

# The Mathematical Principles of Natural Philosophy



Isaac Newton · 1687

Illustrated audio lectures

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Isaac Newton · 1687

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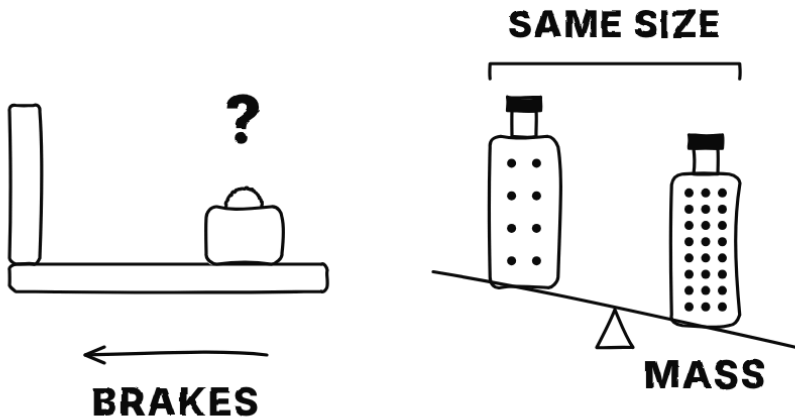
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# What Newton Needed Before His Laws

Brake hard in a car and a loose bag on the seat slides forward, though nothing pushed it. Why does it do that, and what exactly is the thing that slid? Newton's great book opens by answering questions this plain, because without clear answers his laws of motion could not even be stated. So before any law, he writes definitions: what matter is, what motion is, what a force is.



The Bag and the Bottles

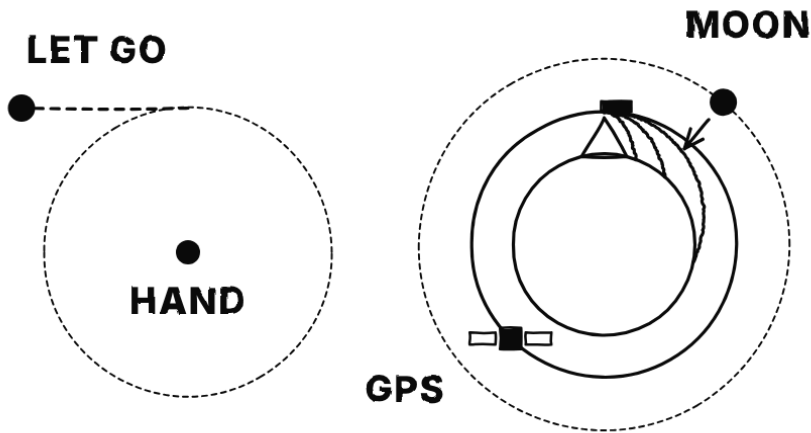
The first idea is the amount of matter, which he calls mass. It is not the size of a thing but how much stuff it holds: how tightly that stuff is

packed, together with how much room it fills. A stiff bottle pumped hard with air and one left at ordinary pressure are the same size but hold very different amounts. Newton also found, by careful pendulum experiments, that mass always goes together with weight. That is why the scale in your bathroom can honestly show your mass in kilograms.

The second idea is the amount of motion, what we now call momentum: matter and speed taken together. Twice the matter at the same speed is twice the motion, and more speed adds more. A loaded van needs far more road to stop than a small car at the same speed, because it carries far more motion.

The third idea is the one that explains the sliding bag: matter is stubborn. Every body keeps doing what it already does, staying still or moving straight ahead at a steady pace, unless something acts on it. Newton calls this the force of inactivity, and our word inertia comes from his Latin name for it. A force, in turn, is only the action that changes what a body does, and it lasts only while it acts. The bag slid because the car was made to stop and the bag was not: inertia, caught by the brakes and by the accelerometer in your phone at the same moment.

The fourth idea is the force the whole book is about: a pull towards a centre, which he calls centripetal. Gravity, magnetism, and whatever holds the planets on their curved paths are all of this kind. A stone in a sling strains to fly off and the sling holds it in; a cannonball fired fast enough from a mountain would fall around the earth forever instead of landing; and the moon is held on its path by a pull of just the right strength. Every GPS satellite overhead is that cannonball, falling around the earth without ever reaching it.



Pulled Toward a Centre

From these few ideas the rest of the book follows. Its task, Newton says, is to find the pull from the path a body takes, and the path from the pull it feels. He also begins to measure pulls in two ways, by the strength of their source and by how fast they speed things up, and he notes that at the same distance from the earth, gravity speeds up everything equally, heavy or light. This part leaves out his worked numbers, such as the ball carried two miles and then ten times as far, and the fine points of his measures of force, which the more detailed versions of this chapter go through.

## Remember

Newton begins with definitions because laws of motion need clear words for matter, motion and force. Mass is how much stuff a body holds, and it matches weight; momentum is matter and speed together. Inertia is the stubbornness of matter, and a force is only the action that changes it. A pull towards a centre turns straight motion into an orbit, which is how the moon and every satellite stay up, and finding paths from pulls and pulls from paths is the work of the whole book.

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## Questions

1. Why does a loose bag slide forward when a car brakes hard?
  - a. Something in the car pushes it forward
  - b. It keeps moving while the car is made to stop
  - c. Gravity pulls it towards the front
  - d. The brakes give it extra speed
2. What does Newton's amount of matter, mass, measure?
  - a. How big a thing is
  - b. How fast it is moving
  - c. How much stuff it holds, packing and room together
  - d. How hard it presses on the ground at any height
3. What does a pull towards a centre do to a moving body?
  - a. It turns straight motion into an orbit

- b. It stops the body at the centre at once
- c. It makes the body move in a straight line
- d. It changes the body's mass

## Answers

1. **b)** The bag slid because the car was made to stop and the bag was not: matter is stubborn and keeps doing what it already does.
2. **c)** Mass is not the size of a thing but how much stuff it holds: how tightly that stuff is packed, together with how much room it fills.
3. **a)** A pull towards a centre turns straight motion into an orbit, which is how the moon and every satellite stay up.

## Practice

1. Explain in your own words why Newton wrote definitions before any law of motion.

**Hint:** Think about what a law of motion has to talk about.

**Answer:** A law of motion speaks about matter, motion and force, so those words need clear meanings first. Newton fixes them in his definitions: mass is how much stuff a body holds, momentum is matter and speed together, inertia is matter's stubbornness, and a force is the action that changes what a body does.

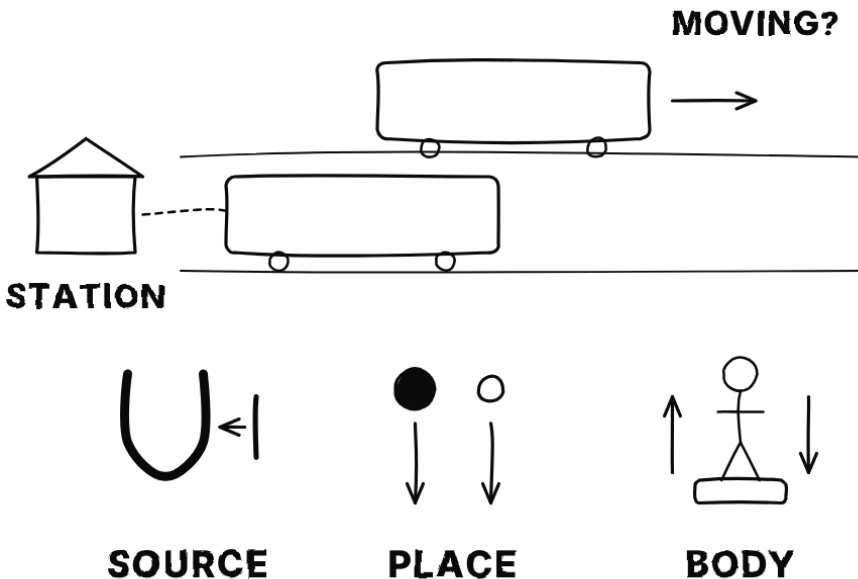
2. Why does a loaded van need more road to stop than a small car going at the same speed?

**Hint:** Use the amount of motion: what goes into it?

**Answer:** The amount of motion counts matter and speed together. At the same speed, the loaded van has much more matter, so it carries much more motion, and more motion takes longer to take away.

# Measuring Pulls, and the World Behind the Clock

Sit in a train that is waiting at a platform, and watch the train on the next track pull away. For a moment you are sure that you are the one moving, until you glance at the station building and see that you have not moved at all. Our sense of motion depends entirely on what we compare ourselves with. This stretch of Newton's book asks two questions: how exactly to measure a pull, and what it means for something to move at all.

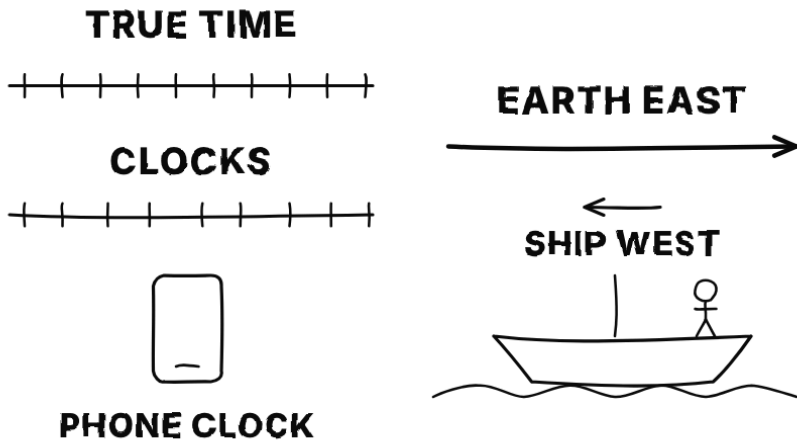


Who Is Moving? How Strong a Pull?

On the first question, Newton separates three ways of measuring the same pull. One looks at the source, as a big magnet is stronger than a small one. One looks at a place, at how quickly the pull speeds things up there, which is the same for every body at the same height above the earth. And one looks at a whole body, at how much motion the pull gives it, which is simply the body's weight. Weight is the speed-up applied to all of a body's matter, so near the earth, where the speed-up is the same for everyone, weight just follows the amount of matter. That is why a bathroom scale works: it pushes up exactly as hard as the earth pulls you, and the earth pulls each kilogram of you the same.

Newton insists that these are mathematical measures only. He does not claim to know what gravity really is or how it acts at a distance, and he warns readers not to read any such claim into his words.

On the second question, he refuses to define time, space, place and motion, which everyone thinks they know, but he warns that people know them only through the things they see, and that leads them astray. So he splits each into a true version and a measured one. True time flows evenly on its own; hours and days are only motions we use to measure it, and they can be slightly uneven. True space never moves; the spaces we measure, a room, the air, the sky above a town, are fixed by bodies and can move along with them. Your phone's clock counts the vibrations of a tiny crystal: it measures time, but it is not time itself.



True and Measured

The same split applies to motion, and this is where the train comes in. True motion is passage from one place in true space to another; relative motion is passage measured against the things around us. Newton's example is a ship: a passenger sitting still in the cabin is at rest relative to the ship, yet moves with it, and if the earth also moves, the passenger's true motion adds the earth's motion, the ship's and the passenger's own. A sailor who seems to drift slowly west compared with the shore may really be rushing east through space, carried by the turning earth. The speed your phone shows while you walk along a moving train is only measured against the ground; the earth's own huge motion is never in the number.

What follows from all this is the programme of the book: to measure forces precisely, and to find true motions behind the relative ones we observe. This part leaves out Newton's worked numbers, such as the weights of bodies where gravity is halved and the exact speeds in the ship example, and his remarks on place as the sum of the places of a body's parts; the more detailed versions of this chapter go through them.

### **Remember**

A pull can be measured by the strength of its source, by how fast it speeds things up at a place, or by the weight it gives a whole body. Weight is the local speed-up applied to all of a body's matter, so near the earth it simply follows the amount of matter. Newton separates true time and true space, which do not change or move, from the clocks and spaces we use to measure them. A body resting on a moving ship, or on a moving earth, is truly moving, and finding true motion behind the relative motion we see is the task ahead.

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## **Questions**

1. Why do you sometimes feel your train is moving when it is the train beside you that leaves?
  - a. Because our sense of motion depends on what we compare ourselves with
  - b. Because trains vibrate when standing

- c. Because the station is moving
  - d. Because true time slows down
2. Which of Newton's three measures of a pull is the body's weight?
- a. The strength of the source
  - b. How fast the pull speeds things up at a place
  - c. How much motion the pull gives the whole body
  - d. The distance from the centre
3. A passenger rests in the cabin of a moving ship on a moving earth. What does Newton conclude?
- a. The passenger is truly at rest
  - b. The passenger is truly moving, sharing the motions of ship and earth
  - c. The passenger moves only relative to the cabin
  - d. Nothing can be said about the passenger's motion

## Answers

1. a) Our sense of motion depends entirely on what we compare ourselves with, and the other train was the only thing in view.
2. c) One measure looks at a whole body, at how much motion the pull gives it, which is simply the body's weight.
3. b) A body resting on a moving ship, or on a moving earth, is truly moving, and its true motion adds the earth's motion, the ship's and its own.

## Practice

**1.** In your own words, what is the difference between true time and the time your phone shows?

**Hint:** Ask what the phone actually counts.

**Answer:** True time flows evenly on its own, whatever happens. The phone counts the vibrations of a tiny crystal, a motion, and uses it to measure that flow; it measures time but is not time itself, and any such measure may be slightly uneven.

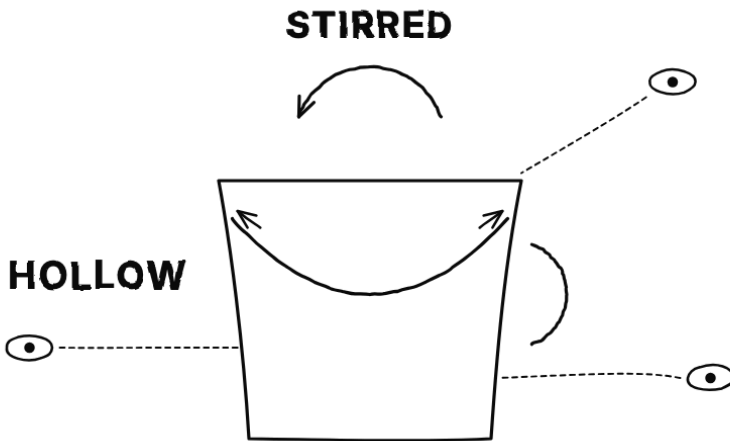
**2.** Why does Newton say these pulls are mathematical measures only?

**Hint:** Think about what he does and does not claim to know about gravity.

**Answer:** He measures pulls by their source, their speed-up at a place and the weight they give a body, but he does not claim to know what gravity really is or how it acts at a distance, so he presents only the measures, not the physical causes.

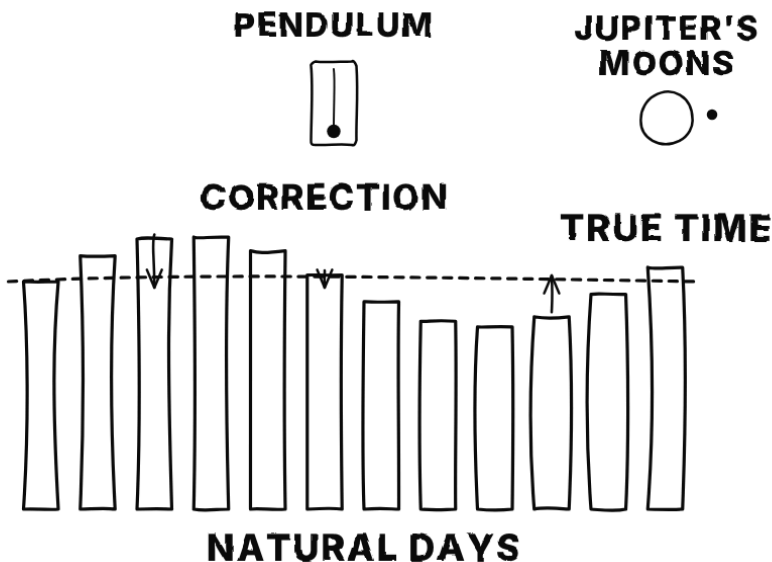
# How to Tell That Something Really Moves

Stir a cup of tea briskly and lift the spoon out. The surface dips in the middle and rises at the rim. It looks the same whether you watch from your chair, from across the room, or from the window of a passing train. That hollow surface is not a matter of point of view; it is a real effect of a real spinning. This stretch of Newton's book asks a question that sounds almost impossible: since we can only ever see things moving relative to other things, how can we know what truly moves? The hollow surface of a spinning liquid turns out to be his answer.



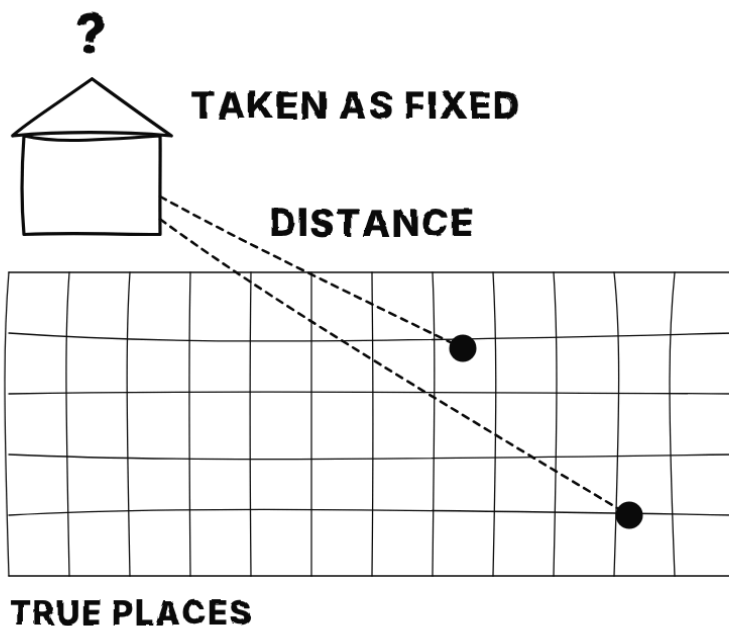
Stirred Tea

Newton begins with time, because time has the same problem. We measure time by days, by the sun going from noon to noon, and everybody treats those days as equal. But they are not quite equal, and astronomers have to correct them before they can calculate the motions of the planets accurately. Newton goes further: perhaps no motion anywhere is perfectly even, so that no clock can measure time exactly. Every motion can speed up or slow down, but true time itself flows on unchanged, and things last just as long whether the motions around them are fast, slow, or absent. So lasting must be kept apart from the clocks that measure it, and astronomers recover it from the uneven days by their correction, which pendulum clocks and the timing of the moons of Jupiter show to be necessary. A sundial in a garden drifts by about a quarter of an hour either way against your phone's steady clock over the year: that drift is the very unevenness Newton means.



Natural Days Are Unequal

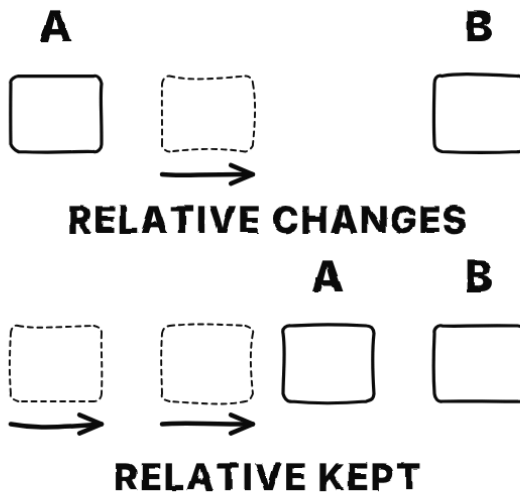
Space comes next. The parts of space, like the parts of time, keep their order forever; they are the true places of everything, and passing from one of them to another is true motion. The trouble is that we cannot see the parts of space or tell them apart. So in practice we pick some body that we treat as standing still, the ground, the shore, a station, and measure every place and every motion from it. For ordinary life that is perfectly good. But Newton warns that in understanding nature we must look past our senses, because there may be no body anywhere that is truly at rest to measure from. The map on your phone measures everything from the ground under your feet as if it were fixed, which is exactly right for finding your way and says nothing about how the earth itself is moving.



Places Measured From a Body

How then can true rest or true motion ever be recognised? Newton promises three kinds of clue: properties, causes and effects. The properties turn out to be disappointing. Things truly at rest stay put with respect to each other, but a truly resting body might sit somewhere far among the stars, and nothing about how nearby things sit relative to each other can tell whether they keep still towards it. And parts always share the motion of the whole they belong to: if a shell moves, its kernel moves with it without ever leaving the shell. So resting quietly among other things proves nothing about true rest. A phone lying still in your pocket on a moving bus never shifts inside the pocket, yet it is carried across town.

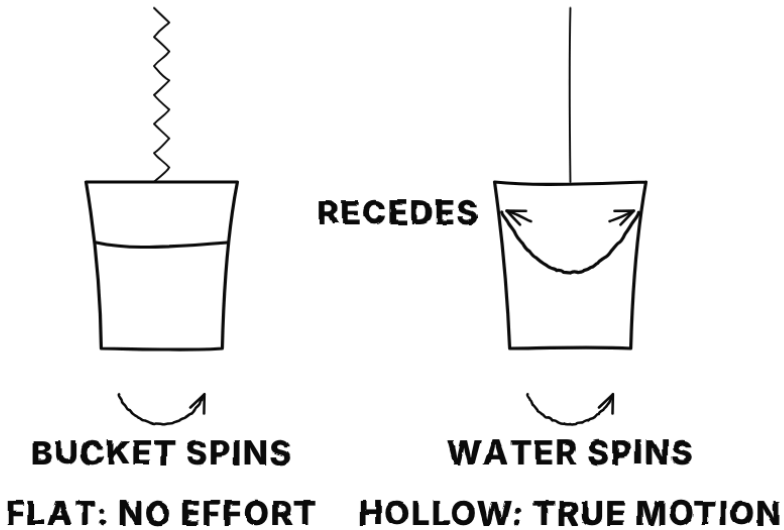
The causes are more helpful. True motion only ever changes when a force acts on the body itself. Relative motion is different: it can change with no force on the body at all, simply because the things it is compared with are pushed away, and it can stay exactly the same while the body is pushed, if the things around it are pushed along with it. When the bus brakes hard, you and the phone in your pocket slow down together, so the phone does not move at all relative to you, though its true motion has plainly changed. So true motion cannot be the same thing as a relation to other bodies.



Push the Other Body

The effects are the decisive clue, and here Newton describes an experiment he did himself. He hung a bucket of water from a long cord, twisted the cord many times, held the bucket still, and then let it

go. As the cord unwound, the bucket spun faster and faster, but at first the water inside stayed perfectly flat, though the bucket was turning around it. Then, little by little, the water caught the motion, began to turn, sank in the middle and climbed up the sides, higher and higher, until water and bucket spun together.



Newton's Bucket

Look at what this means. At the start, the water was moving fastest relative to the bucket, yet it showed no sign of spinning at all. At the end, the water was not moving at all relative to the bucket, yet it climbed the sides highest. The climbing, the outward strain away from the axis, follows the water's true spinning and has nothing to do with its motion relative to the bucket around it. So here is a motion that reveals itself through a real effect, whatever we compare it with.

### **Try it yourself**

Fill a glass jar halfway with water and drop in a few grains of rice. Spin the jar steadily on a turntable or a lazy Susan, if you have one, and watch from the side. At first the water stays flat while the jar turns; after a while it spins along with the jar and its surface becomes a hollow bowl. You have repeated Newton's bucket: the hollow appears with the water's own spinning, not with its motion past the glass.

What follows from this is a whole way of thinking about motion. Every spinning body has only one true spinning, with one real outward strain to match it. But it has countless relative motions, one for every other body you might compare it with, and those relative motions have no real effects of their own. Newton even uses this to settle an old picture of the heavens, in which planets rest inside turning crystal spheres that carry them round: such planets would truly move, because they change their positions towards one another, which truly resting things never do, and as parts of a spinning whole they would strain outwards from its axis like the water in the bucket. The spin cycle of a washing machine shows the same outward strain every week, pressing the clothes against the drum as it truly spins.

This part leaves out the finer steps of Newton's argument, such as how a true motion is built up from a body's motion in its place, that place's motion in its own place, and so on down to unmoving space, and his remarks on why the parts of time and space can never change their order; the more detailed versions of this chapter go through them.

## Remember

Days and clocks only measure time, which itself flows on unchanged, and astronomers correct the uneven days to find it. We measure every place from some body we treat as still, but nearby positions can never reveal true rest, because parts share the motion of the whole around them. True motion changes only when a force acts on the body itself, while relative motion can change or stay the same regardless. A true spinning shows itself by a real effect, the outward strain, which is why the water in Newton's bucket climbs the sides only when it truly spins. Each spinning body has one true motion with real effects among countless relative motions with none.

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## Questions

1. Why does the hollow surface of stirred tea show a real spinning rather than a point of view?
  - a. It looks the same from every place you watch it
  - b. It disappears when you move to another chair
  - c. It only appears in a moving train
  - d. It depends on the colour of the cup
2. Why does resting quietly among other things prove nothing about true rest?
  - a. Because nothing can rest
  - b. Because parts share the motion of the whole they belong to

- c. Because rest needs a force
  - d. Because clocks are uneven
3. In Newton's bucket, when did the water climb the sides highest?
- a. At the start, when it moved fastest relative to the bucket
  - b. At the end, when it was not moving relative to the bucket at all
  - c. When the bucket was held still
  - d. It never climbed

## Answers

1. **a)** The hollow looks the same from your chair, across the room or a passing train; it is a real effect of a real spinning, not a matter of point of view.
2. **b)** Parts always share the motion of the whole they belong to, as a kernel moves with its shell, so resting among other things proves nothing about true rest.
3. **b)** At the end the water was not moving at all relative to the bucket, yet it climbed highest: the climbing follows its true spinning.

## Practice

1. In your own words, why does true motion not consist in relations to other bodies?

**Hint:** Compare what a force on the body does to its true motion and to its relative motion.

**Answer:** True motion changes only when a force acts on the body itself, while relative motion can change without any force on it, just because the bodies around it move, and can stay the same when it is pushed together

with them. Since the two can change independently, true motion cannot be a relation.

**2.** Describe what you would see in a spinning glass jar of water, and what it shows.

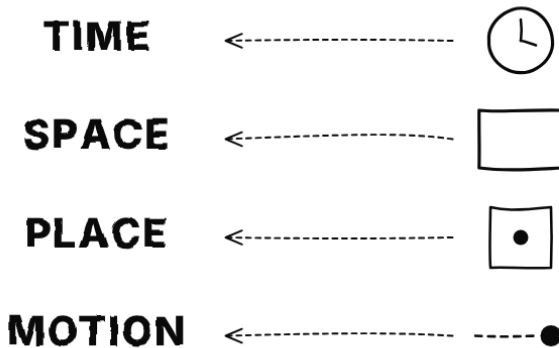
**Hint:** Think of the start and the end of the spinning.

**Answer:** At first the water stays flat while the jar turns around it; later it spins along with the jar and its surface becomes a hollow bowl. The hollow appears with the water's own spinning, not with its motion past the glass, just as in Newton's bucket.

# What Newton Wrote the Book For

Imagine two heavy balls tied by a cord, spinning round their middle in a completely empty sky. Newton ends his definitions by claiming we could still tell that they spin.

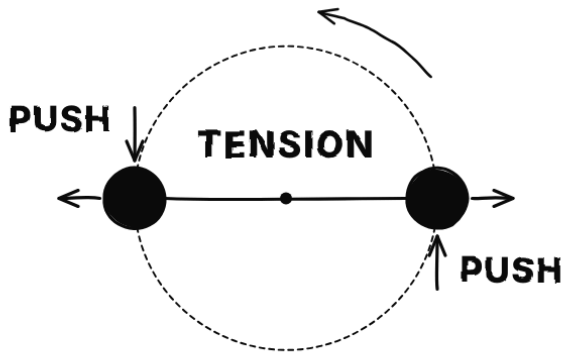
**QUANTITIES     $\neq$     MEASURES**



Quantities and Their Measures

He first settles a question of words. Our hours, our measured places, our speeds against the ground are only measures of time, space and motion, not the things themselves. In everyday talk those words mean the measures, which is fine, as long as nobody mistakes a measure for the real quantity behind it.

Then he faces the hard part: true motion can never be seen directly, because unmoving space is invisible. But it leaves clues in the motions we observe and, above all, in forces. The cord between the two balls is pulled tight by their outward strain, so its tightness reveals how fast they spin, and a push on matching sides of both balls reveals which way, even with nothing else in sight. Your phone's gyroscope works in this spirit, sensing its own turning from forces inside the chip.



## EMPTY SPACE

Two Globes on a Cord

What follows is the whole programme of the book: to work out true motions from their causes and effects, and to work out causes from the motions we see. This part leaves out the detail of how added far-off bodies, like the fixed stars, could be shown to rest while the balls move, which the more detailed versions of this chapter explain. With

matter, motion and force defined and true motion separated from appearance, Newton's laws of motion can begin.

### **Remember**

Everyday times, places and speeds are measures, not the true quantities themselves. True motion cannot be seen, but forces reveal it, as the cord's tightness reveals two spinning balls. Finding true motions from causes and effects, and causes from motions, is what the whole book is for.

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## **Questions**

1. Why can't true motion be seen directly?
  - a. Because it is too fast
  - b. Because unmoving space is invisible
  - c. Because clocks are uneven
  - d. Because only planets have true motion
2. What does the tightness of the cord between two spinning balls reveal?
  - a. How heavy the cord is
  - b. How fast the balls spin
  - c. How far away the stars are
  - d. The time of day
3. What does Newton say the whole book is for?

- a. To list the stars
- b. To work out true motions from causes and effects, and causes from observed motions
- c. To define words for scripture
- d. To build better clocks

## Answers

- 1. **b)** True motion can never be seen directly, because unmoving space is invisible; it leaves clues in observed motions and in forces.
- 2. **b)** The cord is pulled tight by the balls' outward strain, so its tightness reveals how fast they spin.
- 3. **b)** The programme of the book is to work out true motions from their causes and effects, and causes from the motions we see.

## Practice

- 1. In your own words, what is the difference between a measure, like an hour, and the true quantity it measures?

**Hint:** Ask what an hour is made of.

**Answer:** An hour is a measure made by some motion, such as a clock's ticks or the turning earth, used in place of time itself. True time flows on by itself; the hour only measures it, and the two must not be mistaken for each other.

- 2. How could two balls spinning in an empty sky be shown to spin, with nothing to compare them with?

**Hint:** Think about the cord between them.

**Answer:** Their outward strain pulls the cord tight, so the cord's tightness shows that they spin and how fast; a push on matching sides that tightens the cord shows which way.