

The Mathematical Principles of Natural Philosophy



Isaac Newton · 1687

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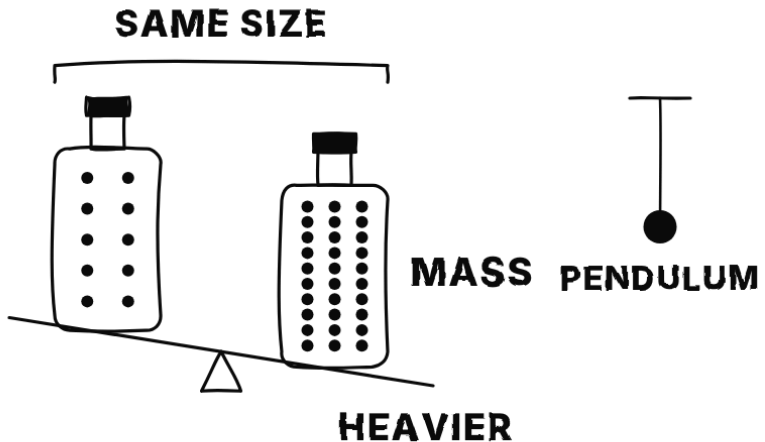
Weight, Time and Where Things Really Are

The Spinning Bucket and the Uneven Days

Finding True Motion

Stuff, Motion and the Pull to the Centre

Pick up two identical stiff water bottles, one holding air at ordinary pressure, the other pumped hard with a bicycle pump. They are the same size, yet one holds much more air than the other. So how big a thing is does not say how much stuff is in it. Newton opens his great book exactly here, with the plain question of how much matter a body has, before a single law of motion appears.

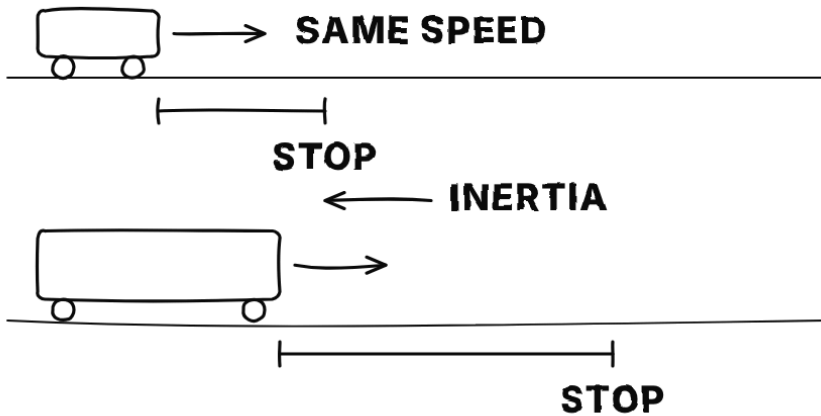


Same Size, More Stuff

His answer is that the amount of matter depends on two things together: how tightly the matter is packed, and how much room it

fills. Pack the same air tighter, or let it fill more room, and there is more of it; do both and the effects stack up. Snow pressed into ice, or flour squeezed into a cake, changes its size and its packing but not the amount of matter in it. This amount is what Newton calls mass, and he notices something surprising: it always goes together with weight. He checked it by very careful experiments with pendulums, which swing at the same pace whatever material their weights are made of. That is why the bathroom scale, which feels only how hard you press on it, can honestly tell you your mass in kilograms.

Next comes the amount of motion, which we now call momentum: it depends on how much matter is moving and how fast it moves. A body is simply the sum of its parts, so twice the matter moving at the same pace carries twice the motion, and making it faster adds still more. A loaded van rolling at the same speed as a small car carries far more motion, and that is exactly why it needs so much more road to stop.

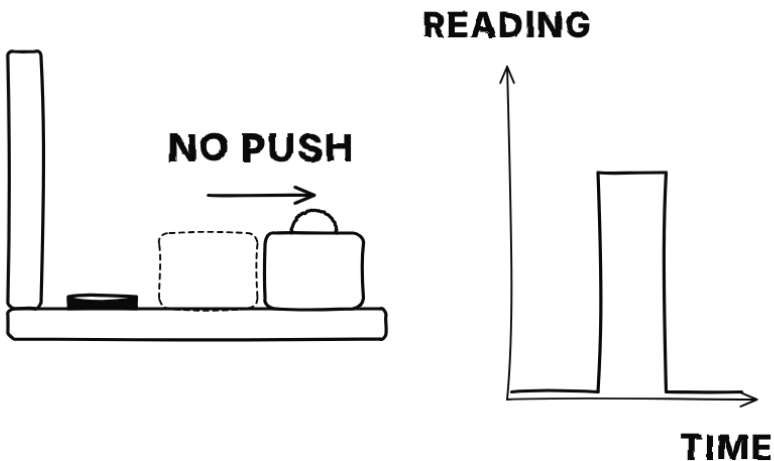


Heavy and Fast Is Hard to Stop

Then Newton gives a name to something everyone has felt: the stubbornness of matter. Every body, as far as it can, keeps doing what it is already doing, staying put if it is still and gliding straight on at a steady pace if it is moving. Newton calls it the force of inactivity, and from his Latin name for it comes our word inertia. It only shows itself when something tries to change what the body is doing; then it feels like resistance from one side and like a shove from the other. And he slips in a warning that will grow important later: whether something is "at rest" or "moving" is usually judged only by comparing it with things around it, and a thing that looks still may not truly be still.

Try it yourself

Put your phone flat on the passenger seat of a car and open any free sensor app that shows the accelerometer. On a straight road at steady speed the reading barely moves. When the driver brakes, the reading jumps and a loose bag on the seat slides forward. Nothing pushed the bag; it simply kept going while the car was made to stop, which is inertia caught in the act.



The Bag Keeps Going

A force, for Newton, is the action that changes what a body is doing, and it exists only while it acts. Once the push is over, the body does not keep any of it inside; it just carries on in its new state by inertia.

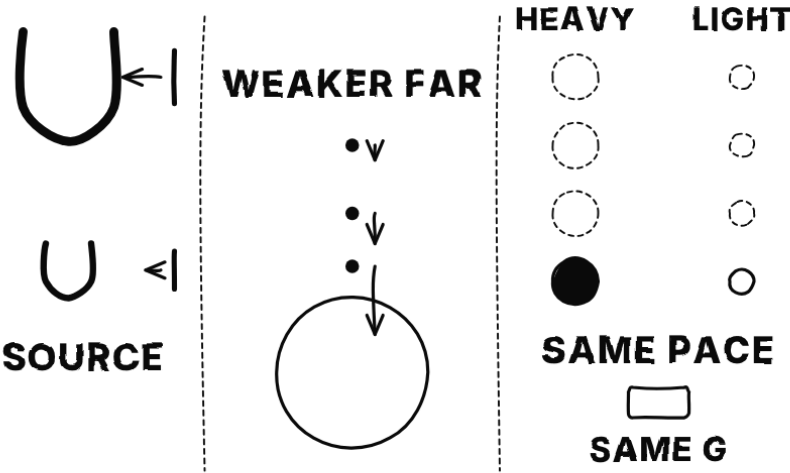
Pushes come from blows, from pressure, and from pulls towards a centre. The brakes work only while your foot presses the pedal; once the car has slowed, it keeps its new speed with nothing pressing at all.

The pull towards a centre is the force his whole book is about, and he calls it centripetal, centre-seeking. Gravity pulls things towards the middle of the earth, a magnet pulls iron towards itself, and some force keeps bending the planets out of straight paths into their orbits. His picture is a stone whirled in a sling: it strains to fly off, the harder the faster it goes, and the sling's pull back towards the hand is what keeps it going round. Let go, and the stone shoots off in a straight line, as every orbiting body would without such a pull.

Then comes his most famous image: a cannon on a mountain top, firing a ball level with the ground. Fire it faster and it lands farther away; faster still and the curve of its fall matches the curve of the earth, so it never lands and keeps falling around the planet. The moon is held the same way, by a pull of exactly the right strength: too weak and it would drift off, too strong and it would fall in. Every GPS satellite your phone listens to is Newton's cannonball, moving so fast sideways that it falls around the earth forever. Newton declares that the job of mathematics is to find the pull from the path, and the path from the pull, and that is what the rest of the book does.

Finally he begins to measure such pulls in more than one way. One way looks at the source: a big, strong magnet has more pulling power than a small, weak one. Another looks at how quickly the pull speeds things up where they are: a magnet pulls harder up close than far away, and gravity is a little stronger in a valley than on a very high mountain, and weaker still far out in space. Yet at the same distance from the earth's centre, gravity speeds up everything equally, heavy or

light, big or small, once the air is taken out of the way. The same accelerometer that caught the bag sliding reads the same pull of gravity whether the phone sits alone or under a heavy book.



Measuring a Pull

Remember

Newton starts by counting matter, by how tightly packed and how much room together, and calls that amount mass, which always matches weight. Motion counts matter and speed together, which is why heavy, fast things are so hard to stop. Inertia is the stubbornness of matter, and a force is only the action that changes it, lasting while it acts. A pull towards a centre turns straight motion into an orbit, which is how the sling holds the stone, the earth holds the moon, and satellites stay up. Such a pull can be measured by the strength of its source or by how quickly it speeds things up, and at the same distance gravity speeds up everything alike.

Questions

1. Two stiff bottles of the same size hold different amounts of air. What does this show about Newton's amount of matter?
 - a. It depends on packing and room together, not on size alone
 - b. It depends only on the size of the container
 - c. It depends only on how heavy the container is
 - d. It cannot be measured for air
2. After the brakes have slowed the car and the pedal is released, why does the car keep its new speed?
 - a. The brakes leave some force stored in the car
 - b. Gravity pulls it along at that speed

- c. It carries on in its new state by inertia
 - d. The road pushes it forward
3. What keeps the moon in its orbit, according to Newton's picture?
- a. Its inertia alone, which makes it move in circles
 - b. The air that surrounds the earth
 - c. A pull towards the earth of just the right strength
 - d. A push from the sun behind it
4. At the same distance from the earth's centre, with the air out of the way, how do a heavy and a light body fall?
- a. The heavy one speeds up faster
 - b. The light one speeds up faster
 - c. They speed up equally
 - d. Only the heavy one is pulled at all

Answers

- 1. a)** The amount of matter depends on two things together: how tightly it is packed and how much room it fills, so size alone does not tell.
- 2. c)** A force lasts only while it acts; once the push is over, the body keeps none of it and carries on in its new state by inertia.
- 3. c)** The moon is held by a pull of exactly the right strength: too weak and it would drift off, too strong and it would fall in.
- 4. c)** At the same distance from the earth's centre, gravity speeds up everything equally, heavy or light, big or small, once the air is out of the way.

Practice

1. A friend says a loaded van is harder to stop than a small car only because it is bigger. Using Newton's amount of motion, how would you correct them?

Hint: Ask what the amount of motion depends on, besides speed.

Answer: What matters is not size but the amount of matter moving, together with its speed. A loaded van at the same speed as a small car carries far more matter, so it carries far more motion, and that is why it needs so much more road to stop.

2. Why can a cannonball fired fast enough from a mountain never land, even though gravity never stops pulling it?

Hint: Think about what happens to its curved path as it goes faster.

Answer: The faster it goes, the flatter its curve of fall. Fast enough, the curve of its fall matches the curve of the earth, so it keeps falling around the planet without ever reaching the ground, which is how satellites stay up.

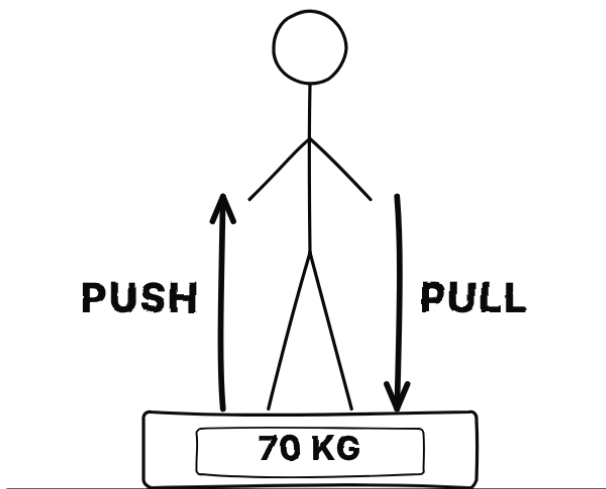
3. Newton measures a pull by the strength of its source and by how quickly it speeds things up at a place. Why are these two different measures?

Hint: Think of one magnet held near and far from a paper clip.

Answer: The same source can speed things up a lot close by and only a little far away, so how strong the source is and how quickly it speeds things up at a particular place are not the same thing; a big magnet has more pulling power than a small one, but either pulls harder up close.

Weight, Time and Where Things Really Are

Step onto a bathroom scale and watch the number settle. The scale cannot see your body; all it does is push up just hard enough to stop you sinking, and report how hard that push is. Newton defines weight in exactly this way, and it is the last of his three ways of measuring a pull.

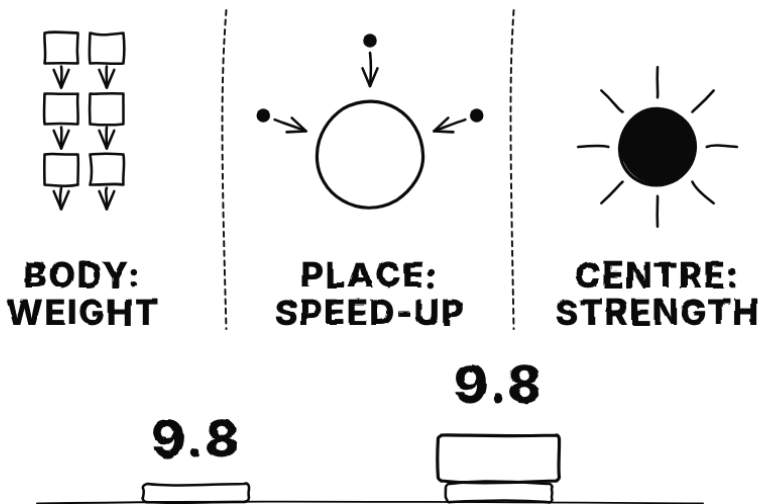


The Scale Pushes Back

We have already met two: the strength of the source, as a big magnet beats a small one, and how quickly the pull speeds things up at a given

spot. The third looks at a whole body and asks how much motion the pull gives it, and Newton says this is simply its weight, the whole body's leaning towards the centre. A bigger body weighs more, and the same body weighs more close to the earth and less far away. And weight is always known by the equal push in the opposite direction that just stops the body from falling, which is the scale's job. Your scale pushes up on you exactly as hard as the earth pulls you down, and that balance is the number it shows.

Newton gives each measure a home. Weight belongs to the body, as the sum of the pulls on all its little parts. The speed-up belongs to the place, like a power spread out from the centre to every spot around it. The strength belongs to the centre itself, to whatever causes the pull: a magnet, the earth, or something nobody has found yet. He is careful to say that he only gives these pulls a mathematical meaning; what they really are, and how they act, he leaves open.

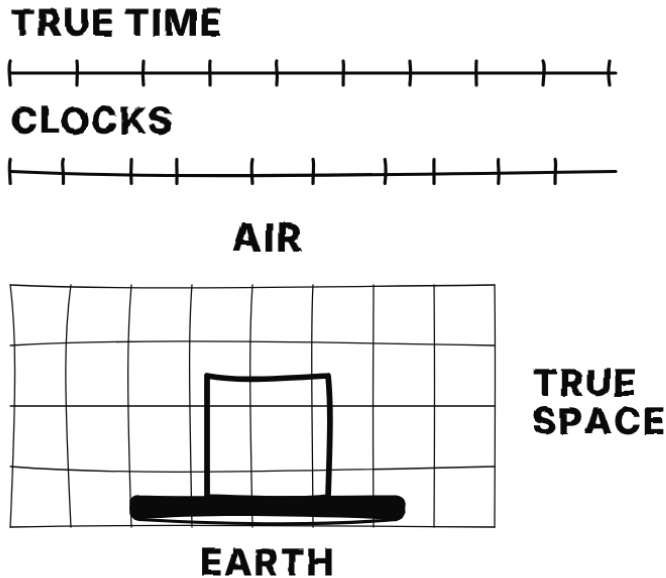


Three Homes for a Pull

The three fit together neatly. Just as a body's motion is its speed applied to all of its matter, its weight is the local speed-up applied to all of its matter. So close to the earth, where the speed-up is the same for everything, weight simply follows the amount of matter. Climb high enough that gravity's speed-up weakens, and every body's weight weakens with it, while its matter stays exactly the same. Your phone's accelerometer shows this sameness every day: lying on a table, it reads the same pull of gravity whether it is bare or sitting under a heavy book.

Here the definitions end, and Newton adds a long explanatory note about four words he refuses to define, because everybody thinks they know them: time, space, place and motion. The trouble, he says, is

that people know them only through the things they see and touch, and that breeds false ideas. So he splits each one in two: a true, absolute version, and a relative version we can actually measure.

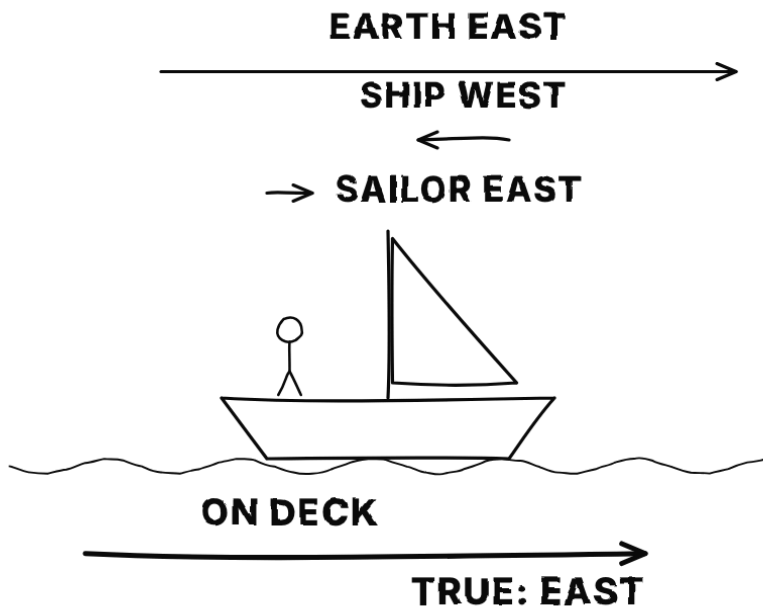


True and Measured Time and Space

True time flows evenly all by itself, whatever happens in the world, and its other name is duration. The time we use, hours, days, months and years, is only a measure of that flow made by some motion we can watch, the turning earth or the swinging pendulum, and such measures can be slightly uneven. Your phone's clock is relative time in exactly this sense: it counts the vibrations of a tiny crystal and is kept in step by the satellites' clocks, but it only measures time, it is not time itself.

True space stays the same everywhere and never moves. The space we measure, the room, the air above the town, the sky as seen from here, is fixed by its position among bodies, so it can move along with them. If the earth moves, the patch of air that always surrounds us is, in true space, passing through one place after another. A body's place is the part of space it fills, not merely its outer surface or its position, and the place of a whole is the sum of the places of its parts.

Motion splits the same way. True motion is a body's passage from one true place to another; relative motion is its passage from one place to another among the things around it. Newton's picture is a ship under sail: a passenger sitting still in a cabin is at rest relative to the ship, yet if the ship moves, the passenger moves with it. And if the earth itself is moving, the passenger's true motion is made up of the earth's motion, the ship's sailing over the earth, and any walking the passenger does on deck, all added together. His example has the earth carrying the ship eastward very fast, the ship sailing slowly west, and a sailor strolling a little to the east, so that the sailor, who seems to be drifting west compared with the shore, is really rushing east through space. Your phone tells you how fast you walk along a moving train only relative to the ground; your speed through true space would add the earth's own enormous motion too.



The Ship and the Sailor

Remember

Weight is the third measure of a pull: the motion it gives a whole body, known by the push that stops the body falling, as a bathroom scale does. Weight belongs to the body, the speed-up to the place, and the strength to the centre, and Newton gives them only a mathematical meaning. Near the earth the speed-up is the same for everything, so weight follows the amount of matter. True time flows evenly on its own, and clocks only measure it; true space never moves, while the spaces we measure travel with the bodies around us. A body at rest on a sailing ship is still moving through true space, and its true motion adds up the motions of the earth, the ship and the body itself.

Questions

1. How does a bathroom scale know your weight, in Newton's sense?
 - a. It sees how much matter you contain
 - b. It pushes up just hard enough to stop you falling, and that push equals your weight
 - c. It measures how fast you are moving
 - d. It measures your height
2. To what does Newton assign the speed-up of a pull?
 - a. To the body being pulled
 - b. To the place where the body is

- c. To the source at the centre
 - d. To the observer
3. Why is your phone's clock only relative time?
- a. Because it shows the wrong hour
 - b. Because it measures time by counting a motion, which only measures true time
 - c. Because true time stops when the phone is off
 - d. Because it is set by the satellites
4. A passenger sits still in the cabin of a sailing ship. What is true of their motion?
- a. They are truly at rest
 - b. They are at rest relative to the ship but move with it
 - c. They move only if they walk
 - d. They move only relative to the cabin

Answers

- 1. b)** Weight is always known by the equal push in the opposite direction that just stops the body from falling, which is what the scale does.
- 2. b)** The speed-up belongs to the place, like a power spread out from the centre to every spot around it.
- 3. b)** The time we use is only a measure of the true flow made by some motion we can watch; the phone counts a crystal's vibrations, so it measures time but is not time itself.
- 4. b)** A passenger sitting still is at rest relative to the ship, yet if the ship moves, the passenger moves with it.

Practice

1. Explain why, close to the earth, a body's weight follows its amount of matter.

Hint: Ask what is the same for every body at one place.

Answer: Weight is the local speed-up applied to all of a body's matter. Close to the earth the speed-up is the same for everything, so the only thing that differs from body to body is the amount of matter, and weight follows it.

2. Why might a sailor who seems to drift west compared with the shore really be rushing east through space?

Hint: Think about everything that carries the sailor.

Answer: The sailor's true motion adds up the earth's motion, the ship's motion over the earth and the sailor's own walking. If the earth carries everything east very fast, that can outweigh the slow westward drift seen from the shore.

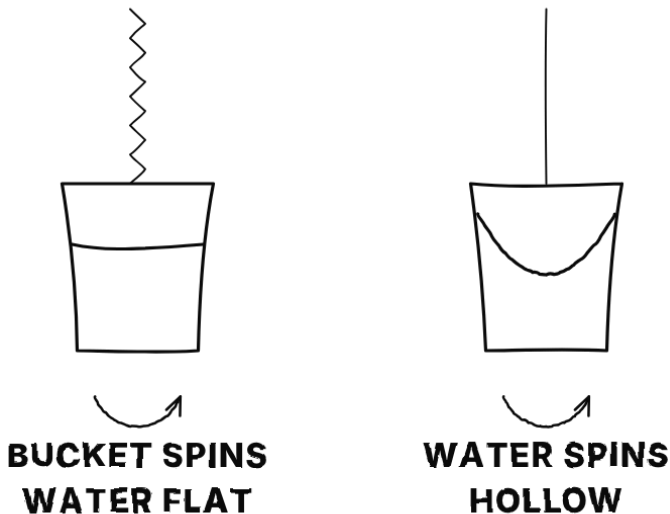
3. Why does Newton say the patch of air around us may be changing in true space, though it seems to stay put?

Hint: Ask what that patch of air is measured from.

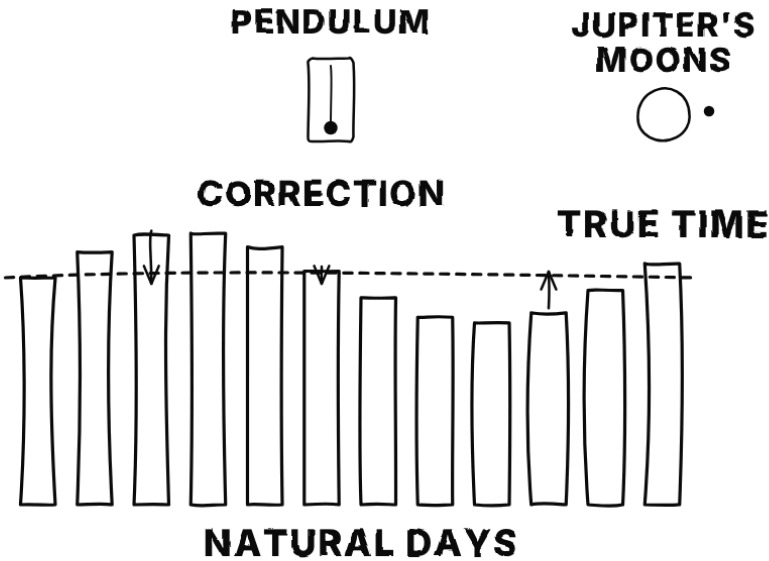
Answer: We fix that patch of air by its position relative to the earth. If the earth moves, the air moves with it, so in true space it passes through one place after another, even though relative to us it stays the same.

The Spinning Bucket and the Uneven Days

Hang a small bucket of water from a long string, twist the string many times, hold the bucket still, then let it go. As the string unwinds, the bucket spins faster and faster, yet at first the water inside stays flat and calm. Slowly the water starts to turn with the bucket, its middle sinks, and it climbs the sides into a hollow, until bucket and water spin together. Newton did this himself, and it is his best evidence that some motion is real and not just a matter of what you compare it with.

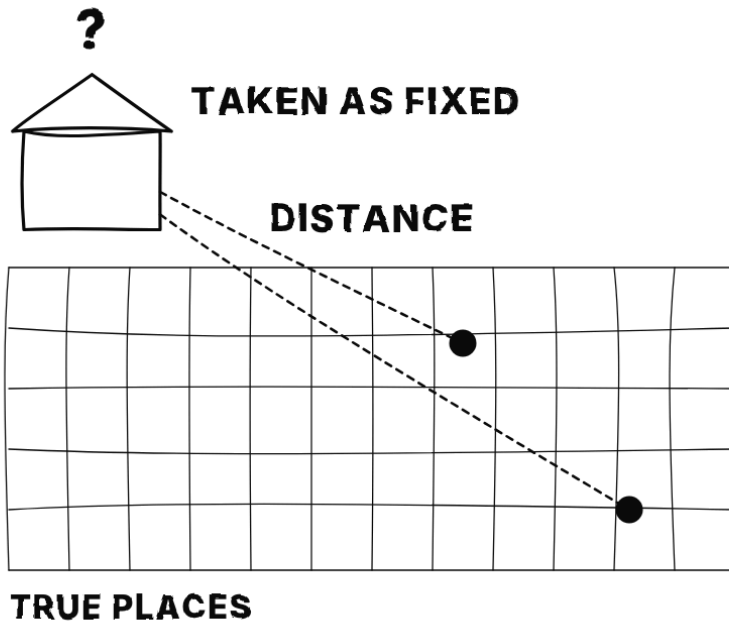


Before the bucket, he deals with time. Astronomers, he says, separate true time from everyday time by a correction, because the natural days, from noon to noon by the sun, are not all the same length, though everyone treats them as equal. Perhaps no motion anywhere is perfectly even, so that time can never be measured exactly; every motion can speed up or slow down, but true time flows on unchanged. Things last just as long whether the motions around them are fast, slow, or absent, so lasting must be kept apart from the clocks that measure it. Pendulum clocks and the timing of the moons of Jupiter both show that the correction is needed. A sundial in your garden drifts by about a quarter of an hour either way against your phone's steady clock through the year, which is that same unevenness of the days.



Natural Days Are Unequal

Space is like time: its parts keep their order forever, and moving a part of space out of its place would be moving it out of itself. Those unmoving parts are the true places, and passing out of them is the only true motion. But we cannot see the parts of space, so we measure from some body we treat as still, the ground, the shore, the station. That is perfectly fine for daily life, but in natural philosophy we must look past our senses, because there may be no body anywhere truly at rest to measure from. The map on your phone measures every position from the ground, taken as fixed, which is exactly right for finding a café and says nothing about how the earth itself moves.

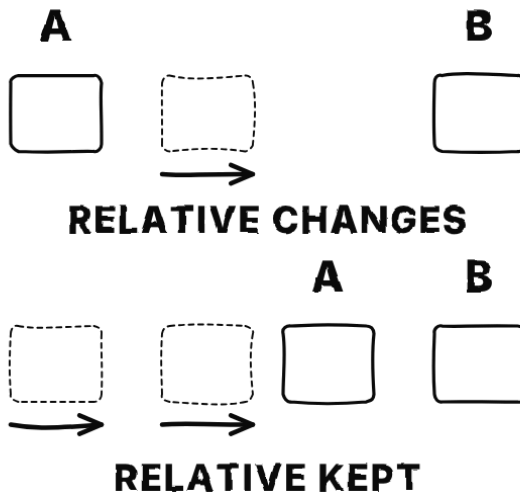


Places Measured From a Body

Newton then promises to tell true from apparent motion in three ways: by their properties, their causes and their effects. One property

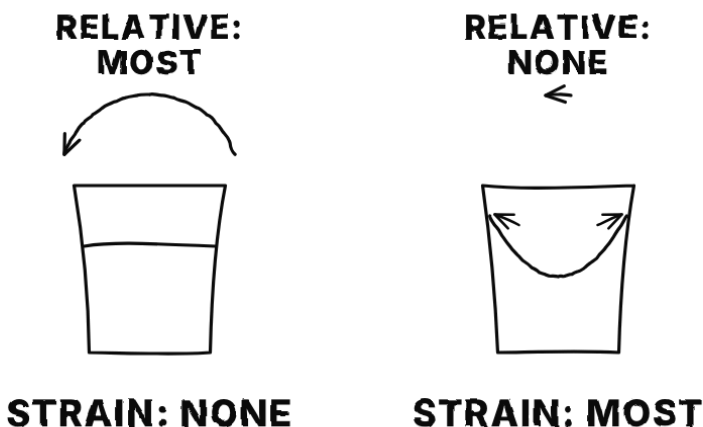
is that things truly at rest stay put with respect to one another; but a truly resting body might sit far off among the stars, and nothing about how nearby things sit relative to each other can tell whether they keep still towards it. Another property is that parts share the motion of their whole: if the shell moves, the kernel moves with it, without ever leaving the shell. So resting among moving things is not true rest, and true motion is built up, layer by layer, from a body's motion in its place, that place's motion, and so on, down to a place that does not move. A phone lying still in your pocket on a moving bus never shifts in the pocket, yet travels across town with the bus.

The cause that separates true from relative motion is force. True motion only changes when a force acts on the body itself. Relative motion can change with no force on the body at all, simply because the things it is compared with are pushed away; and it can stay the same while the body is pushed, if the things around it are pushed in the same way. When the bus brakes, you and the phone in your pocket slow down together, so the phone seems not to move relative to you, though its true motion has clearly changed. So true motion cannot consist in relations to other things.



Push the Other Body

And the effect that gives true motion away is the outward strain of spinning. A merely relative circling produces no outward strain; a true spinning does, more the faster it spins. This is where the bucket comes in. At first, the water is turning fastest relative to the bucket, since the bucket is spinning around still water, yet its surface stays flat: no strain, so no true spinning yet. Later, as the water catches up and spins less relative to the bucket, it climbs the sides more and more, until it rides highest exactly when it is at rest relative to the bucket. The strain follows the water's true spinning, not its motion relative to the bucket around it.



Strain Follows True Spinning

Try it yourself

Stir a cup of tea briskly and lift the spoon out. While the tea spins, its surface dips in the middle and rises at the rim. That hollow is the outward strain of a true spinning, the same thing Newton saw in his bucket.

Every spinning body has just one true spinning, with one outward strain to match, but countless relative motions, one for every body you compare it with, and those have no real effects of their own. That settles an old idea of the heavens, in which planets rest inside turning crystal spheres that carry them round: such planets would truly move,

because they change their positions towards one another, which truly resting things never do, and as parts of a spinning whole they would strain outwards from its axis. The spin cycle of a washing machine shows the same strain every week: the drum's true spinning pins the clothes against its wall.

Remember

Natural days are uneven, so astronomers correct them, and true time flows on unchanged whatever clocks do. We measure places from bodies we treat as still, but nearby positions can never reveal true rest, and parts always share the motion of their whole. Only a force on a body changes its true motion, while relative motion can change or stay the same regardless, so true motion is not a relation to other things. A true spinning shows itself by an outward strain, which is why the water in Newton's bucket climbs the sides only when it truly spins. Each spinning body has one true motion and countless relative ones, and only the true one has real effects.

Questions

1. Why do astronomers correct everyday time?
 - a. Because clocks are always slow
 - b. Because the natural days by the sun are not all the same length
 - c. Because true time speeds up in summer

- d. Because the moon changes the length of the year
2. What alone changes a body's true motion?
- a. A change in the things around it
 - b. A force acting on the body itself
 - c. The passing of time
 - d. Its place among other bodies
3. In Newton's bucket, when does the water climb the sides highest?
- a. When the bucket first starts spinning around still water
 - b. When the water spins along with the bucket and is at rest relative to it
 - c. When the bucket stops
 - d. Never; the surface stays flat

Answers

- 1. b)** Astronomers separate true time from everyday time by a correction, because the natural days, from noon to noon by the sun, are not all the same length.
- 2. b)** True motion only changes when a force acts on the body itself, while relative motion can change with no force on the body at all.
- 3. b)** As the water catches up and spins less relative to the bucket, it climbs higher, riding highest exactly when it is at rest relative to the bucket.

Practice

- 1.** Explain why the bucket experiment shows that true spinning is not the same as spinning relative to the bucket.

Hint: Compare the moments when the water moves most relative to the bucket and when it climbs highest.

Answer: At first the water moves fastest relative to the bucket yet stays flat, so it is not truly spinning. At the end it does not move relative to the bucket at all, yet it climbs highest. The outward strain follows the water's true spinning, not its motion relative to the bucket.

2. Why can resting quietly among other things never prove that a body is truly at rest?

Hint: Think of a phone in your pocket on a bus.

Answer: Parts share the motion of the whole they belong to, as a kernel moves with its shell. A phone resting in your pocket never moves relative to the pocket, yet it is carried across town by the bus, so resting among things says nothing about true rest.

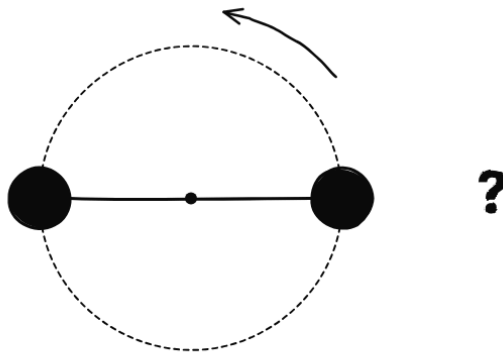
3. What did the old picture of planets carried by turning heavenly spheres get wrong, according to Newton?

Hint: Ask whether such planets would show the signs of true motion.

Answer: Such planets would only be at rest relative to their spheres. They would still change their positions towards one another, which truly resting things never do, and as parts of a spinning whole they would strain outwards from its axis, so they would truly move.

Finding True Motion

Picture two heavy balls tied by a strong cord, spinning round their middle in an empty sky with nothing else in sight. Newton's last word in this chapter is that even there, we could tell they spin.

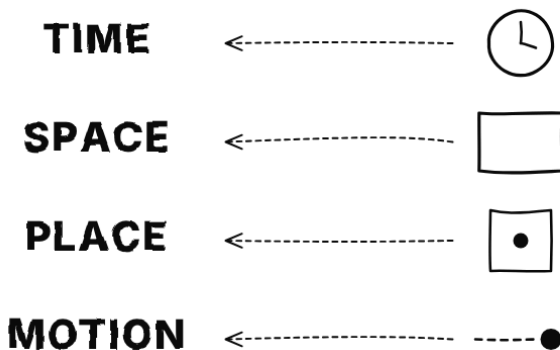


EMPTY SPACE

Two Globes in Empty Space

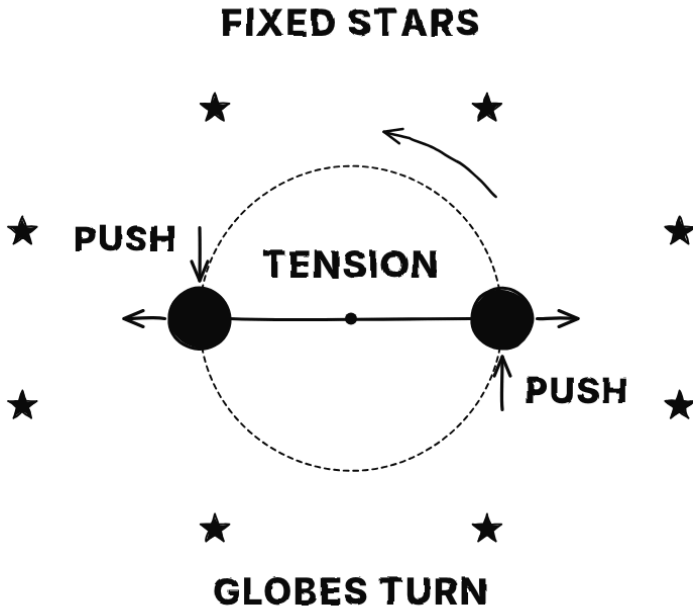
First he settles words. Our hours, measured places and speeds against the ground are not the true things they are named after, only the measures we use instead. In everyday speech, time and motion mean those measures, and that is fine, as long as we never mistake the measures for the real quantities.

QUANTITIES $\neq$ MEASURES



Quantities and Their Measures

Finding a body's true motion is hard, he admits, because unmoving space cannot be seen. But there are clues: the motions we observe, which are the differences between true motions, and the forces, which cause them and show them. The cord between the two balls is pulled tight by their outward strain, so its tightness tells how fast they spin. Push both balls at once on matching sides: if the cord tightens, the push sped them up, and that reveals which way they are turning. Your phone's gyroscope senses its own turning in just this spirit, from forces inside the chip, with no need to look at anything outside.



Tension, Pushes and the Stars

Put some far-off bodies around, keeping still towards each other like the stars, and watching alone could not tell whether the balls turn or the stars do. But if the cord's tightness matches what the balls' spinning needs, then the balls move and the stars stand still. Learning true motions from their causes and effects, and causes from motions, is what the rest of Newton's book sets out to do, and with it the laws of motion begin.

Remember

Our everyday times and places are only measures of the true quantities, and must not be mistaken for them. True motion cannot be seen directly, but observed motions and forces give it away. Two balls on a cord reveal their spinning by the cord's tightness, even with nothing around them. Finding true motions from causes and effects is the task of the whole book.

Questions

1. What reveals how fast the two balls on a cord are spinning?
 - a. How tight the cord is
 - b. The colour of the balls
 - c. The stars around them
 - d. The time of day
2. With far-off bodies around, when may we say the balls move and the bodies stand still?
 - a. When the balls look bigger
 - b. When the cord's tightness matches what the balls' spinning needs
 - c. Whenever the balls pass a star
 - d. Never
3. What does Newton say about our everyday hours and measured places?

- a. They are the true quantities
- b. They are only measures, fine to use if not mistaken for the real quantities
- c. They should never be used
- d. They are forces

Answers

- 1. **a)** The cord between the two balls is pulled tight by their outward strain, so its tightness tells how fast they spin.
- 2. **b)** If the cord's tightness matches what the balls' spinning needs, then the balls move and the stars stand still.
- 3. **b)** Our hours and measured places are not the true things, only measures; using them is fine as long as we never mistake them for the real quantities.

Practice

- 1. In your own words, how could pushing both balls reveal which way they turn?

Hint: Ask what a tighter cord tells you about the push.

Answer: If pushing matching sides makes the cord tighter, the push sped the balls up, so those sides were trailing in the spin; that tells which way the balls are turning.

- 2. Why can't true motion be seen directly, and what gives it away instead?

Hint: Think about unmoving space and about forces.

Answer: Unmoving space is invisible, so we cannot watch a body pass through it. But forces cause and show true motion, as the tightness of a cord shows that two balls are spinning.