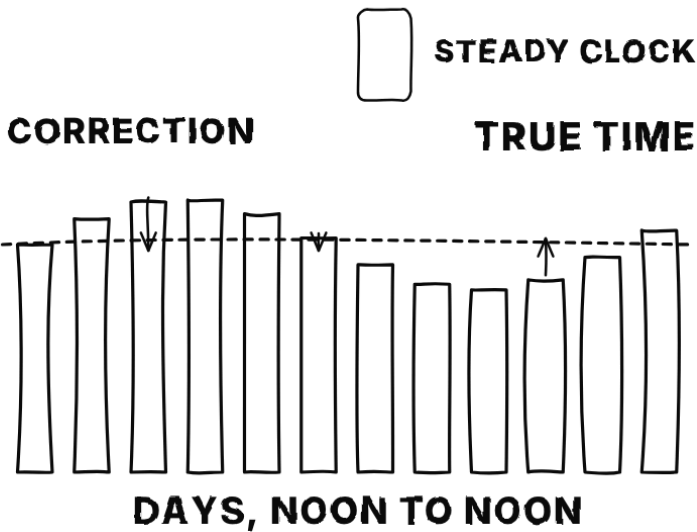
The Sailor's True Motion

Newton's numbers make this concrete. Let the stretch of earth under the ship move truly eastwards with ten thousand and ten parts of speed, let the ship sail westwards with ten parts, and let a sailor stroll eastwards on deck with one part. Speeds along one line add, each measured from what carries it: ten thousand and ten, take away ten, add one, gives ten thousand and one parts eastwards, the sailor's true motion. Relative to the earth, the earth's share drops out: the ship's ten westwards against the sailor's one eastwards leaves nine parts westwards. The same sailor rushes east through space while drifting west along the shore. Your phone's GPS reports only the second kind of number, your speed over the ground, never the earth's own motion.

Try it yourself

On a moving train, open a GPS speed app and walk forwards along the aisle, then backwards. Walking forwards, the app shows the train's speed plus your walking pace; walking backwards, the train's speed minus your pace. It is adding speeds along a line exactly as Newton does for the sailor, and it stops at the ground.

Time needs the same care. The natural days, from noon to noon by the sun, are not all equally long, though everyone treats them as equal; astronomers correct this to compute the motions of the heavens. Perhaps no motion is perfectly even, so time can never be measured exactly; but true time flows on unchanged, and the lasting of things is the same whether their motions are fast, slow, or none at all. Pendulum clocks and the eclipses of the moons of Jupiter both show that the correction is needed. A sundial's noon drifts by about a quarter of an hour either way against your phone's steady clock over the year, which is that same inequality of the days.

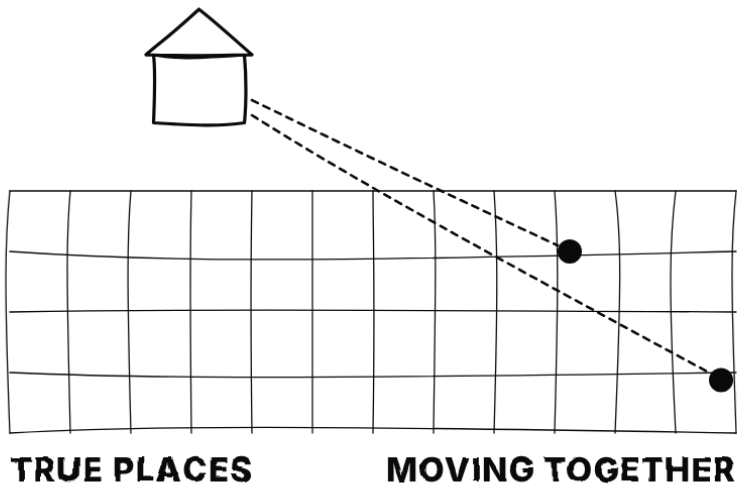


Natural Days Are Unequal

The parts of time keep their order forever, and so do the parts of space. To move a part of space out of its place would be to move it out of itself, since times and spaces are the places both of themselves and of everything else: everything sits in time in an order of succession, and in space in an order of situation. So these are the true places, and passages out of them are the only true motions.

But we cannot see the parts of space or tell them apart, so we use measures we can see instead. We pick some body that we treat as unmoving, define every place by its distance and direction from that body, and judge every motion against those places. That causes no trouble in daily life, but in natural philosophy we must look past our

senses to the things themselves, because there may be no body anywhere that is truly at rest to measure from.



Places Measured From a Body

Newton then promises to tell true from relative motion by three things: their properties, their causes and their effects. The first property: bodies truly at rest are at rest with respect to one another. Somewhere among the distant fixed stars, or beyond them, there might be a body that is truly at rest; but nothing in how the bodies around us sit with respect to each other can tell whether they keep the same position towards that far-off body. So true rest can never be found from the positions of the bodies in our part of the world. Like the water in the glass on the smooth train, everything around us can look perfectly still while all of it is moving together.

Remember

A place is the part of space a body fills, and the place of a whole is the places of its parts together. True motion is passage through true space; relative motion is measured against surrounding things, and a passenger at rest in a sailing ship is truly moving. Newton's sailor moves ten thousand and one parts east in truth and nine parts west relative to the earth, because speeds along a line add. True time flows unchanged while days and clocks only measure it, and astronomers correct the unequal days. Since we can only measure from bodies we treat as still, and none may be truly still, true rest cannot be found from the positions of bodies around us.

Questions

1. A passenger sits still in the cabin of a sailing ship. Relative to what are they at rest?
 - a. Relative to unmoving space
 - b. Relative to the ship, whose part they keep
 - c. Relative to the stars
 - d. Relative to the earth's centre
2. Why does the earth's motion drop out when finding the sailor's motion relative to the earth?
 - a. Because the earth does not move

- b. Because motion relative to the earth is measured from the earth, so its own share is not counted
 - c. Because the earth moves too slowly to matter
 - d. Because only the ship's motion counts
3. Why do we use measures we can see instead of true places?
- a. Because true places are always moving
 - b. Because we cannot see the parts of space or tell them apart
 - c. Because true places are too far away
 - d. Because measured places are more accurate
4. Why can true rest never be found from the positions of the bodies around us?
- a. Because nothing in the universe is at rest
 - b. Because a truly resting body might be far away, and our bodies' positions to each other cannot show whether they keep position towards it
 - c. Because positions cannot be measured
 - d. Because rest is the same as motion

Answers

- 1. b)** A passenger sitting in a cabin keeps the same part of the ship, and so is at rest relative to the ship; true rest would mean staying in the same part of unmoving space.
- 2. b)** Relative to the earth, the earth's share drops out, leaving the ship's ten westwards against the sailor's one eastwards: nine westwards.
- 3. b)** We cannot see the parts of space or tell them apart, so we use measures we can see instead, taken from some body treated as unmoving.

4. b) There might be a body truly at rest among or beyond the fixed stars, but nothing in how the bodies around us sit with respect to each other can tell whether they keep the same position towards it.

Worked examples

The sailor's true and relative motion The earth under the ship moves east with ten thousand and ten parts of speed, the ship sails west with ten, and a sailor walks east on deck with one. Find the sailor's true motion and his motion relative to the earth.

Speeds along one line add, each measured from what carries it.

True motion: ten thousand and ten, take away ten, add one, gives ten thousand and one parts east.

Relative to the earth, leave out the earth's share: ten west against one east leaves nine parts west.

Answer: ten thousand and one parts east truly; nine parts west relative to the earth

Practice

1. Keep the earth's ten thousand and ten parts east and the ship's ten west, but let the sailor walk west with one part. What are his true and relative motions?

Hint: Add the speeds along the line, east counting forward and west counting back.

Answer: Truly, ten thousand and ten take away ten take away one, nine thousand nine hundred and ninety-nine parts east; relative to the earth, ten west and one more west, eleven parts west.

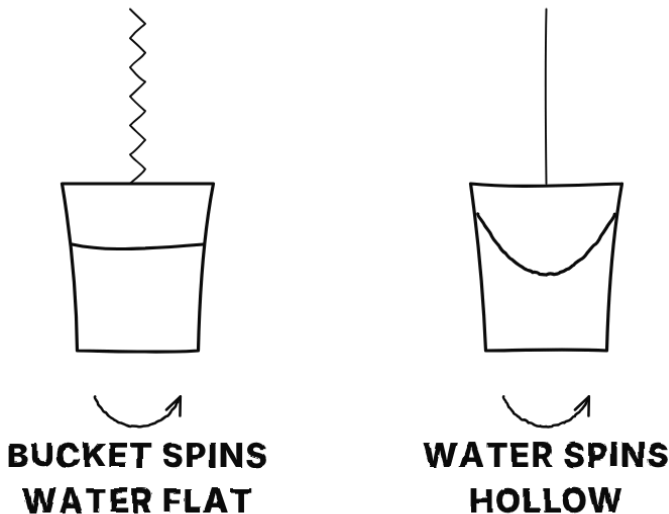
2. In your own words, why does a glass of water on a smoothly moving train not tell you whether you are moving?

Hint: Think about what you compare the glass with.

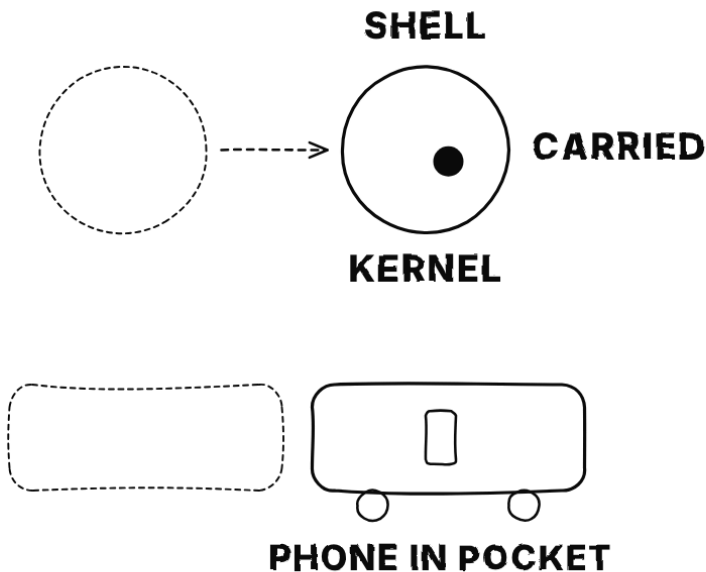
Answer: The glass, the table and you all move together, so relative to each other you are all at rest. Comparing bodies that move together cannot reveal their common motion, just as the positions of bodies around us cannot reveal true rest.

The Kernel, the Shell and the Spinning Bucket

Hang a small bucket of water from a long string, twist the string many times, hold the bucket still, then let go. As the string unwinds, the bucket spins faster and faster. At first the water inside stays flat and calm while the bucket turns around it. Then, little by little, the water starts to turn with the bucket, its middle sinks, and it climbs up the sides into a hollow, until water and bucket spin together. Newton did this experiment himself, and it is the key to everything in this part.

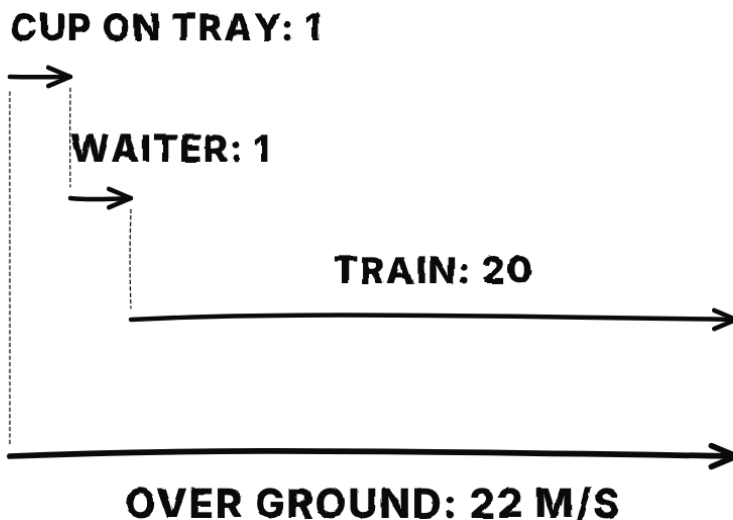


First, though, a property of motion: parts that keep fixed positions within a whole share the motion of the whole. Every part of a spinning body strains outwards, away from the axis, and the forward push of a moving body is all the pushes of its parts together. So if the things around a body move, the body resting among them moves with them. That is why true motion cannot be found by watching a body against things that only seem to be at rest: they would have to be truly at rest. Newton's picture is a shell and its kernel: if the shell moves, the kernel moves too, without ever moving away from the shell. A phone lying still in your pocket on a moving bus is the kernel: it never shifts inside the pocket, yet it travels across town with the bus.



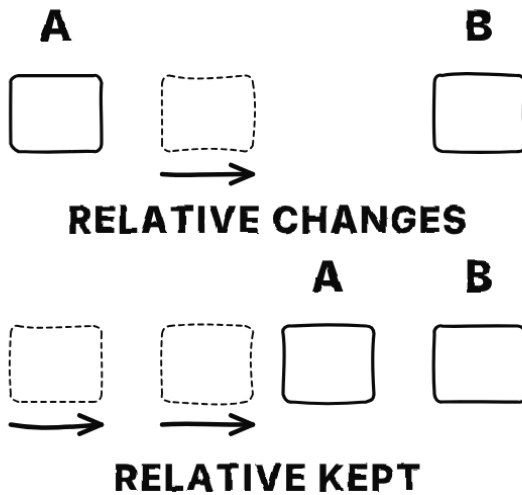
If the Shell Moves, the Kernel Moves

A close relative of this property: if a place moves, everything in it moves with it. So any motion measured from a moving place is only part of the true motion. The true motion is built up step by step: the body's motion in its place, plus that place's motion in its own place, and so on, until an unmoving place is reached, as with the sailor whose true motion added the earth's, the ship's and his own. A calculation in words: a cup slides forward on a tray at one metre per second, the waiter walks forward at one metre per second along the train, and the train runs at twenty metres per second, so the cup moves at twenty-two metres per second over the ground, and even that is not its true motion, because the ground is carried along too. So whole, true motions can only be fixed against unmoving places, and the only unmoving places are those that, from infinity to infinity, keep the same positions towards one another, which together make up unmoving space.



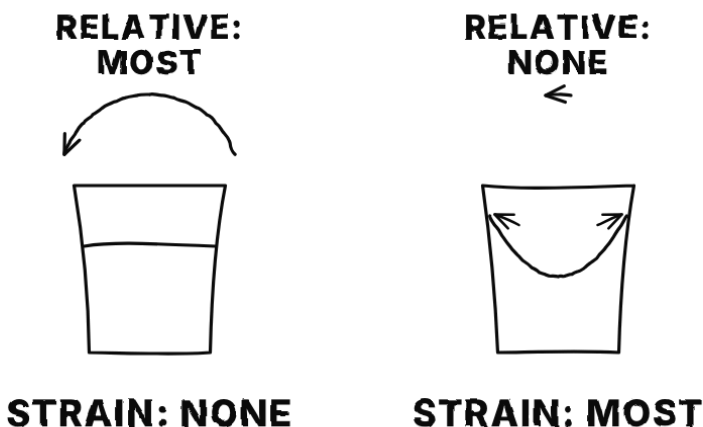
Motions Added Up

Next, the causes. True motion is never created or changed except by a force pressed on the body itself. Relative motion can change with no force on the body at all: push the things it is compared with instead, and the relation changes. And a force on the body always changes its true motion, while its relative motion may stay the same, if the things it is compared with are pushed equally and keep their positions. When the bus brakes, you and the phone in your pocket slow down together: relative to you the phone does not move, yet its true motion has changed. So relative motion can change while true motion stays the same, and stay the same while true motion changes; true motion does not consist in such relations.



Push the Other Body

Then the effects, and here the bucket returns. What tells true circular motion from merely relative circular motion is the strain to move away from the axis. A purely relative circling produces no such strain; a true circling produces more or less of it, according to how much circular motion there is.



Strain Follows True Spinning

Look again at the bucket. At the start, the water moves fastest relative to the bucket, since the bucket is turning around still water; yet the surface stays flat, with no strain outwards at all. So the water's true circling had not yet begun. Later, as the water catches up and its motion relative to the bucket shrinks, it climbs the sides, showing its strain outwards, and that strain grows until the water spins along with the bucket and is at rest relative to it, the moment it climbs highest. The strain follows the true circling, not the motion relative to the surrounding bucket, so true circular motion cannot be defined by such relations.

Try it yourself

Stir a cup of tea briskly with a spoon, then take the spoon out. While the tea spins, its surface dips in the middle and rises at the rim, and the tea leaves or crumbs crowd towards the centre at the bottom. The hollow surface is the strain away from the axis, the sign of a true spinning, exactly as in Newton's bucket.

Every spinning body has only one true circular motion, matched by one strain away from its axis, its proper effect. But it has countless relative motions, one for every body it might be compared with, and these, like any relations, have no real effect of their own. That settles an old picture of the heavens, in which the planets are carried round by revolving spheres while resting within them: such planets would truly move, because they change their positions towards one another, which bodies truly at rest never do, and as parts of a spinning whole they strain away from its axis. A washing machine's spin cycle shows the same strain: the drum's true spinning presses the clothes outwards against the wall, whatever the room around it is doing.

Remember

Parts share the motion of their whole, as a kernel moves with its shell or a phone with a bus, so resting among moving things is not true rest. True motion is built from nested motions down to unmoving space, and only a force on the body itself changes it. Relative motion can change without any force on the body, or stay the same despite one, so true motion is not a relation to other bodies. The effect that reveals true circling is the strain away from the axis: in Newton's bucket the water climbs the sides only when it truly spins, not when it merely moves relative to the bucket. Each spinning body has one true circular motion and countless relative ones, and only the true one has a real effect.

Questions

1. At the start of the bucket experiment, the water moves fastest relative to the bucket, yet its surface stays flat. What does that show?
 - a. That the bucket is not really spinning
 - b. That the water's true circling has not yet begun
 - c. That water cannot spin
 - d. That the string is not twisted
2. When does the water climb highest up the sides of the bucket?
 - a. When it moves fastest relative to the bucket

- b. When it spins along with the bucket and is at rest relative to it
 - c. Just after the bucket is let go
 - d. When the bucket stops
3. A cup slides forward at one metre per second on a tray, the waiter walks forward at one metre per second along the train, and the train runs at twenty. How fast does the cup move over the ground?
- a. Twenty metres per second
 - b. Twenty-one metres per second
 - c. Twenty-two metres per second
 - d. Two metres per second
4. Why can't a phone resting in your pocket on a moving bus be called truly at rest?
- a. Because it vibrates
 - b. Because parts that keep their positions within a whole share the whole's motion
 - c. Because pockets are not places
 - d. Because it is too small to be at rest

Answers

- 1. b)** A purely relative circling produces no strain away from the axis; the flat surface shows that the water's true circling had not yet begun.
- 2. b)** As the water catches up, it climbs the sides more and more, until it spins along with the bucket and is at rest relative to it, the moment it climbs highest.

- 3. c)** True motion is built up step by step from nested motions: one plus one plus twenty gives twenty-two metres per second over the ground.
- 4. b)** Parts that keep fixed positions within a whole share the motion of the whole; the phone is the kernel and the bus the shell.

Worked examples

Nested motions on a train A cup slides forward on a tray at one metre per second, the waiter walks forward along the train at one metre per second, and the train runs at twenty metres per second over the ground. How fast does the cup move over the ground?

Each motion is measured from the place that carries it.

The cup's motion on the tray, the waiter's along the train and the train's over the ground all point forward, so they add.

One plus one plus twenty gives twenty-two.

Answer: twenty-two metres per second over the ground

Practice

1. Now the waiter walks backwards along the train at one metre per second while the cup still slides forward at one. The train runs at twenty. How fast does the cup move over the ground?

Hint: Forward motions add, backward ones take away.

Answer: Twenty metres per second: one forward and one backward cancel, leaving the train's twenty.

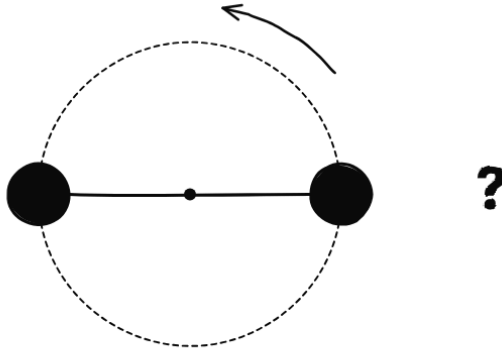
2. In your own words, why does the hollow surface of stirred tea show a true spinning rather than a relative one?

Hint: Ask whether the hollow depends on what you compare the tea with.

Answer: The hollow is the strain away from the axis, which only a true spinning produces; a merely relative circling has no such effect. It appears whether you watch from the table or a moving train, so it does not depend on any comparison.

Two Globes in an Empty Sky

Picture two heavy balls tied together by a strong cord, spinning around their common middle far out in empty space, with nothing else anywhere to look at. Could anyone tell that they are spinning? Newton says yes, and this last part shows how.



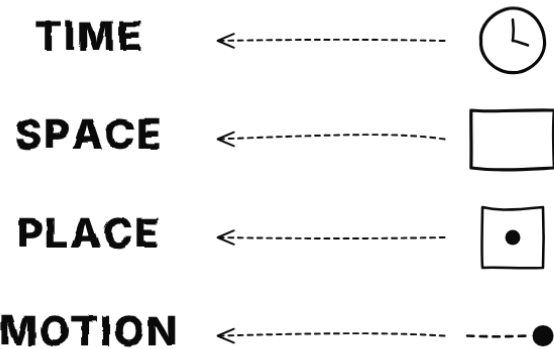
EMPTY SPACE

Two Globes in Empty Space

First, a point about words. The relative quantities, our hours, our measured places, our speeds against the ground, are not the true quantities they are named after, but the visible measures we commonly use instead, accurate or not. If words mean what everyday

use makes them mean, then time, space, place and motion name those measures, and using them for the true quantities is unusual and purely mathematical. Newton warns that confusing real quantities with their relations and everyday measures spoils the purity of mathematical and philosophical truth, and that reading the true quantities into scripture strains it.

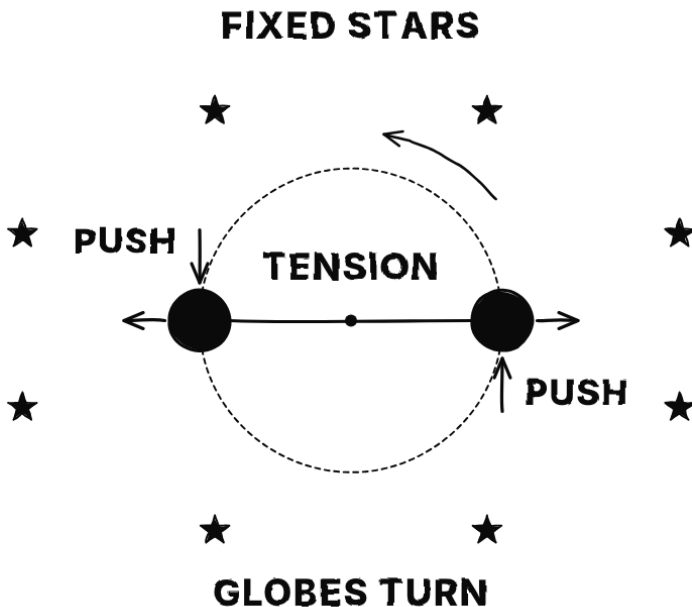
QUANTITIES ≠ MEASURES



Quantities and Their Measures

Finding the true motions of particular bodies is very hard, he admits, because the parts of unmoving space can never be seen. But it is not hopeless, because there are two kinds of clue: the apparent motions, which are the differences between true motions, and the forces, which are their causes and effects.

Back to the two balls. The cord between them is pulled tight by their outward strain, the effort of anything truly spinning to move away from its axis. So from how tight the cord is, we can tell how much the balls are circling. Now press equal pushes at the same moment on matching faces of the two balls: if the cord grows tighter, those pushes sped the balls up; if it grows slacker, they slowed them down. That tells which faces trail behind and which lead, and so which way the balls are turning, even in an immense empty space with nothing to compare them with. A phone's gyroscope does something similar: it senses its own true turning from forces inside the chip, which is how your screen knows you rotated it, even in a dark room.



Tension, Pushes and the Stars

Now place some far-off bodies around them that always keep the same positions towards one another, as the fixed stars do. Watching the balls move against those bodies, we could not tell whether the balls were turning or the far bodies were circling round them. But if the cord's tightness is just what the balls' spinning requires, we may conclude that the balls move and the far bodies are at rest; and then, from the balls' passage among them, we can find which way they turn.

How to gather true motions from their causes, effects and apparent differences, and how, the other way round, to learn the causes and effects from the motions, is what the rest of the book explains; for that, Newton says, he wrote it. With matter, motion and force defined, and true motion separated from appearance, the laws of motion can begin.

Remember

Our everyday hours, places and speeds are measures, not the true quantities they are named after, and the two must not be confused. True motions are hard to find, because unmoving space cannot be seen, but apparent motions and forces give clues. Two balls on a cord in empty space reveal their spinning through the cord's tightness, and extra pushes reveal its direction. Finding true motions from causes and effects, and causes from motions, is the purpose of the whole book.

Questions

1. How can the spinning of two balls on a cord be detected in an empty sky?
 - a. By watching them pass the stars
 - b. By how tight the cord is, pulled by their outward strain
 - c. By the sound they make
 - d. It cannot be detected
2. If equal pushes on matching faces of the two balls make the cord slacker, what did the pushes do?
 - a. They sped the balls up
 - b. They slowed the balls down
 - c. They stopped the cord from turning
 - d. Nothing at all
3. What are our everyday hours and measured places, according to Newton?
 - a. The true quantities themselves
 - b. Visible measures used instead of the true quantities
 - c. Parts of unmoving space
 - d. Forces acting on bodies

Answers

- 1. b)** The cord is pulled tight by the balls' outward strain, so from how tight it is we can tell how much they are circling.
- 2. b)** If the cord grows tighter, the pushes sped the balls up; if it grows slacker, they slowed them down.
- 3. b)** The relative quantities are not the true quantities they are named after, but the visible measures we commonly use instead.

Practice

1. In your own words, how could the pushes on the two balls reveal which way they turn?

Hint: Think about which faces, when pushed, make the cord tighter.

Answer: Pushes that speed the balls up make the cord tighter. Finding the faces where a push tightens the cord shows which faces trail behind in the circling and which lead, and that gives the direction of turning.

2. Why could watching the balls against the fixed stars alone not tell whether the balls or the stars move?

Hint: Think about what a change of relative position shows.

Answer: A change in relative position looks the same whether the balls turn or the stars circle round them. Only the cord's tightness, matching what the balls' spinning requires, shows that the balls move and the stars rest.