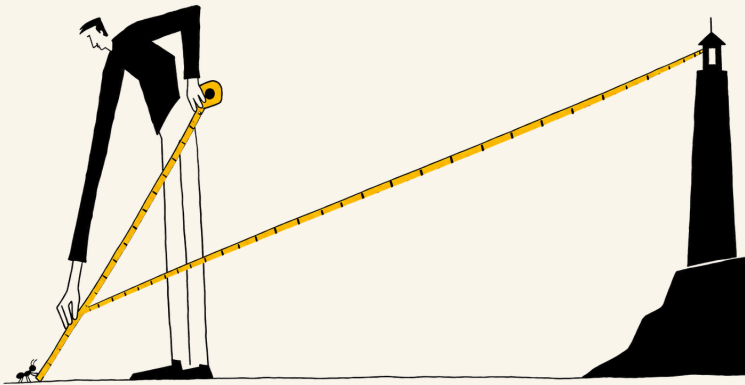


College Physics



Paul Peter Urone, Roger Hinrichs · 2022

Illustrated audio lectures

Lecturized

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Physics Is Closer Than You Think

What do you feel when you hear the word "physics"? For many people it is a small chill: pages of hard equations, formulas learned for an exam and never used again. The book begins by admitting exactly that, and then sets out to change your mind. Its claim is simple: physics plays a far larger part in your life than you think, whatever you do for a living.

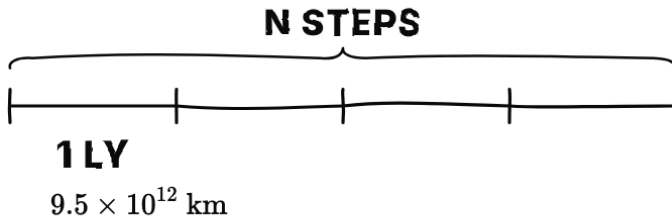


How the Veil Nebula formed

The first piece of evidence is a photograph from the sky. It shows the Veil Nebula, a vast cloud of heated dust and gas, drawn in thin twisted

ropes and wide glowing sheets. Here is how it came to be. A star about twenty times larger than our Sun exploded; such an explosion is called a supernova. The book says it happened about eight thousand years ago. The blast sent out a shock wave, and that wave is still moving, slamming into clouds of gas and dust and making them glow in those rope-like and sheet-like shapes. Today the whole cloud stretches over more than one hundred and ten light years.

That word needs a moment. A light year sounds like a stretch of time, but it is a distance: how far light travels in one year. The book gives its size: about nine and a half trillion kilometres. So a distance in light years turns into kilometres by one multiplication: the number of light years times nine and a half trillion kilometres. In symbols, $d = n \times 9.5 \times 10^{12} \text{ km}$, where d is the distance and n is the number of light years. Why does it work? Each light year is one equal step of nine and a half trillion kilometres, so n steps are n times as long, just as ten paces of one metre make ten metres. Now the calculation aloud. The Veil Nebula is about two thousand four hundred light years away. Two thousand four hundred times nine and a half trillion kilometres is about twenty-two thousand eight hundred trillion kilometres, or $2.3 \times 10^{16} \text{ km}$. Your car's speedometer may show a hundred and twenty thousand kilometres after many years of driving; you would need to drive that much about two hundred billion times to reach the nebula. And there is a second way to read the same number: the light reaching your eye from the Veil tonight set off about two thousand four hundred years ago.



$$d = n \times 9.5 \times 10^{12} \text{ km}$$

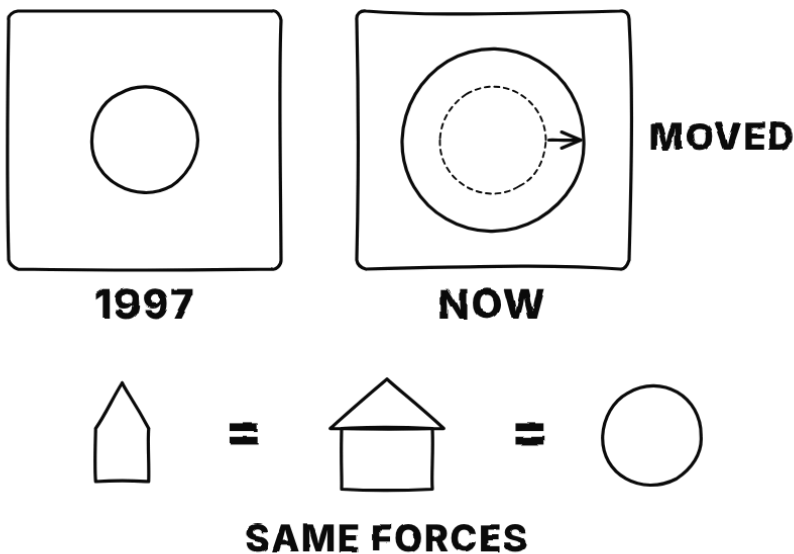
$$n = 2400$$

$$d \approx 2.3 \times 10^{16} \text{ km}$$

Light years into kilometres

Try it yourself

Tonight, step outside and look up at the stars for a minute. Pick the brightest one you can see. The light entering your eye left that star years, perhaps centuries, ago, and has been crossing space all that time. What you see is not the star as it is now, but the star as it was when the light set out.



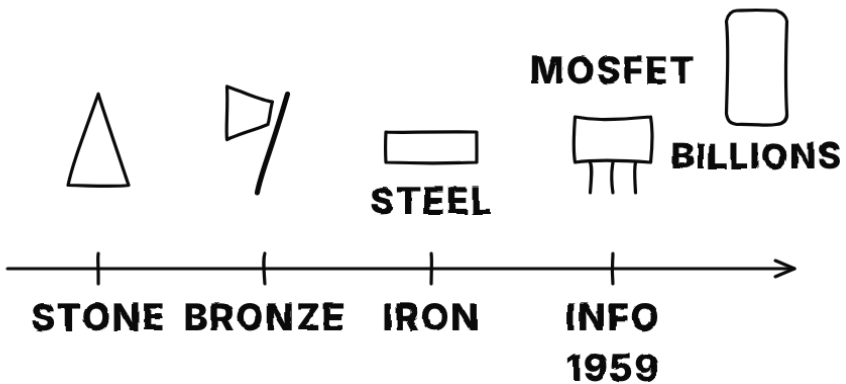
Two photos, years apart

How do scientists study something so far away? They watch it change. The Hubble Space Telescope took detailed images of the Veil in 1997. Newer images are laid beside the old ones, and the difference shows how far the shock wave has travelled and how the cloud has spread. It is like comparing two photos of the same spot on your phone, taken years apart: what has moved tells you what happened in between.

Now comes the point of the whole picture. The forces that make this supernova remnant behave as it does are the same forces we deal with here on Earth. Engineers meet them when they plan to send a rocket into space, and also when they simply work out how to heat a new home. The scale is wildly different, but the gases in the Veil push and mix much as gases do on Earth. Even the colours are familiar: they

are the glows of hydrogen, sulfur, oxygen and similar elements, the same elements that make up everything around us. In the book's words, "the forces out there are the same as the ones here on Earth." Said another way, there is no separate physics for the sky; one set of rules covers the stars and your kitchen alike.

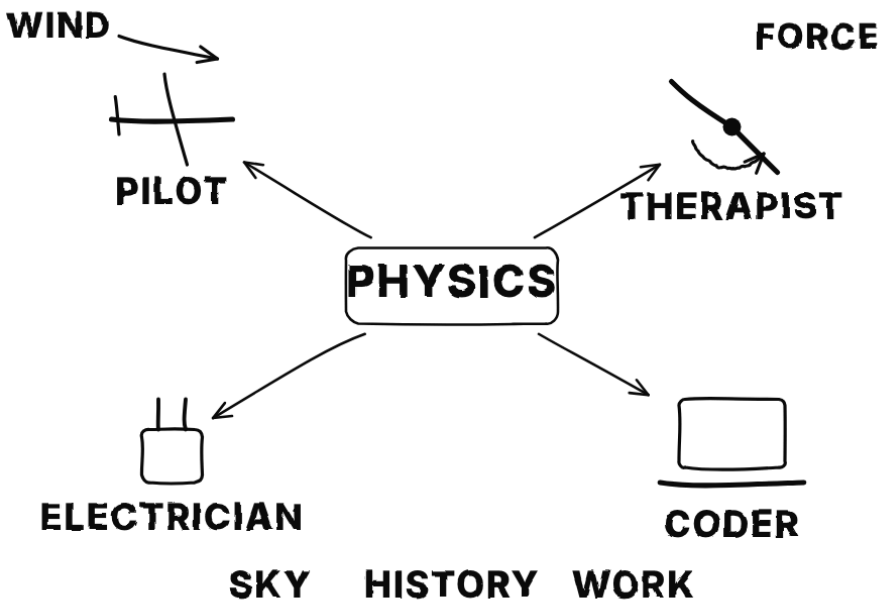
The second piece of evidence is the history of our tools. We name ages of history after materials: the Stone Age, the Bronze Age, the Iron Age. Each age was marked by knowing the physical properties of some material and being able to work it. That knowledge is physics, whether it was how a stone flakes when struck to make a spear point, how carbon worked into iron in furnaces of South India and Sri Lanka made the earliest high-quality steel, or how to grind and polish glass and put the pieces together into optical instruments.



Ages named after materials

Our own age, the Information Age, builds on all of that, and on one small invention in particular. In 1959, in a laboratory in New Jersey, Mohamed Atalla, born in Egypt, and Dawon Kahng, born in Korea, invented the MOSFET, a metal-oxide-semiconductor field-effect transistor: a tiny electrical switch. Almost nobody has heard of it, yet more of them have been made than any other object in human history. The phone in your pocket holds billions of them, and so do the chips behind your car's controls and the microwave in your kitchen. The book uses this story to make a further point: the great innovations of our time came from people of many backgrounds working together.

The third piece of evidence is people's work. Trains that float above their tracks, cloaks that bend light around themselves, and tiny robots that fight cancer cells all rest on the principles of physics. But so do ordinary jobs. Engineers, pilots, doctors, physical therapists, electricians and computer programmers use physics every day. A pilot must understand how the wind's forces push the plane off its path; a physical therapist must understand the forces that muscles feel as the body moves and bends. Even the dose on a medicine label rests on careful measurement, which is where this chapter is heading.



Jobs that use physics

So the book's case is made three times over: by the sky, by history and by daily work. Physics is not a set of formulas cut off from life; it describes the world you already live in.

Remember

The Veil Nebula is the glowing remains of a star that exploded, and its shock wave is still spreading through gas and dust. A light year is a distance, about nine and a half trillion kilometres, so the nebula's two thousand four hundred light years are about two point three times ten to the sixteenth kilometres. The forces that shape that cloud are the same as those on Earth, in rockets and in heating a house. Every technological age, from stone tools to the transistors in your phone, rested on knowing how materials behave. Many careers, from pilot to physical therapist, use physics every day.

Questions

1. In $d = n \times 9.5 \times 10^{12} \text{ km}$, why is the number of light years multiplied rather than added?
 - a. Because light speeds up over long distances
 - b. Because each light year is one equal step of the same length, and n equal steps make n times the length
 - c. Because light years and kilometres are the same unit
 - d. Because the formula gives a time, not a distance
2. Light from the Veil Nebula, 2,400 light years away, reaches your eye tonight. What does that tell you?
 - a. The light set off about 2,400 years ago
 - b. The nebula is 2,400 years old

- c. The light took one year to arrive
 - d. The nebula exploded 2,400 years ago
3. What do the colours in the photograph of the Veil Nebula show?
- a. Paint added to make the picture attractive
 - b. Elements found only in distant space
 - c. The glows of hydrogen, sulfur, oxygen and similar familiar elements
 - d. Heat from the original star still burning
4. Why does the lecture call the history of tools evidence for physics?
- a. Because ancient smiths wrote down physical laws
 - b. Because tools were always invented by physicists
 - c. Because stone, bronze and iron are physical constants
 - d. Because each age rested on knowing how a material behaves and how to work it

Answers

1. **b)** Each light year is one equal step of nine and a half trillion kilometres, so n steps are n times as long.
2. **a)** A light year is how far light goes in a year, so light from 2,400 light years away has been travelling for about 2,400 years.
3. **c)** The colours are the glows of familiar elements, the same that make up everything around us.
4. **d)** Each age was marked by knowing the physical properties of some material and being able to work it, and that knowledge is physics.

Worked examples

From light years to kilometres The Veil Nebula is about 2,400 light years from Earth, and one light year is about 9.5×10^{12} km. How far is it in kilometres?

Use $d = n \times 9.5 \times 10^{12}$ km, because each light year is one equal step of 9.5×10^{12} km.

Here $n = 2,400$, so $d = 2,400 \times 9.5 \times 10^{12}$ km.

Multiply the plain numbers first: $2,400 \times 9.5 = 22,800$.

So $d = 22,800 \times 10^{12}$ km, which is 2.28×10^{16} km, about 2.3×10^{16} km.

Answer: about 2.3×10^{16} km

Practice

1. The Veil Nebula stretches about 110 light years across. Using $d = n \times 9.5 \times 10^{12}$ km, how wide is it in kilometres?

Hint: Put $n = 110$ and multiply the plain numbers first.

Answer: About 1.0×10^{15} km: $110 \times 9.5 = 1,045$, so $d = 1,045 \times 10^{12}$ km $\approx 1.0 \times 10^{15}$ km.

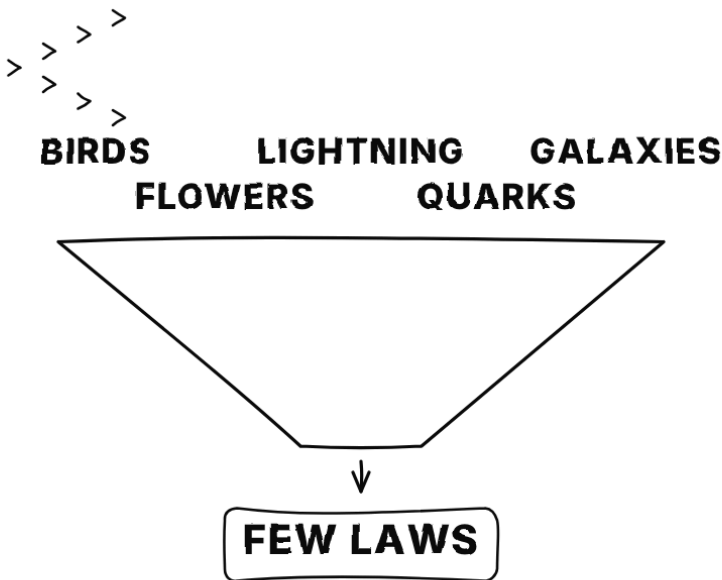
2. Explain in your own words why a pilot and a physical therapist both need physics.

Hint: Name the forces each of them must understand.

Answer: A pilot must understand how the wind's forces push the plane off its path, and a physical therapist must understand the forces muscles feel as the body moves and bends. Both work with forces, so both need the same physics.

Crisps, a Battery and One Law

Look at the flight of a flock of geese heading south in autumn: a neat V across the sky. The book shows such a picture and says, without fuss, that this formation is governed by the laws of physics. That is a bold claim, and the opening of the chapter explains why it can be made.

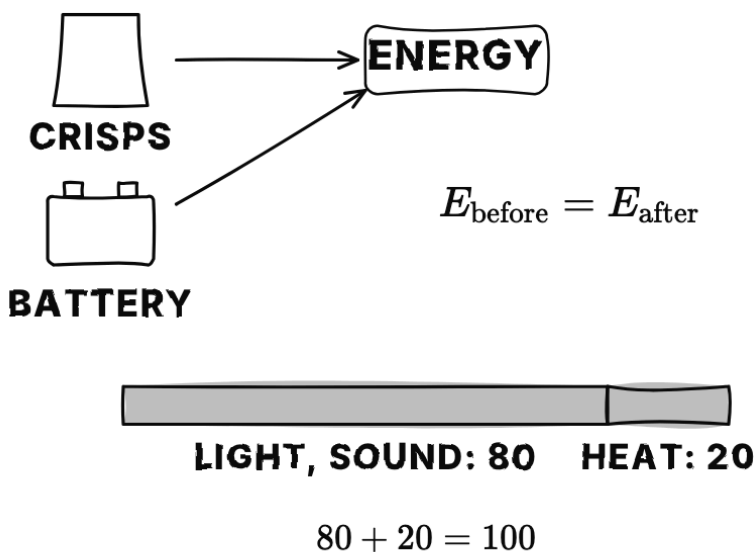


Many facts, few laws

The world is enormously complicated in its detail. Every day we see a great variety of things, and over centuries people have collected a vast store of facts: about the flight of birds and the colours of flowers,

lightning and gravity, the tiny particles called quarks and huge clusters of galaxies, the flow of time and the mystery of how the universe began. Faced with all that detail, people found something unexpected: a surprisingly small and unified set of physical laws explains what we observe. We look for order, and nature, in the book's words, is "remarkably cooperative".

Here is the book's own example. What do a bag of crisps and a car battery have in common? Both hold energy that can be turned into other forms. The law that ties them together is called the conservation of energy: energy can change its form, but it is never lost. Written as a formula, $E_{\text{before}} = E_{\text{after}}$, where E stands for the total energy, counted in all its forms at once. In words: whatever forms the energy takes, add them all up before a change and after it, and the two totals are equal. Where does the idea come from, by intuition? Think of money moved between pockets: coins can pass from one pocket to another, but if none leave you, the total stays the same. Say your phone's battery gives out one hundred units of energy while you watch a film; perhaps eighty of them end up as light and sound and twenty as the warmth you feel on the back of the phone. Eighty plus twenty is one hundred, the same total as before; nothing has vanished, it has only changed form.



Energy is never lost

One law, and it ties together food calories, batteries, heat, light and the wound-up spring of a watch. Understanding it makes each of those topics easier, because you see how the forms of energy relate to one another. Said another way: topics that look unrelated are joined by laws that apply very widely, and that lets you understand rather than memorise lists of facts.

These two ideas, that a few laws unify everything and that nature is at heart simple, run through the whole book. While you learn to apply the laws you will meet the main topics of physics, of course. But the book stresses something more important: the habits of analysis that let you use these laws far beyond anything a single book can hold.

Those habits help you succeed in study, and they help you think critically in whatever career you choose.

The book then sets out the road ahead. It will trace the history of the careful study of nature, beginning with natural philosophy and the ancient thinkers of the Middle East and the Mediterranean, and leading up to Sir Isaac Newton and the laws that carry his name. It will introduce the standards scientists use to measure physical quantities, a shared system of measurement that lets scientists everywhere speak one mathematical language. It is the same agreement that lets a kitchen scale in one country and a medicine dose printed in another mean exactly the same grams. And it will look at the limits of measurement: how accurate and how precise we can be, and why scientists take great pains to say clearly what they cannot know. Finally, this first section promises three things: to say what physics is, to show how it serves other fields, and to explain what a physical law really is and why experiment matters so much to theory.

Remember

A small, unified set of laws explains an enormously varied world, and nature turns out to be orderly and simple underneath. The conservation of energy says that the total energy before a change equals the total after it: energy changes form but is never lost. That one law links food, batteries, heat, light and springs, so understanding replaces memorising. Ahead lie the history of physics, shared standards of measurement, and the honest limits of how precisely we can measure.

Questions

1. What does the formula $E_{\text{before}} = E_{\text{after}}$ state?
 - a. Energy is created during every change
 - b. The total energy, counted in all its forms, is the same before and after a change
 - c. Energy before a change is always larger
 - d. Only heat energy is conserved
2. A phone battery gives out 100 units of energy; 80 end up as light and sound. Where are the other 20?
 - a. They are destroyed
 - b. They return to the battery
 - c. They become warmth in the phone
 - d. They never existed
3. Why does the lecture compare energy to coins moved between pockets?
 - a. Because coins can move from pocket to pocket while the total stays the same if none leave
 - b. Because energy costs money
 - c. Because coins and energy are both made of metal
 - d. Because energy is lost like spent coins
4. What does the book consider more important than learning the main topics of physics?
 - a. Memorising the values of constants
 - b. Learning the names of famous physicists

- c. Reading as many applications as possible
- d. Gaining habits of analysis that apply the laws far beyond a single book

Answers

- 1. **b)** Whatever forms the energy takes, the totals before and after a change are equal: energy changes form but is never lost.
- 2. **c)** Eighty plus twenty is one hundred: the rest becomes warmth, so the total is unchanged.
- 3. **a)** Energy moves between forms like coins between pockets; the total stays the same.
- 4. **d)** The book stresses analytical habits that let you use the laws far beyond what any one book can hold.

Worked examples

Following energy through a change A battery gives out 100 units of energy; 80 units become light and sound. Using $E_{\text{before}} = E_{\text{after}}$, how much becomes heat?

Energy is conserved, so the total after must equal the 100 units before.

Write $100 = 80 + \text{heat}$.

Subtract: $\text{heat} = 100 - 80 = 20$.

Answer: 20 units

Practice

- 1. A lamp takes in 60 units of energy each second and gives out 6 units as light. If no energy is lost, how much leaves as heat each second?

Hint: The total after must equal the total before: $60 = \text{light} + \text{heat}$.

Answer: 54 units: $60 - 6 = 54$, because the energy changes form but its total stays 60.

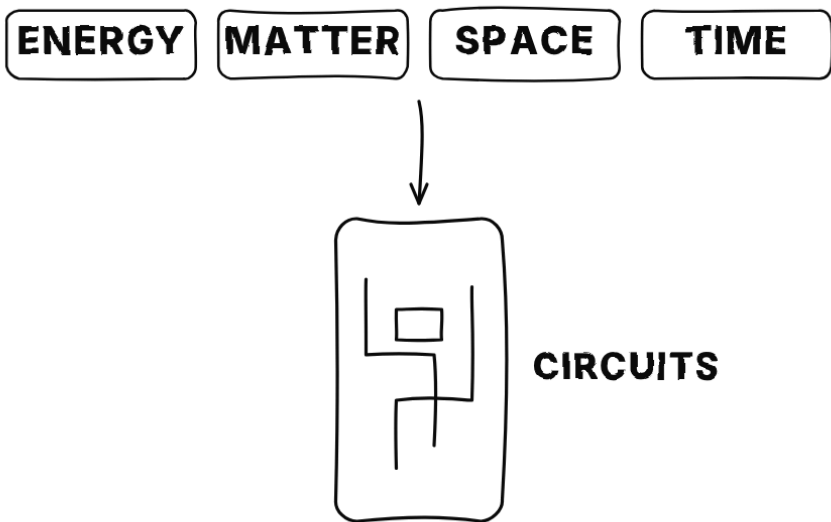
2. In your own words, explain why the book calls nature "remarkably cooperative".

Hint: Compare the number of facts with the number of laws.

Answer: We have collected enormous numbers of facts about the world, yet a surprisingly small and unified set of laws explains them. Nature shows an underlying order and simplicity, which is what we hope for when we seek generalizations.

Speed, Distance, Time and Your Phone

Open the map on your phone and ask for a route across town. Within a second it tells you the distance and how long the drive will take. How can a small glass slab in your hand know that? The answer begins with what physics is, and with one of its simplest relations.



What physics describes

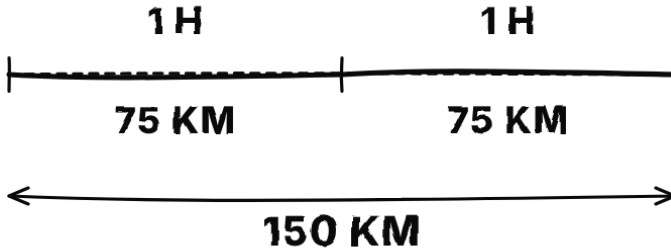
First, the book's definition. Science is made of theories and laws, the general truths of nature, together with all the knowledge they hold; scientists keep adding to that knowledge and keep sharpening the way the laws are stated. Physics is the part of science that describes how

energy, matter, space and time act on one another. Above all, it asks what basic mechanism lies underneath each happening. That concern with the most basic workings of nature is what marks out physics from other sciences. Its reach runs from the movement of tiny charged particles to the motion of people, cars and spaceships, and almost everything around you can be described quite accurately by its laws.

Take the phone itself. Inside your phone, electricity flows through many small circuits, and physics describes exactly how it does so. Engineers use that knowledge to choose the materials and lay out the circuits when they build the phone.

Now the map. Physics describes how the speed of an object, the distance it covers and the time it takes are related. In symbols, $d = vt$. Here d is the distance travelled, v is the speed, and t is the time; vt means v multiplied by t . In words: distance equals speed multiplied by time. Where does it come from? At a steady speed, each hour adds the same stretch of road, so after two hours you have two such stretches, after three hours three; the stretches simply add up, and adding equal pieces is multiplying. Say your car's speedometer stays at seventy-five kilometres an hour for two hours: $d = 75 \text{ km/h} \times 2 \text{ h} = 150 \text{ km}$. Notice that the hours cancel, leaving kilometres, just as a distance should. The same relation turned around gives the time: $t = d/v$. A trip of one hundred and fifty kilometres at seventy-five kilometres an hour takes one hundred and fifty divided by seventy-five, which is two hours; that is the sum your map does when it estimates the driving time.

$$d = v t$$



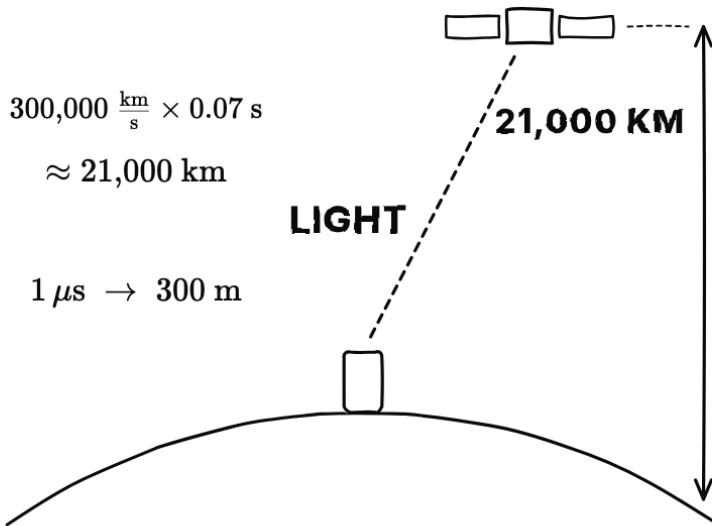
$$\frac{150}{75} = 2 \text{ h}$$

$$t = \frac{d}{v}$$

Distance equals speed times time

The navigation system behind the map, called GPS, uses the same relation in a far grander way. It has to account for the shape of the Earth's surface, the exact distances to satellites circling high above, and even a subtle effect called time dilation, in which clocks that move fast or sit in weaker gravity run at slightly different rates. Your receiver finds its distance from each satellite from how long the satellite's signal takes to arrive, and the signal travels at the speed of light, about three hundred thousand kilometres every second. So $d = vt$ again: a signal that takes seven hundredths of a second has covered about three hundred thousand times seven hundredths, or roughly twenty-one thousand kilometres, which is about how high those satellites fly. Because light is so fast, a clock error of only a millionth of a second would shift the answer by three hundred metres, which is

why such care with time is needed. Most of these calculations rest on methods developed by Gladys West, a mathematician and computer scientist who programmed the first computers capable of highly accurate positioning from afar. When the blue dot on your phone's map shows where you stand, it is her methods, and the relation between distance, speed and time, that put it there.



A satellite signal to your phone

The book then widens the view. You need not be a scientist to use physics: it explains how a microwave oven heats food and why metal must stay out of it, and why a white roof keeps a house cool. It is the base of other sciences too: chemistry rests on the physics of atoms and molecules, most engineering is applied physics, and medicine uses it in x-rays, scanners and radiotherapy.

But here is the point the book wants you to keep. You do not have to study every one of these uses. What matters most is knowing the basic laws and having the skill to apply them. The relation $d = vt$ is a good example: one short law serves a car trip, a map app and a satellite signal alike. Said another way, learn the law and the method, and the applications will follow on their own. The book adds that this kind of study sharpens your problem-solving, and since physics keeps the most basic parts of science, it makes every other science easier to learn.

Remember

Physics describes how energy, matter, space and time act on one another and looks for the basic mechanism under each happening. Distance equals speed multiplied by time, $d = vt$, so seventy-five kilometres an hour for two hours covers one hundred and fifty kilometres. Turned around, $t = d/v$ gives the travel time, which is how a map estimates a drive. Satellite navigation uses the same relation with signals moving at the speed of light, so tiny errors in time matter greatly. What counts most is not a list of applications but the basic laws and the skill to apply them.

Questions

1. What makes physics different from other sciences, according to the lecture?

- a. It uses more mathematics than any other science
 - b. It studies only very large objects
 - c. It describes how energy, matter, space and time act on one another and asks for the basic mechanism under each happening
 - d. It deals only with electricity
2. In $d = vt$, what happens to the distance if you drive at the same speed for three times as long?
- a. It becomes three times as large
 - b. It stays the same
 - c. It becomes a third as large
 - d. It becomes nine times as large
3. Why does a tiny clock error matter so much in satellite navigation?
- a. Because satellites move slowly
 - b. Because the signals travel at the speed of light, so a millionth of a second shifts the distance by about three hundred metres
 - c. Because the Earth's surface is flat
 - d. Because clocks on satellites never run differently
4. What point does the lecture draw from the relation $d = vt$ serving a car trip, a map and a satellite signal?
- a. That every application needs its own law
 - b. That maps are more accurate than speedometers
 - c. That GPS does not need physics

- d. That knowing a basic law and how to apply it matters more than memorising applications

Answers

1. **c)** Physics describes how energy, matter, space and time act on one another, and its concern with the most basic workings of nature marks it out.
2. **a)** At a steady speed each hour adds the same stretch, so three times the time gives three times the distance.
3. **b)** Light covers about three hundred thousand kilometres a second, so even a millionth of a second means about three hundred metres.
4. **d)** One short law serves many uses; learn the law and the method, and the applications follow.

Worked examples

Distance and time on a car trip Your speedometer stays at 75 km/h for 2 h. How far do you go? And how long would a 150 km trip take at that speed?

Use $d = vt$, distance equals speed multiplied by time.

$d = 75 \text{ km/h} \times 2 \text{ h} = 150 \text{ km}$; the hours cancel, leaving kilometres.

Turn the relation around for time: $t = d/v$.

$t = 150 \text{ km} \div 75 \text{ km/h} = 2 \text{ h}$.

Answer: 150 km, and 2 h

Practice

1. You drive 210 km at a steady 70 km/h. How long does the trip take?

Hint: Use $t = d/v$.

Answer: 3 h: $t = 210 \text{ km} \div 70 \text{ km/h} = 3 \text{ h}$.

2. A signal travels at about 300,000 km each second. How far does it go in 0.05 s?

Hint: Use $d = vt$ with $v = 300,000 \text{ km/s}$ and $t = 0.05 \text{ s}$.

Answer: About 15,000 km: $300,000 \times 0.05 = 15,000$, and the seconds cancel, leaving kilometres.

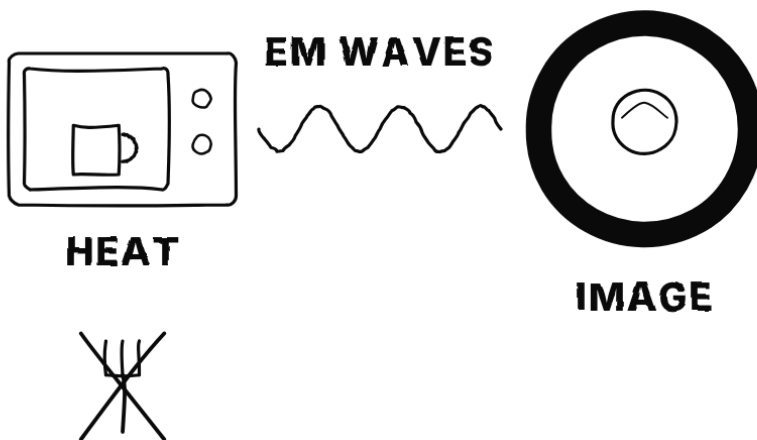
3. A trip of 120 km takes 1.5 h. What was the average speed?

Hint: Rearrange $d = vt$ to find v .

Answer: 80 km/h: $v = d/t = 120 \text{ km} \div 1.5 \text{ h} = 80 \text{ km/h}$.

From Microwaves to Models

Put a cup of cold coffee in the microwave, press start, and wait a minute. The cup comes out warm, but the oven's walls stay cool and nothing inside glowed like a stove. Now picture something quite different: a patient sliding into the tunnel of a hospital scanner, which produces a sharp image of the brain. What could a kitchen appliance and a hospital scanner possibly have in common?

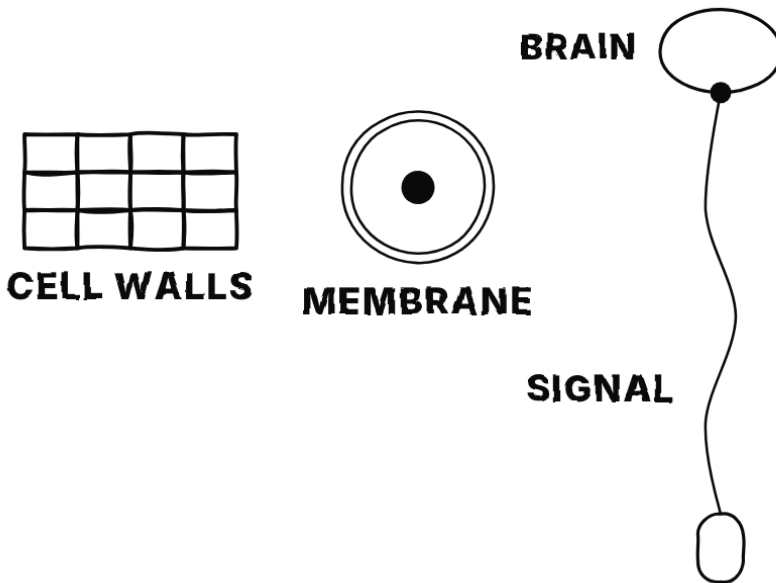


One kind of wave, two machines

The book answers with a pair of pictures. Both machines use electromagnetic waves, the same family of waves to which light and

radio belong. The microwave oven uses them to heat food. The scanner, called magnetic resonance imaging, or MRI, uses them to build an image of the brain detailed enough to show exactly where a tumour sits. Said another way, one piece of physics serves both dinner and diagnosis. The same laws also explain why metal objects must never go into a microwave oven, which is why the book calls the laws of physics a key to common appliances.

Physics reaches into living things as well. Look at a thin layer of onion skin under a microscope and you see neat rows of boxes: the walls of plant cells. Physics, together with chemistry and biology, helps describe how those walls behave. Animal cells have no rigid walls; instead each is wrapped in a thin, complex membrane that marks its boundary. Many of the most basic properties of life depend on such membranes, among them the firing of nerve cells, which carries every signal from your fingertips to your brain. Biology, chemistry and physics together are needed to understand them. The vibration of your phone when a message arrives, which you feel in your fingers, travels to your brain along exactly such nerve cells.



Cell walls and membranes

So far we have seen what physics explains. Now the book turns to how it explains: what laws, models and theories are, and why experiment rules over them all.

Start with the laws of nature. The book calls them concise descriptions of the universe: human statements of the underlying rules that every natural process follows. The rules themselves belong to the universe. People did not make them and cannot change them; we can only discover and understand them. And the discovery is a very human effort, full of mystery, imagination, struggle, triumph and disappointment, like any creative work.

Two pictures in the book make that human side vivid. Isaac Newton was so unwilling to publish his revolutionary work that others had to convince him. In later life he took charge of the Royal Mint, and he took the post seriously enough to introduce ridges around the edges of coins, so that dishonest people could no longer trim silver off them before spending them. Marie Curie spent her own money to pay for her early research and harmed her health through exposure to radiation; she is the only person ever to win Nobel Prizes in both physics and chemistry, and one of her daughters also won a Nobel Prize.

Whatever the struggle, one thing anchors the whole effort: observation. In the book's words, science must describe the universe "as it is, not as we may imagine it to be."

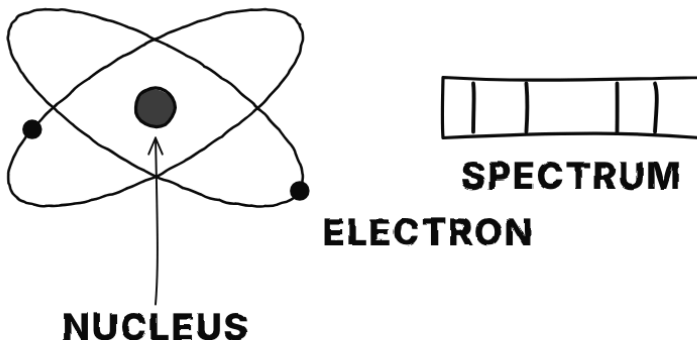
Try it yourself

For a week, glance at the sky each morning and note the kind of clouds you see. Then write down whether it rained that day. At the end of the week, look for a pattern: did certain clouds come before rain? You have just done what the book describes as the first, informal step of science: looking, generalising and trying to understand.

That is how all of us start. When we get serious, the book says, we become more organised: we collect and analyse data formally, aim for greater precision, run controlled experiments where we can, and write down how the data might be fitted together. Then we build models, theories and laws, to sum up the results and to share them with others.

First, a model. A model is a representation of something too difficult, or impossible, to show directly. The book's example is the planetary model of the atom: electrons pictured as circling a nucleus, the way planets circle the Sun. No one can watch an electron's orbit, but the picture helps explain things we can observe, such as the particular colours of light that hot gases give off, called atomic spectra. Here is the crucial limit: a model is supported by experiment, yet it is accurate only in limited situations. Your phone's map app is a fair comparison: a simplified picture that is excellent for finding a street, but tells you nothing about how high the buildings are. Physicists use models to analyse a situation and perform a calculation, or to represent it as a computer simulation.

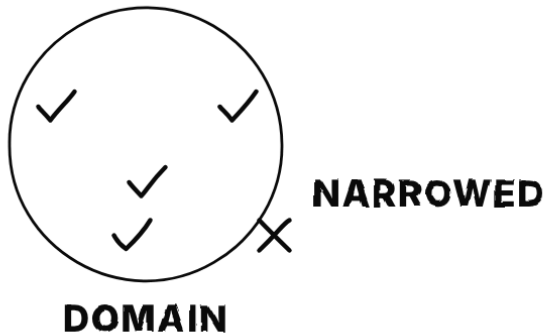
MODEL LIMITED



The planetary model of the atom

Next, a theory. In everyday speech a theory is a guess, an idea, at most a hypothesis, and people dismiss things as "just a theory". To scientists that phrase sounds meaningless, because science itself is built on theories. A scientific theory is a testable explanation of patterns in nature, supported by evidence and confirmed many times by different groups of researchers. Some theories use models and some do not; Newton's theory of gravity needs no mental picture, because the falling and orbiting bodies it describes can be seen directly.

THEORY



EXPERIMENT DECIDES

A theory is tested in its domain

The difference between a model and a theory lies in their reach. A model is meant to describe only certain aspects of a system accurately. A theory should describe every aspect of any system that falls within

its domain of applicability, the range of cases it claims to cover. That claim is a promise that can be checked: every consequence of a theory that can be tested by experiment should be verified. And if an experiment ever shows one consequence to be false, the theory is either thrown out or adjusted, for example by narrowing its domain. Said another way, experiment draws the borders of every model and every theory.

Remember

Microwave ovens and MRI scanners both use electromagnetic waves, one to heat food and the other to picture the brain.

Physics, with chemistry and biology, explains cell walls and the membranes on which nerve signals depend. Laws of nature belong to the universe; we only discover them, and observation of the world as it is anchors science. A model is a picture of what cannot be shown directly, like the planetary atom, and it is accurate only in limited situations. A scientific theory is a well-tested explanation that must cover everything in its domain and be changed or dropped if an experiment contradicts it.

Questions

1. What do a microwave oven and an MRI scanner have in common, according to the lecture?
 - a. Both use electromagnetic waves
 - b. Both heat food

- c. Both work only on metal
 - d. Both use radioactive material
2. Why is the planetary picture of the atom called a model rather than a description of what we see?
- a. Because it was drawn by an artist
 - b. Because it is proven true in every situation
 - c. Because electron orbits cannot be observed, though the picture explains what we can observe
 - d. Because atoms really are tiny solar systems
3. What happens to a theory if an experiment shows one of its consequences to be false?
- a. It becomes a model
 - b. It is thrown out or adjusted, for example by narrowing its domain
 - c. Nothing, since theories are only guesses
 - d. It is kept because it has been tested before
4. How does a theory's reach differ from a model's?
- a. A model covers everything; a theory covers only some aspects
 - b. Both cover exactly the same cases
 - c. Neither can be tested by experiment
 - d. A model describes only certain aspects; a theory should describe every aspect of any system in its domain

Answers

1. **a)** The microwave uses electromagnetic waves to heat food; the MRI scanner uses them to picture the brain.
2. **c)** No one can watch an electron's orbit, but the picture helps explain observations such as atomic spectra.
3. **b)** Every testable consequence should be verified; if one fails, the theory is dropped or its domain narrowed.
4. **d)** A model is meant to describe certain aspects accurately, while a theory should cover all aspects within its domain of applicability.

Practice

1. Someone says evolution is "just a theory". Using the lecture, explain why a scientist finds this remark meaningless.

Hint: Compare the everyday meaning of theory with the scientific one.

Answer: In everyday speech a theory is a guess, but to scientists a theory is a testable explanation of patterns in nature, supported by evidence and confirmed many times by different researchers. Science is built on theories, so calling something "just a theory" misunderstands the word.

2. Why does Newton's theory of gravity need no model, while the planetary atom is a model? Explain in your own words.

Hint: Ask whether the objects involved can be seen directly.

Answer: The falling and orbiting bodies in Newton's theory can be observed directly with our senses, so no mental picture is needed. Electrons in an atom cannot be seen, so we use a picture, the planetary model, to explain what we can observe.

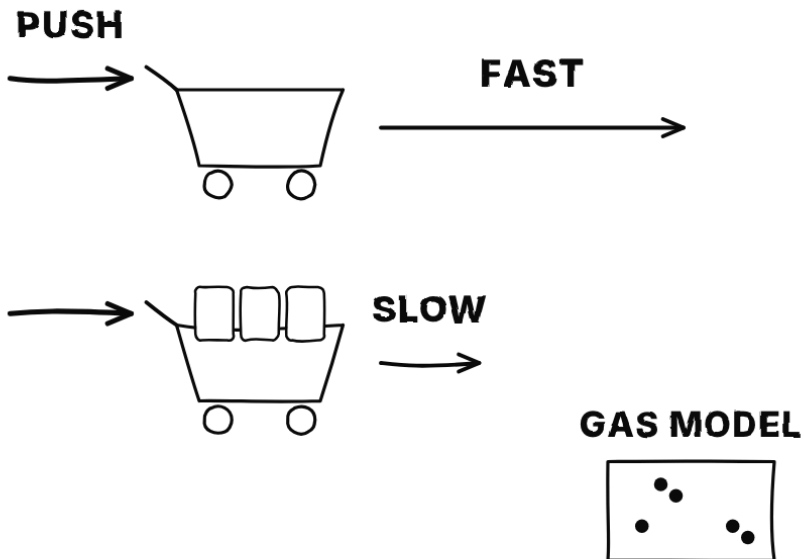
3. The lecture says the cornerstone of discovering laws is observation. What does that mean for an elegant idea that disagrees with careful observations?

Hint: Recall what science must describe.

Answer: Science must describe the universe as it is, not as we imagine it, so the idea must give way: if careful observation contradicts it, the idea is wrong or must be changed, however elegant it is.

Laws, Predictions and the Scientific Method

Push an empty shopping trolley across a car park, and it rolls off easily. Now load it with heavy bags and give it the same push: it gathers speed much more slowly. To make the full trolley speed up as fast as the empty one, you have to push harder. That everyday feeling is captured by one of the most famous laws in physics, and it is a good place to see what makes a law different from a theory.



Same push, different load

First, a word about pictures. Some theories come with a model, a mental image of what cannot be seen; others need none. Newton's theory of gravity needs no model, because the falling and orbiting bodies it describes can be seen directly. The kinetic theory of gases, on the other hand, pictures a gas as made of atoms and molecules, far too small for our senses, and we imagine them so that we can make sense of what our instruments tell us about how gases behave.

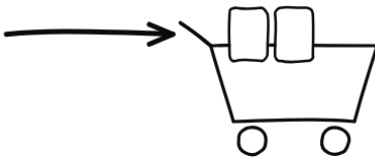
Now the law. A law describes a general pattern in nature in concise language, backed by evidence and repeated experiments, and it can often be written as a single equation. The book's example is Newton's second law of motion: $F = ma$. Here F is the force, the push or pull, measured in newtons; m is the mass, in kilograms; and a is the acceleration, how quickly the speed changes, in metres per second gained every second. In words: the force needed equals the mass multiplied by the acceleration. Where does it come from, by intuition? Twice the mass is like two trolleys side by side, and each needs its own push, so the force doubles; and to speed the same trolley up twice as fast, you must push twice as hard. Now a calculation aloud. A trolley of twenty kilograms that gains two metres per second of speed every second needs $F = 20 \text{ kg} \times 2 \text{ m/s}^2 = 40 \text{ N}$. Load it to forty kilograms and keep the same acceleration, and the force doubles to eighty newtons. Another law the book names is the conservation of energy: in any process, energy is conserved.

$$F = m a$$

$$\text{N} \qquad \text{kg} \qquad \text{m/s}^2$$



$$20 \text{ kg} \times 2 \text{ m/s}^2 = 40 \text{ N}$$

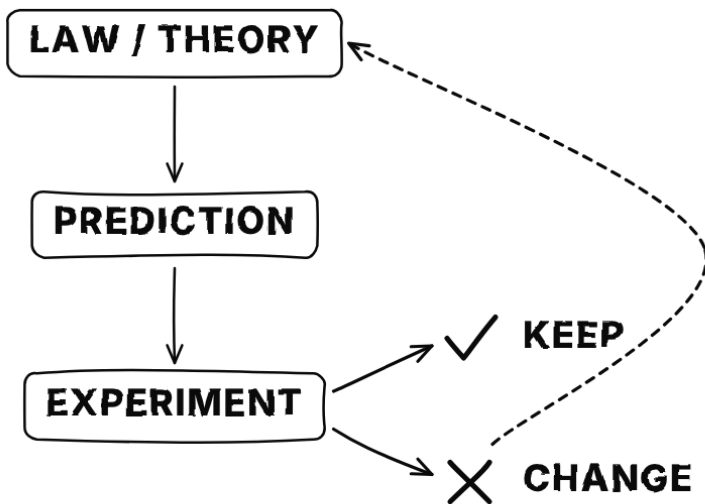


$$40 \text{ kg} \times 2 \text{ m/s}^2 = 80 \text{ N}$$

Newton's second law, $F = ma$

How do laws and theories differ? They are alike in one way: both are scientific statements that grew out of tested hypotheses and are backed by evidence. But the word law is kept for a statement that is short and very general, like $F = ma$. A theory is a less concise account; the theory of evolution and the theory of relativity cannot be squeezed into a single short statement, so they are not called laws. A law describes a single kind of action, while a theory explains a whole group of related happenings, and a theory is far more complex and changeable. The book puts the contrast in one line: a law is a starting assumption on which the scientific method builds, and a theory is the end result of that method. Statements that apply more narrowly are usually called principles, such as Pascal's principle, which holds only in fluids, though people do not always keep the words strictly apart.

Models, theories and laws are not only for tidying up data already collected. Often, once one is in place, it points scientists toward discoveries they would never otherwise have made. Sometimes they imply that something exists which no one has yet seen, and when the thing is then found, it is a remarkable triumph of science. The book credits the underlying order of the universe for such spectacular predictions. But the rule cuts both ways: if experiment does not confirm a prediction, the theory or law is wrong, no matter how elegant or convenient it is.



Prediction meets experiment

That is why a law can never be known with absolute certainty. To be sure, we would have to run every imaginable experiment in every possible situation, and no one can. So physicists treat laws and

theories as valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, the law must be changed or thrown out completely. Said another way, a law is trusted, but never beyond the reach of the next careful experiment.

The book calls science an adventure, like exploring an uncharted ocean: discoveries are made, models, theories and laws are formed, and the universe looks all the more beautiful for what we learn.

The way of working behind this adventure has a history. Ibn al-Haytham, sometimes called Alhazen, worked in Cairo in the tenth and eleventh centuries and greatly advanced the understanding of optics and vision. His larger contribution was a way of thinking. In showing that earlier ideas were wrong, he insisted that a scientist must be ready to reject existing knowledge and become "the enemy" of everything they read, trusting only objective evidence. He stressed repeating experiments and checking results, and he warned that our senses and our expectations can lead us to poor conclusions. His work was a forerunner of the scientific method we use today.

The book sets the method out as a sequence of steps. It usually begins with an observation and a question to investigate. Next comes some research into the topic, and then a hypothesis, a proposed answer that can be tested. Then the hypothesis is tested by an experiment. Finally the results are analysed and a conclusion is drawn. Weighing flour on a kitchen scale to check whether a recipe's "one cup" really is the same every time would be a tiny example: question, guess, test, conclusion. And the book adds that this method is not only for science; it can be used in many situations and adjusted to fit them.

Remember

Newton's second law, $F = ma$, says the force needed equals mass times acceleration, so a twenty-kilogram trolley speeding up by two metres per second each second needs forty newtons. A law is a short, general statement of one pattern; a theory is a broader, more complex explanation of a whole group of happenings, such as evolution or relativity. Laws and theories can predict things not yet seen, but any prediction that fails in a careful experiment shows them to be wrong. No law is ever certain, because no one can perform every possible experiment. The scientific method runs from observation and question, through research and hypothesis, to experiment, analysis and conclusion.

Questions

1. In $F = ma$, what happens to the force needed if the mass is doubled and the acceleration stays the same?
 - a. It doubles
 - b. It halves
 - c. It stays the same
 - d. It becomes four times larger
2. Why is relativity called a theory and not a law?
 - a. Because it has never been tested
 - b. Because it is too new

- c. Because it explains a whole group of happenings and cannot be squeezed into one short statement
 - d. Because it is only a guess
- 3. Why can a law never be known with absolute certainty?
 - a. Because laws keep changing every year
 - b. Because no one can run every imaginable experiment in every possible situation
 - c. Because laws are only guesses
 - d. Because scientists disagree about them
- 4. In the scientific method, what comes right after the hypothesis?
 - a. The conclusion
 - b. The observation
 - c. The research
 - d. An experiment that tests it

Answers

- 1. **a)** Twice the mass is like two trolleys side by side, each needing its own push, so the force doubles.
- 2. **c)** A theory is a less concise account of a whole group of related happenings; relativity cannot be put into one short statement.
- 3. **b)** To be certain we would have to run every possible experiment, so laws are treated as valid until a counterexample appears.
- 4. **d)** The method runs from observation and question to research and hypothesis, then the hypothesis is tested by an experiment.

Worked examples

Force on a loaded trolley A trolley of 20 kg gains 2 m/s of speed every second. What force does it need? What if its mass is 40 kg with the same acceleration?

Use Newton's second law, $F = ma$.

$$F = 20 \text{ kg} \times 2 \text{ m/s}^2 = 40 \text{ N}.$$

$$\text{With } m = 40 \text{ kg: } F = 40 \text{ kg} \times 2 \text{ m/s}^2 = 80 \text{ N}.$$

Doubling the mass at the same acceleration doubles the force.

Answer: 40 N, and 80 N

Practice

1. A box of 12 kg is pushed so that it gains 0.5 m/s of speed every second. What force acts on it?

Hint: Multiply mass by acceleration.

$$\textbf{Answer: } 6 \text{ N: } F = 12 \text{ kg} \times 0.5 \text{ m/s}^2 = 6 \text{ N}.$$

2. A force of 90 N acts on a cart of 30 kg. What is its acceleration?

Hint: Turn $F = ma$ around: $a = F/m$.

Answer: 3 m/s²: $a = 90 \text{ N} \div 30 \text{ kg} = 3 \text{ m/s}^2$, so it gains 3 m/s of speed every second.

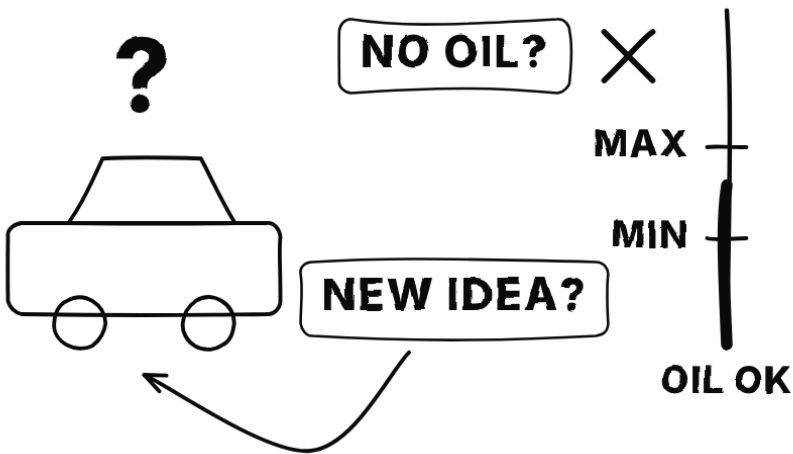
3. A beautiful theory predicts something that a careful, repeatable experiment does not find. What does the lecture say must follow?

Hint: Recall what happens when experiment does not confirm a prediction.

Answer: The theory is wrong, however elegant or convenient it is, and must be changed or thrown out. Experiment, not elegance, decides.

From Natural Philosophy to Classical Physics

You turn the key on a cold morning, and the car will not start. What do you do? The book uses this very scene to show the scientific method at work in ordinary life.

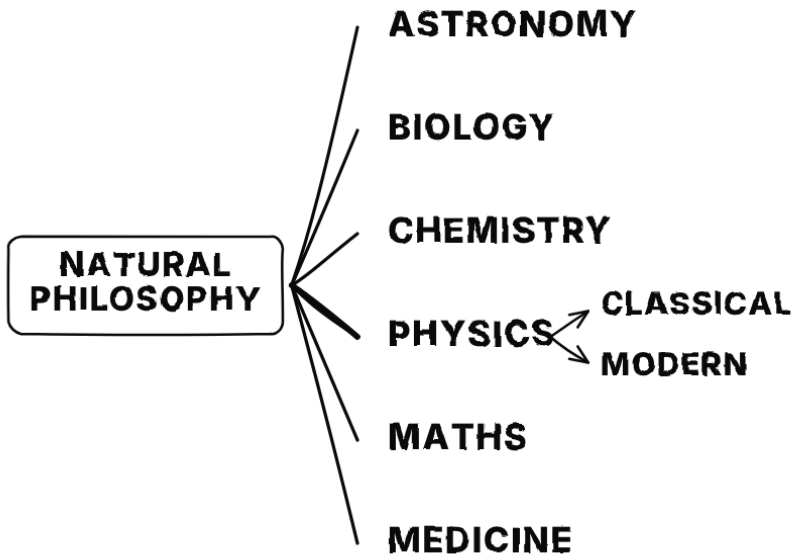


Why won't the car start?

The question is plain: why will the car not start? First you look into the possible reasons a car might fail. Then you state a hypothesis: perhaps it has no engine oil. To test it, you open the bonnet and check the oil level. The oil turns out to be at an acceptable level, so you

conclude that oil is not the cause. And then the important step: you form a new hypothesis, test it, and repeat the process until the cause is found. Said another way, a failed guess is not wasted; it crosses one suspect off the list. Fixing a phone that will not charge goes the same way: you suspect the cable, try another one, and if the phone still will not charge, you move on to the next suspect, the plug or the socket.

Now the book steps back into history. Physics was not always a separate subject, and even today it stays connected to the other sciences. The word physics comes from the Greek word for nature, and the study of nature was long called natural philosophy. From ancient times through the Renaissance, natural philosophy took in astronomy, biology, chemistry, physics, mathematics and medicine all at once. The Greek philosopher Aristotle, for example, wrote on physics, animals, the soul, politics and poetry. Over the last few centuries, as knowledge grew, natural philosophy branched into ever more specialised fields, and physics kept the most basic parts of all. The book's pictures show some of the great minds along the way: Ibn al-Haytham and Galileo Galilei, so important to scientific practice that a later scientist put both on the cover of his major work, and Niels Bohr, who helped build quantum mechanics in the twentieth century.

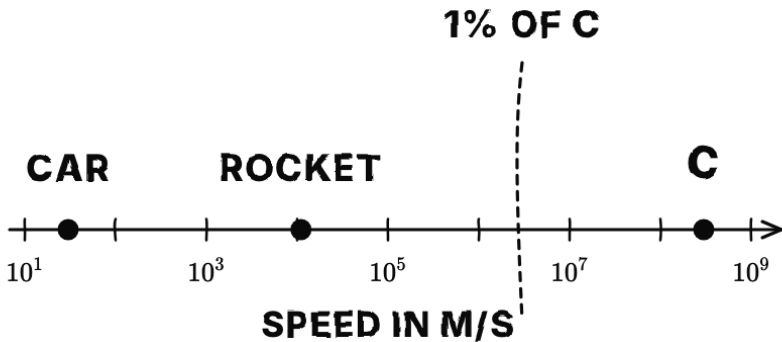


Natural philosophy branches out

Physics as it grew from the Renaissance to the end of the nineteenth century is called classical physics. At the start of the twentieth century, revolutionary discoveries transformed it into modern physics.

Here is a key point. Classical physics is not an exact description of the universe, but it is an excellent approximation under three conditions. First, matter must move at less than about one per cent of the speed of light. Second, the objects must be large enough to be seen with a microscope. Third, only weak gravity may be involved, such as the gravity of the Earth.

$$v < 0.01 c$$

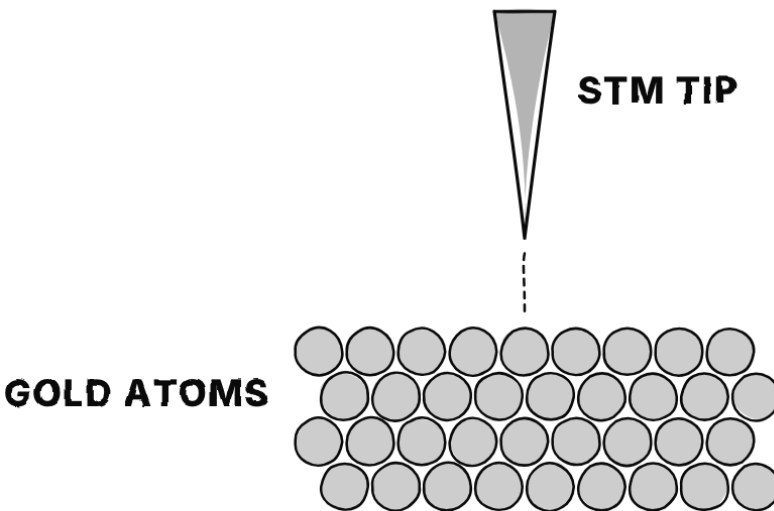


One per cent of light speed

The first condition can be written as a formula: $v < 0.01c$. Here v is the speed of the object and c is the speed of light, about 3×10^8 m/s. In words: the object's speed must be less than one hundredth of the speed of light. Where does the one per cent come from, by intuition? The errors of classical physics grow with speed; far below the speed of light they are so tiny that no ordinary measurement notices them. Now the calculation aloud. One hundredth of three times ten to the eighth metres per second is 3×10^6 m/s, which is three thousand kilometres every second. A car on the motorway at about thirty metres per second, roughly a hundred kilometres an hour, is a hundred thousand times slower than that limit. Even a rocket leaving the Earth at about eleven kilometres per second moves at less than half of one per cent of the speed of light, so classical physics serves it well.

Try it yourself

Hold your thumb up at arm's length and look past it at the room. Everything you can see moves slowly, is large enough to see, and sits in the Earth's weak gravity. All three conditions of classical physics are met at once, which is why it feels like common sense.



Seeing atoms

Because we live in exactly these conditions, classical physics seems natural, while much of modern physics seems bizarre. That is why models are so useful in modern physics: they let us picture things we never experience, such as objects moving at enormous speeds, or objects too small for our senses. We understand the properties of an

atom because we can picture one in our minds, even though no one has seen an atom with the naked eye. New tools help, too. A scanning tunnelling microscope now lets scientists "see" the individual atoms of a sheet of gold, lined up like oranges in a crate.

Modern physics has given some of the most spectacular advances in science. Many classical laws have been changed or rejected, and technology, society and our view of the universe have changed with them. Modern physics is full of objects stranger than science fiction, with the advantage of being real.

Why, then, does most of the book still teach classical physics? The book gives two reasons. First, classical physics describes the universe extremely accurately across a wide range of everyday situations; our car on the motorway is a perfect example. Second, you need classical physics in order to understand modern physics at all. Said another way, classical physics is both the everyday tool and the foundation on which the newer physics is built.

Remember

Testing a hypothesis, finding it wrong and forming a new one, as with a car that will not start, is the scientific method in daily life. Physics grew out of natural philosophy, which once covered all the sciences; physics kept the most basic parts. Classical physics works when speeds are below about one per cent of the speed of light, objects are visible under a microscope, and gravity is weak. One per cent of the speed of light is three thousand kilometres per second, so cars and even rockets fall well within classical physics. Most of the book teaches classical physics because it is very accurate for everyday life and is the foundation for modern physics.

Questions

1. After finding that the oil level is fine in a car that will not start, what does the scientific method say to do?
 - a. Conclude the car is broken beyond repair
 - b. Form a new hypothesis and test it
 - c. Check the oil again until it fails
 - d. Stop, because the test failed
2. What happened to natural philosophy as knowledge grew?
 - a. It split into specialised fields, with physics keeping the most basic parts
 - b. It was abandoned completely

- c. It became the same as medicine
 - d. It was replaced by astronomy alone
3. Why does classical physics feel like common sense to us?
- a. Because it is the newest physics
 - b. Because it was discovered by Aristotle
 - c. Because it is an exact description of the universe
 - d. Because we live among slow speeds, visible objects and weak gravity
4. A rocket leaves Earth at about 11 km/s. Is classical physics good enough to describe it?
- a. No, because rockets need relativity
 - b. No, because it is too large
 - c. Yes, because 11 km/s is less than half of one per cent of the speed of light
 - d. Yes, because rockets are visible from Earth only

Answers

1. **b)** A failed guess crosses one suspect off; you form a new hypothesis and repeat until the cause is found.
2. **a)** Natural philosophy branched into specialised fields over the last few centuries, and physics kept the most basic parts.
3. **d)** We live exactly where the three conditions of classical physics hold, so it seems natural.
4. **c)** The limit is about 3,000 km/s; 11 km/s is far below it, so classical physics serves well.

Worked examples

The speed limit of classical physics Using $v < 0.01c$ with $c = 3 \times 10^8$ m/s, find the speed below which classical physics works well, in m/s and km/s.

The condition says v must be less than one hundredth of c .

$$0.01 \times 3 \times 10^8 \text{ m/s} = 3 \times 10^6 \text{ m/s}.$$

Divide by 1,000 to change metres to kilometres: 3×10^3 km/s.

So the limit is about 3,000 km every second.

Answer: 3×10^6 m/s, or 3,000 km/s

Practice

1. A car travels at 30 m/s. How many times slower is it than the classical-physics limit of 3×10^6 m/s?

Hint: Divide the limit by the car's speed.

Answer: 10^5 times, a hundred thousand: $3 \times 10^6 \div 30 = 10^5$.

2. A particle in an accelerator moves at 6×10^7 m/s. Does it satisfy $v < 0.01c$, with $c = 3 \times 10^8$ m/s?

Hint: Compare it with $0.01c = 3 \times 10^6$ m/s.

Answer: No: 6×10^7 m/s is twenty times the limit of 3×10^6 m/s, which is 20 per cent of c , so classical physics is not good enough.

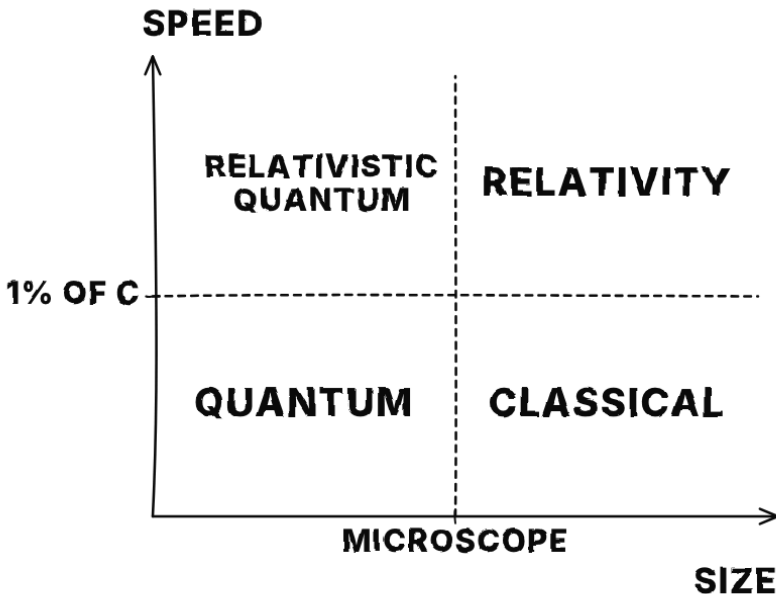
3. Give the two reasons the book offers for teaching mostly classical physics.

Hint: One concerns everyday accuracy, the other concerns modern physics.

Answer: Classical physics describes the universe extremely accurately in a wide range of everyday situations, and it is needed in order to understand modern physics.

Very Fast, Very Small, and the Need for Units

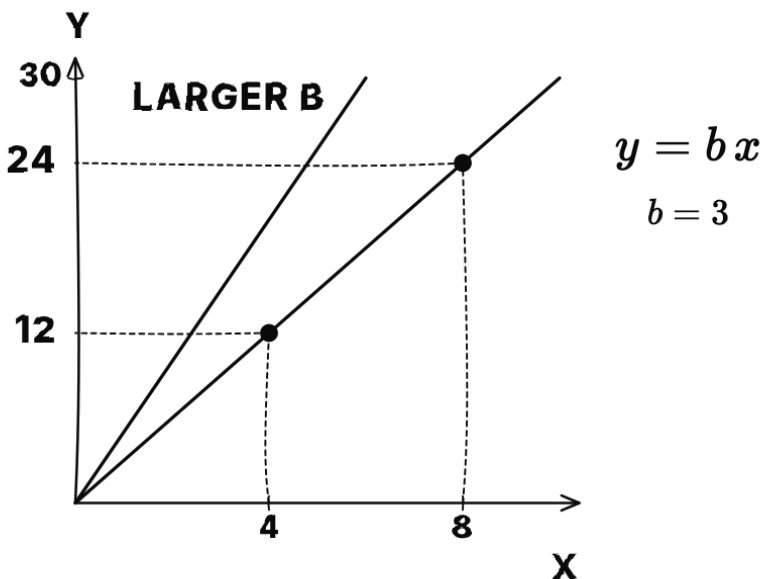
Imagine a friend phones you and says: "I walked twelve today." Twelve what? Twelve steps, twelve blocks, twelve kilometres? The number alone is useless, and the book even has a picture with that caption: distances given in unknown units are maddeningly useless. Before we reach units, though, the book finishes its story about where modern physics takes over.



Which theory to use

Modern physics rests on two revolutionary theories: relativity and quantum mechanics. Relativity deals with the very fast, quantum mechanics with the very small. Relativity must be used whenever an object moves faster than about one per cent of the speed of light, or feels a strong gravitational field, such as that near the Sun. Quantum mechanics must be used for objects smaller than can be seen with a microscope. Combine the two and you get relativistic quantum mechanics, which describes small objects that move very fast or feel strong gravity. It is the best theory we have that applies everywhere, but it is so complicated mathematically that it is used only when necessary; simpler theories are used whenever they are accurate enough. And the book promises that a great deal of modern physics can still be done with ordinary algebra and trigonometry.

Notice that relativity and quantum mechanics are called theories. The book tests that idea with a small puzzle. A friend says they have learned a new law of nature. Even before hearing it, you know it will be a concise description of the universe, a statement of a rule all natural processes follow. Had the friend learned a theory instead, you would know it was a large-scale, broadly applicable generalisation.

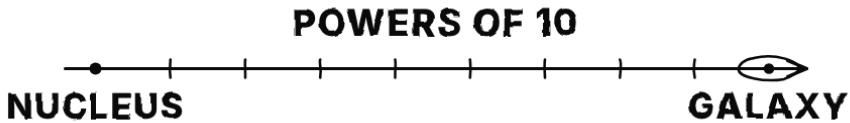


A straight line, $y = bx$

Next to this the book points to an online grapher for curves built from several terms, called polynomials. One of its simplest pieces is a straight line, $y = bx$, where b is a fixed number and x can vary. In words: y is always b times x , so doubling x doubles y . For example, with $b = 3$ and $x = 4$, $y = 12$; with $x = 8$, $y = 24$. Changing b tilts the line, and adding other terms bends it into a curve.

Now the new section: physical quantities and units. The book opens with a striking picture of the Earth and the Moon, and notes that the distance between them, immense as it seems, is a tiny fraction of the distances to other bodies in the sky. The range of things physics studies is enormous: from the incredibly short life of an atomic nucleus to the age of the Earth, from particles smaller than a nucleus

to the edges of the known universe, from the push of a jumping flea to the pull between the Earth and the Sun. There are enough factors of ten, the book says, to stretch the imagination of even the most experienced scientist.



Four fundamental quantities

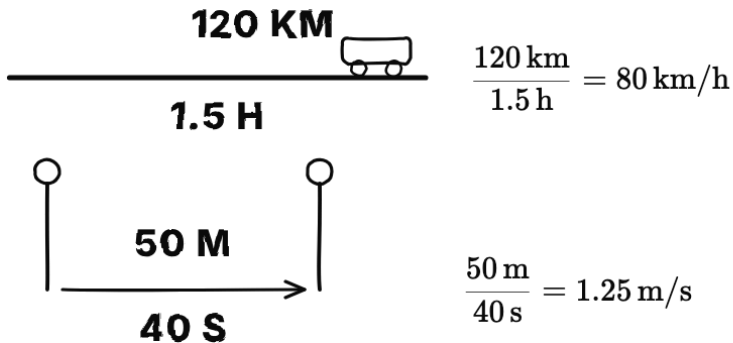
Putting numbers on quantities and equations on principles lets us understand nature far more deeply than words alone. But to handle such vast ranges we need accepted units. And here, the book says, a deep simplicity appears: most physical quantities can be built from just four basic ones: length, mass, time and electric current.

How do we define a physical quantity? In one of two ways: by saying how it is measured, or by saying how it is calculated from other

measurements. Distance and time are defined by how we measure them. Average speed is defined by a calculation: average speed = distance/time. In words: how far you went, shared out over how long it took. Say your car's trip computer shows that you covered one hundred and twenty kilometres in one and a half hours; your average speed was one hundred and twenty divided by one and a half, or eighty kilometres per hour. Notice that the speedometer may have shown more or less at different moments; the average tells you only the overall pace.

$$v = \frac{d}{t}$$

AVERAGE SPEED



Average speed

Try it yourself

Walk a measured stretch, say fifty metres between two lamp posts, and time yourself with your phone's stopwatch. If it takes forty seconds, divide fifty by forty. Your average walking speed is one and a quarter metres per second.

Measurements are given in units, which are standardised values. The length of a race, for instance, is given in metres for sprinters and in kilometres for distance runners. Without agreed units it would be extremely hard for scientists to report and compare what they measure; hence the "maddeningly useless" distances in unknown units.

Two big systems of units are used in the world. One is the SI system, also called the metric system. The other is the system of English units, also called customary or imperial units, used historically in countries once ruled by the British Empire and still widely used in the United States. Almost every other country now uses SI units as its standard, and the metric system is the standard agreed upon by scientists and mathematicians. The letters SI come from the French name, *Système International*, the International System. It is why the grams on your kitchen scale and the milligrams on a medicine label mean the same thing in any country.

Remember

Relativity is needed for speeds above about one per cent of the speed of light or strong gravity; quantum mechanics for things too small to see with a microscope. A law is a concise statement of a rule; a theory is a large-scale, broadly applicable generalisation. Most physical quantities can be built from four basic ones: length, mass, time and electric current. A quantity is defined either by how it is measured or by how it is calculated, as average speed is distance divided by time. Agreed units are essential, and the SI, or metric, system is the standard used by scientists and most of the world.

Questions

1. An object is smaller than can be seen with a microscope. Which theory must be used?
 - a. Relativity
 - b. Quantum mechanics
 - c. Classical physics
 - d. The theory of gases
2. In $y = bx$ with $b = 3$, what happens to y when x doubles from 4 to 8?
 - a. It doubles from 12 to 24
 - b. It stays at 12
 - c. It increases by 3

- d. It becomes four times as large
3. Which pair below shows the two ways a physical quantity can be defined?
- a. By its size or by its colour
 - b. By its unit or by its symbol
 - c. By how it is measured or by how it is calculated from other measurements
 - d. By experiment or by theory
4. Your trip computer shows 120 km covered in 1.5 hours. What does the average speed of 80 km/h tell you?
- a. The speedometer showed 80 km/h the whole time
 - b. The fastest speed was 80 km/h
 - c. The car never stopped
 - d. The overall pace: distance shared out over the whole time

Answers

1. **b)** Quantum mechanics must be used for objects smaller than can be seen with a microscope.
2. **a)** y is always b times x , so doubling x doubles y .
3. **c)** Distance and time are defined by how we measure them; average speed by a calculation.
4. **d)** Average speed is distance divided by time; the speedometer may have shown more or less at different moments.

Worked examples

Average speed of a trip A car covers 120 km in 1.5 h. What is its average speed?

Average speed = distance / time.

Substitute: 120 km / 1.5 h.

$$120 \div 1.5 = 80.$$

Answer: 80 km/h

Practice

1. You walk 50 m between two lamp posts in 40 s. What is your average speed?

Hint: Divide distance by time.

Answer: 1.25 m/s: $50 \text{ m} \div 40 \text{ s} = 1.25 \text{ m/s}$.

2. A cyclist's average speed is 18 km/h over a 2.5 h ride. How far did they ride?

Hint: Average speed = distance/time, so distance = average speed $\times$ time.

Answer: 45 km: $18 \text{ km/h} \times 2.5 \text{ h} = 45 \text{ km}$.

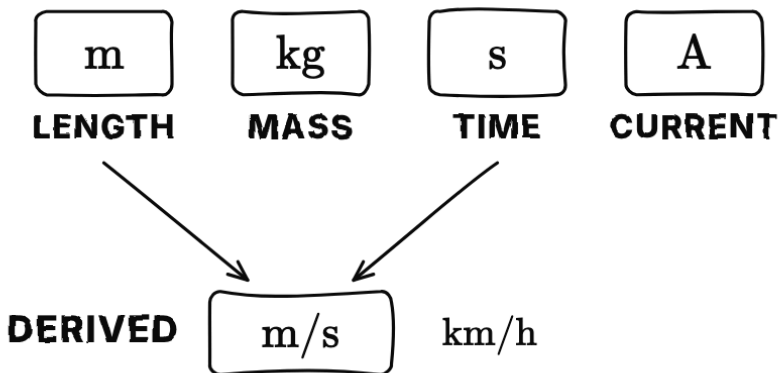
3. In $y = bx$, $y = 20$ when $x = 5$. What is b , and what is y when $x = 7$?

Hint: Find b by dividing y by x .

Answer: $b = 4$, since $20 \div 5 = 4$; then $y = 4 \times 7 = 28$.

Four Building Blocks and the Second

How long is one second, really? You might say: the time it takes to say "one thousand". Science needs something far better, and the story of the second shows why.



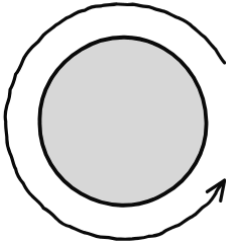
Four fundamental SI units

First, the building blocks. The book's table lists four fundamental SI units: the metre (m) for length, the kilogram (kg) for mass, the second (s) for time, and the ampere (A) for electric current, which comes much later in the book. The most fundamental quantities can be defined only by the procedure used to measure them, and their units

are called fundamental units. Every other quantity, such as force or electric charge, is an algebraic combination of these four, and its unit is a derived unit. Speed is the simplest example: speed = length/time, so its unit is metres per second, m/s. The kilometres per hour on your car's speedometer is a derived unit of the same kind: a length divided by a time. The book uses a few units outside SI where they are very common, such as millimetres of mercury, mm Hg, for blood pressure, and always ties them back to SI units by conversion.

Now the second. For many years it was defined as a fraction of a day: $1\text{ s} = 1/86,400$ of a mean solar day. Where does eighty-six thousand four hundred come from? A day has twenty-four hours, each hour sixty minutes, each minute sixty seconds: twenty-four times sixty is one thousand four hundred and forty minutes, and times sixty again is eighty-six thousand four hundred seconds. But the Earth's spin is very gradually slowing, so the solar day is slowly getting longer, and a unit tied to it would slowly drift.

1 DAY



$$24 \times 60 \times 60 = 86\,400$$

CAESIUM



9,192,631,770

VIBRATIONS = 1 S

SLOWING

Defining the second

So a new standard was chosen, based on something that does not change. Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be counted. In 1967 the second was redefined as the time taken by exactly 9,192,631,770 of these vibrations. In words: count a little over nine billion cesium vibrations, and one second has passed. An atomic clock built on this keeps time to better than a millionth of a second per year. The clock on your phone is regularly corrected by signals that trace back to such atomic clocks.

Why such fuss over accuracy? Because every measurement is finally expressed in fundamental units, and no measurement can be more accurate than the units themselves. Said another way, a ruler can be

no better than the metre it copies, and a stopwatch no better than the second it counts.

Remember

The four fundamental SI units here are the metre, kilogram, second and ampere; all other units, like metres per second, are derived from them. The second was once one eighty-six thousand four hundredth of a day, but the day slowly lengthens. Since 1967 the second is the time of 9,192,631,770 vibrations of a cesium atom. All measurements rest on the fundamental units, so they can be no more accurate than those units.

Questions

1. Why was the second redefined using cesium atoms rather than the length of the day?
 - a. Because the Earth's spin is slowly slowing, so the day gradually gets longer
 - b. Because days are too long to measure
 - c. Because cesium is easier to see
 - d. Because the day has exactly 86,400 seconds
2. Why is the unit of speed called a derived unit?
 - a. Because speed is measured with a speedometer
 - b. Because it is built from fundamental units: a length divided by a time

- c. Because it is used only in cars
 - d. Because it was defined in 1967
3. Why does the accuracy of the fundamental units matter so much?
- a. Because only scientists use them
 - b. Because they are expensive
 - c. Because every measurement is expressed in them and can be no more accurate than they are
 - d. Because they change every year
4. Which of these is NOT one of the four fundamental SI units listed in the book?
- a. The metre
 - b. The ampere
 - c. The second
 - d. The newton

Answers

1. **a)** A unit tied to the slowly lengthening day would drift; cesium vibrations do not change.
2. **b)** Speed = length/time, so its unit, metres per second, is an algebraic combination of fundamental units.
3. **c)** A ruler can be no better than the metre it copies, and a stopwatch no better than the second it counts.
4. **d)** The four are the metre, kilogram, second and ampere; units like the newton are derived.

Worked examples

Seconds in a day Show that the old definition, $1 \text{ s} = 1/86,400$ of a day, matches 24 hours of 60 minutes of 60 seconds.

A day has 24 h.

$$24 \text{ h} \times 60 \text{ min/h} = 1,440 \text{ min.}$$

$$1,440 \text{ min} \times 60 \text{ s/min} = 86,400 \text{ s.}$$

So one second is $1/86,400$ of a day.

Answer: 86,400 s in a day

Practice

1. How many seconds are in one week, using 86,400 s per day?

Hint: Multiply by the number of days.

Answer: 604,800 s: $7 \times 86,400 = 604,800$.

2. About how many cesium vibrations occur in one minute, using 9,192,631,770 per second?

Hint: Multiply by 60.

Answer: About 5.5×10^{11} : $9,192,631,770 \times 60 = 551,557,906,200$.

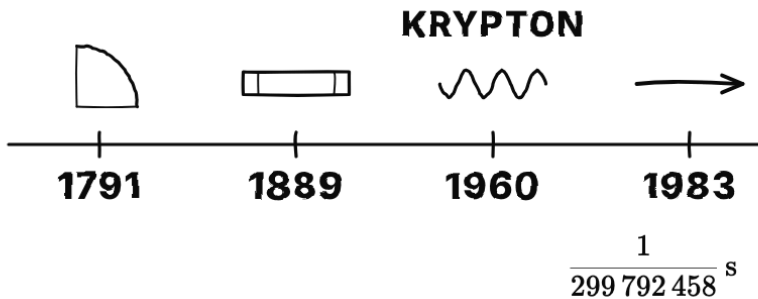
3. Express 10 m/s in kilometres per hour.

Hint: Multiply by 3,600 s per hour, then divide by 1,000 m per kilometre.

Answer: 36 km/h: $10 \times 3,600 = 36,000 \text{ m each hour}$, which is 36 km.

The Metre and the Kilogram

Where, in the end, does the metre on your tape measure come from? For most of history the answer was "from some object kept somewhere safe". Today the answer is "from light", and the change shows how measurement keeps getting sharper.



Four definitions of the metre

The metre (m) was first defined in 1791 as one ten-millionth of the distance from the equator to the North Pole. In 1889 it was redefined as the distance between two lines engraved on a bar of platinum and iridium, now kept near Paris. By 1960 light could do better, and the

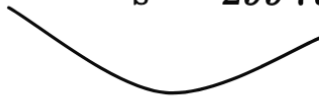
metre became 1,650,763.73 wavelengths of the orange light given off by krypton atoms. Since 1983 it has been the distance light travels in a vacuum in $1/299,792,458$ s.

A caption in the book gives the reasoning: distance travelled is speed multiplied by time, $d = ct$. Here d is the distance, c the speed of light, and t the time. The new definition fixes $c = 299,792,458$ m/s exactly. Put in the time: $d = 299,792,458$ m/s $\times$ $1/299,792,458$ s = 1 m. The large numbers cancel, and exactly one metre is left. So the number can never change again: a better measurement would only let us mark out the metre more precisely. The tape measure in your drawer was checked against copies that trace back, step by step, to this definition.

$$d = ct$$

$$c = 299\,792\,458 \text{ m/s}$$

$$d = 299\,792\,458 \frac{\text{m}}{\text{s}} \times \frac{1}{299\,792\,458} \text{ s} = 1 \text{ m}$$

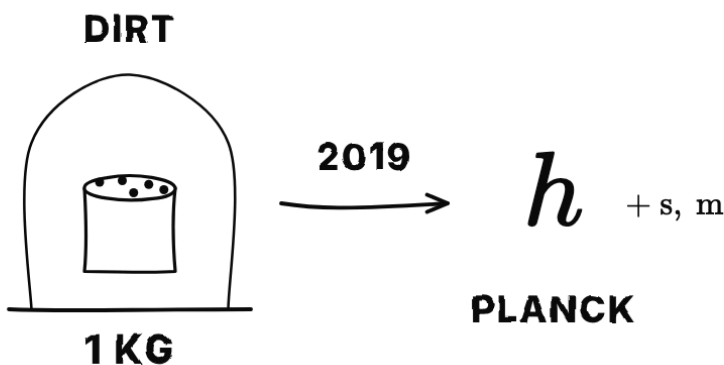


CANCEL

1 m

Why light gives exactly one metre

Now the kilogram (kg). It used to be the mass of one platinum-iridium cylinder, kept with the old metre bar at the International Bureau of Weights and Measures near Paris. Exact copies sat elsewhere, including at the National Institute of Standards and Technology, NIST, in Gaithersburg, Maryland, near Washington. Every other mass could in the end be traced to a comparison with that cylinder. But although the cylinder resisted corrosion, particles from the air stuck to its surface and slowly changed its mass. In May 2019 scientists adopted a more stable definition, built on the second, the metre and Planck's constant, h , a quantum value that links the energy of a particle of light to its frequency. The kilogram on your kitchen scale now rests, through a chain of comparisons, on a constant of nature rather than on a lump of metal.



The fourth fundamental unit, the ampere for electric current, will come much later with electricity and magnetism. The first parts of the book deal with mechanics, fluids, heat and waves, and there every quantity can be built from length, mass and time alone. Said another way, three units, the metre, the kilogram and the second, carry us through the first half of physics.

Remember

The metre went from a fraction of the Earth's size, to an engraved bar, to krypton light, to the distance light travels in $1/299,792,458$ of a second. Because $d = ct$, that definition makes the speed of light exactly $299,792,458$ m/s. The kilogram was a metal cylinder that slowly changed; since 2019 it is defined from the second, the metre and Planck's constant. For mechanics, fluids, heat and waves, length, mass and time are enough.

Questions

1. Since the metre is defined by light, what would a more accurate measurement of light's speed change?
 - a. The number $299,792,458$
 - b. The length of the metre
 - c. Only how precisely labs can mark out a metre
 - d. The length of the second
2. In $d = ct$ with $t = 1/299,792,458$ s, why is d exactly one metre?

- a. Because the large numbers cancel, leaving 1 m
 - b. Because light is very fast
 - c. Because one second is short
 - d. Because the metre was a bar in Paris
3. What problem did the platinum-iridium kilogram have?
- a. It corroded quickly
 - b. It was the wrong size
 - c. Particles from the air stuck to it and slowly changed its mass
 - d. It was lost in 2019
4. Which units are enough for mechanics, fluids, heat and waves?
- a. The ampere alone
 - b. The metre and the ampere
 - c. All four, including the ampere
 - d. The metre, the kilogram and the second

Answers

1. **c)** The speed of light is now exact by definition, so the number cannot change; a better measurement only makes the metre easier to mark out precisely.
2. **a)** $c = 299,792,458 \text{ m/s}$ times $1/299,792,458 \text{ s}$ gives exactly 1 m.
3. **c)** Though resistant to corrosion, the cylinder gathered particles from the air, slowly changing its mass.
4. **d)** In these first subjects, every quantity can be built from length, mass and time.

Worked examples

Light's distance in a given time Using $d = ct$ with $c = 299,792,458$ m/s, how far does light travel in $1/299,792,458$ s?

Distance equals speed times time.

$$d = 299,792,458 \text{ m/s} \times 1/299,792,458 \text{ s}.$$

The numbers cancel and the seconds cancel, leaving metres.

Answer: exactly 1 m

Practice

1. Using $d = ct$ and $c \approx 3.00 \times 10^8$ m/s, how far does light travel in 0.0010 s?

Hint: Multiply the speed by the time.

Answer: 3.0×10^5 m, or 300 km: $3.00 \times 10^8 \times 0.0010 = 3.0 \times 10^5$.

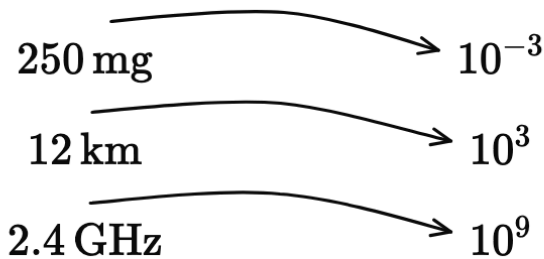
2. In 1791 the metre was one ten-millionth of the distance from the equator to the North Pole. What was that distance in kilometres by this definition?

Hint: That distance is ten million metres.

Answer: 10,000 km: 10,000,000 m divided by 1,000 m per km.

Prefixes and Orders of Magnitude

A medicine label says two hundred and fifty milligrams. Your phone's map says twelve kilometres. A router box says two point four gigahertz. Three different prefixes, milli, kilo and giga, and each one is simply a power of ten stuck onto a unit.



$$100 \text{ cm} = 1 \text{ m}, \quad 1000 \text{ m} = 1 \text{ km}$$

$$12 \text{ in} = 1 \text{ ft}, \quad 5280 \text{ ft} = 1 \text{ mi}$$

Prefixes in everyday labels

The SI units belong to the metric system, which is convenient for science and engineering because its units go up and down in factors of ten. A converting step is always a power of ten: a hundred centimetres in a metre, a thousand metres in a kilometre. In the customary units

still used in the United States, the steps are irregular: twelve inches in a foot, five thousand two hundred and eighty feet in a mile.

The book's table gives each prefix a symbol and a value. Kilo, *k*, is 10^3 , a thousand; so 1 km = 10^3 m, about six tenths of a mile. Milli, *m*, is 10^{-3} , a thousandth; so a millimetre is a thousandth of a metre, about the height of a flea at its shoulders. Giga, *G*, is 10^9 ; a gigahertz is a typical microwave frequency. Mega, *M*, is 10^6 , and micro, μ , is 10^{-6} . Centi, *c*, is 10^{-2} , so a centimetre, about the thickness of a fingertip, is a hundredth of a metre. At the far ends, exa, 10^{18} , gives the exametre, roughly the distance light travels in a century, and atto, 10^{-18} , gives the attosecond, about the time light takes to cross an atom. In between, the femtometre, 10^{-15} m, is the size of a proton.

exa *E* 10^{18}

giga *G* 10^9

mega *M* 10^6

kilo *k* 10^3

centi *c* 10^{-2}

milli *m* 10^{-3}

micro μ 10^{-6}

femto *f* 10^{-15}

atto *a* 10^{-18}

How does a prefix work in a calculation? In words: replace the prefix by its power of ten and multiply. For the medicine: $250 \text{ mg} = 250 \times 10^{-3} \text{ g} = 0.250 \text{ g}$, a quarter of a gram. For the map: $12 \text{ km} = 12 \times 10^3 \text{ m} = 12,000 \text{ m}$. Said another way, a prefix is just a shorthand for a power of ten, and you can always unpack it.

$$\text{prefix} \rightarrow 10^n$$

$$250 \text{ mg} = 250 \times 10^{-3} \text{ g} = 0.250 \text{ g}$$

$$12 \text{ km} = 12 \times 10^3 \text{ m} = 12\,000 \text{ m}$$

nm: optics

m: building

km: flights

Unpacking a prefix

The big advantage is that one unit serves an enormous range. Metres suit building, kilometres suit air travel, and nanometres, 10^{-9} m , suit the design of optical instruments. No new units have to be invented for each job.

Powers of ten also measure the scale of a number, its order of magnitude. Any number can be written as a number times a power of

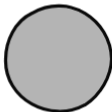
ten. $800 = 8 \times 10^2$, and $450 = 4.5 \times 10^2$. Both carry the factor 10^2 , so they share the same order of magnitude. Order of magnitude is a ballpark: it tells you the step on the ladder of tens, not the exact rung.

$$\left. \begin{array}{l} 800 = 8 \times 10^2 \\ 450 = 4.5 \times 10^2 \end{array} \right\} \text{ **SAME ORDER**}$$



atom 10^{-10} m

$$\frac{10^9}{10^{-10}} = 10^{19}$$



Sun 10^9 m

Orders of magnitude

Now a comparison across the ladder. An atom is on the order of 10^{-10} m across; the Sun is on the order of 10^9 m. To compare, divide: $10^9 / 10^{-10} = 10^{19}$. The rule behind it is that dividing powers of ten subtracts their exponents: nine minus minus ten is nineteen. So the Sun is about nineteen orders of magnitude wider than an atom, ten million million million times.

Try it yourself

Take a millimetre ruler and a long tape measure. Measure your thumbnail in millimetres, then the length of your room in metres. Write both in metres as a number times a power of ten. Count how many powers of ten apart they are: usually three or four.

The book closes with a note on the future of standards. Since matter is built from atoms, physicists would most like units based on tiny, unchanging things in nature, such as the speed of light. Time is already there: the second rests on the oscillations of the cesium atom. Length was once based on the wavelength of light from a certain atom, and now on the more precise speed of light. If the mass of atoms, or of a carefully made silicon sphere, can be measured more precisely than the kilogram standard, mass may follow. Electric charge might one day be based on the charges of electrons and protons, though for now current and charge are tied to large currents and the forces between wires.

Remember

Each metric prefix stands for a power of ten: kilo is ten cubed, milli ten to the minus three, giga ten to the ninth. To use a prefix, replace it by its power of ten: 250 mg is 0.250 g, and 12 km is 12,000 m. The order of magnitude of a number is its power of ten: 800 and 450 are both of order ten squared. Dividing powers of ten subtracts exponents, so the Sun, ten to the ninth metres, is nineteen orders of magnitude wider than an atom, ten to the minus ten. Physicists aim to base all units on tiny, unchanging things; the second already rests on the cesium atom.

Questions

1. How do you use a metric prefix in a calculation?
 - a. Add the prefix's power of ten to the number
 - b. Replace the prefix by its power of ten and multiply
 - c. Divide the number by the prefix's symbol
 - d. Ignore it, since prefixes are only names
2. Why do 800 and 450 share the same order of magnitude?
 - a. Both carry the factor 10^2
 - b. Both are less than 1,000
 - c. Both end in zero
 - d. Both are divisible by 50
3. What rule gives $10^9 / 10^{-10} = 10^{19}$?

- a. Dividing powers of ten multiplies the exponents
 - b. Dividing powers of ten adds the bases
 - c. Dividing powers of ten subtracts the exponents: $9 - (-10) = 19$
 - d. Dividing powers of ten always gives 10^1
4. What is the big advantage of prefixes for everyday and scientific work?
- a. They make numbers more precise
 - b. They replace the need for units
 - c. They work only for lengths
 - d. One unit serves an enormous range, so no new units are needed

Answers

- 1. **b)** A prefix is a shorthand for a power of ten: $250 \text{ mg} = 250 \times 10^{-3} \text{ g}$.
- 2. **a)** $800 = 8 \times 10^2$ and $450 = 4.5 \times 10^2$; the shared power of ten sets the order of magnitude.
- 3. **c)** When powers of ten are divided, their exponents are subtracted.
- 4. **d)** Metres, kilometres and nanometres are one unit with different prefixes, serving building, air travel and optics.

Worked examples

Comparing an atom with the Sun An atom is about 10^{-10} m across and the Sun about 10^9 m . How many orders of magnitude apart are they?

Divide the larger by the smaller: $10^9 / 10^{-10}$.

Dividing powers of ten subtracts exponents: $9 - (-10) = 19$.

So the ratio is 10^{19} .

Answer: 19 orders of magnitude

Practice

1. Write 2.4 GHz in hertz, as a power of ten.

Hint: Giga is 10^9 .

Answer: 2.4×10^9 Hz: replace G by 10^9 .

2. Write 0.075 m in millimetres and 3,500 m in kilometres.

Hint: 1 mm = 10^{-3} m and 1 km = 10^3 m.

Answer: 75 mm and 3.5 km: $0.075 \div 10^{-3} = 75$, and $3,500 \div 10^3 = 3.5$.

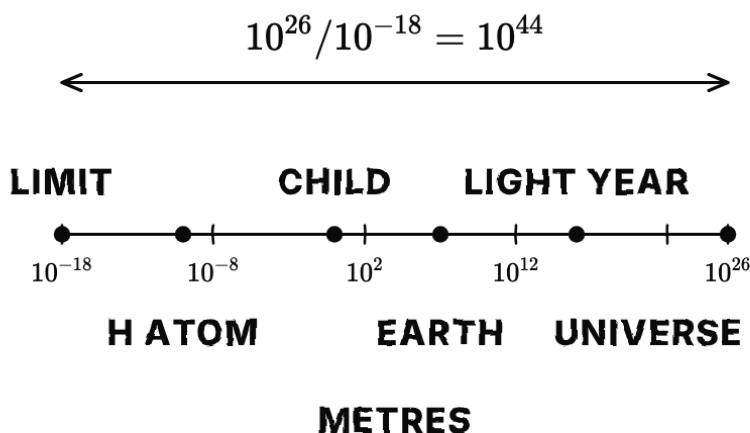
3. A virus is about 10^{-7} m across and a person about 1 m tall (10^0 m).
How many orders of magnitude apart are they?

Hint: Divide and subtract the exponents.

Answer: 7: $10^0 / 10^{-7} = 10^7$, since $0 - (-7) = 7$.

The Range of Nature and the Art of Converting

Two photographs open this section. In the first, tiny phytoplankton drift among ice crystals in the Antarctic sea; they range from a few micrometres, 10^{-6} m each, up to about two millimetres long. In the second, clusters of galaxies collide two point four billion light years away. Between a speck of plankton and a crash of galaxies lies the whole range of physics, and the book's table of lengths, masses and times lets you walk along it.



Forty-four orders of magnitude

Here are a few steps from that table, all as powers of ten. For length: the smallest detail we can currently observe, 10^{-18} m; a hydrogen atom, 10^{-10} m; a four-year-old child, about 1 m; the Earth, 10^7 m across; a light year, 10^{16} m; the edge of the known universe, 10^{26} m. For mass: an electron, 9.11×10^{-31} kg; a mosquito, 10^{-5} kg; a person, 10^2 kg; the Earth, 5.97×10^{24} kg; the Sun, 1.99×10^{30} kg. For time: one oscillation of visible light, 10^{-15} s; one heartbeat, about 1 s; one day, 8.64×10^4 s; one year, 3.16×10^7 s; the age of the universe, about 10^{18} s. From the smallest observable length to the edge of the universe is $10^{26} / 10^{-18} = 10^{44}$: forty-four orders of magnitude. Said another way, no single everyday unit could cope with all this; we need prefixes, and we need a sure way to switch between units.

Switching units is an everyday need too. A European cookbook may give milk in litres when your measuring jug is marked in cups. Walking directions may give feet when you want miles. The book's method works for all of these, and it starts with a simple case: turn 80 metres into kilometres.

$$80 \text{ m} \rightarrow \text{km}$$

$$\frac{1 \text{ km}}{1000 \text{ m}}$$

$$12 \text{ in} = 1 \text{ ft}$$

$$100 \text{ cm} = 1 \text{ m}$$

$$60 \text{ s} = 1 \text{ min}$$

$$80 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}} = 0.080 \text{ km}$$

Converting 80 m to km

Step one: write the unit you have, metres, and the unit you want, kilometres. Step two: find a conversion factor, a ratio that says how many of one unit equal another. There are 12 inches in a foot, 100 centimetres in a metre, 60 seconds in a minute, and 1,000 metres in a kilometre. Step three: multiply what you have by the factor, arranged so the unwanted unit cancels: $80 \text{ m} \times 1 \text{ km}/1000 \text{ m} = 0.080 \text{ km}$.

Let us see each part of that line. The factor $1 \text{ km}/1000 \text{ m}$ has the same length on top and bottom, so it equals exactly one. Multiplying by one never changes the amount; it only changes how the amount is written. The metres in 80 m and the metres under the line cancel, as a number cancels when it stands above and below a fraction bar, so only kilometres remain. The numbers do the rest: 80 divided by 1000 is

0.080. This way of letting the units guide and check the calculation is called dimensional analysis. If the units do not cancel as you want, the factor is upside down, and you know before you finish that something is wrong.

$$\frac{1 \text{ km}}{1000 \text{ m}} = 1$$

$$80 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}} = 0.080 \text{ km}$$

m cancels

DIMENSIONAL ANALYSIS

$$80 \text{ m} \times \frac{1000 \text{ m}}{1 \text{ km}} = 80\,000 \frac{\text{m}^2}{\text{km}} \quad \times$$

The factor equals one

Try it yourself

Write "80 m × 1 km / 1000 m" on paper. Strike out "m" above the line and "m" below it. Read what is left: 80 times 1 km divided by 1000. Now try it wrongly, with "1000 m / 1 km": nothing cancels, and you are left with square metres per kilometre, which is plainly not a distance in kilometres.

The same method handles any pair of units. Your phone's measurement converter works in exactly this way: it multiplies by a stored ratio that equals one. For a fuller list of factors, the book points to its Appendix C.

With these tools, the enormous numbers of the table stop being frightening. Prefixes shrink them to handy sizes, orders of magnitude let you compare them at a glance, and conversion factors let you move between units without losing your way.

Remember

The book's table spans from the smallest observable detail, ten to the minus eighteen metres, to the edge of the universe, ten to the twenty-sixth metres: forty-four orders of magnitude. A conversion factor is a ratio of equal amounts in different units, such as 1 km per 1000 m. Multiply by the factor so the unwanted unit cancels: $80 \text{ m} \times 1 \text{ km}/1000 \text{ m} = 0.080 \text{ km}$. The factor equals one, so the quantity is unchanged; only its unit changes. Letting units cancel to guide and check a calculation is called dimensional analysis.

Questions

1. Why does the factor 1 km/1000 m not change the length being converted?
 - a. Because it has the same length on top and bottom, so it equals one

- b. Because 1000 is a round number
 - c. Because kilometres are fundamental units
 - d. Because it is used only once
2. What is dimensional analysis?
- a. Measuring the dimensions of an object
 - b. Letting the units guide and check a calculation
 - c. Rounding to the right number of digits
 - d. Comparing orders of magnitude
3. If you multiply 80 m by 1000 m/1 km by mistake, what tells you something is wrong?
- a. The answer is too small
 - b. The answer is negative
 - c. The units do not cancel and you get square metres per kilometre
 - d. The answer has too many digits
4. How many orders of magnitude separate the smallest observable length (10^{-18} m) from the edge of the universe (10^{26} m)?
- a. 8
 - b. 18
 - c. 26
 - d. 44

Answers

1. a) Top and bottom are the same length, so the factor equals one; multiplying by one changes only how the amount is written.

- 2. b)** Letting units cancel to guide and check the calculation is called dimensional analysis.
- 3. c)** With the factor upside down nothing cancels, leaving units that are plainly not kilometres.
- 4. d)** $10^{26} / 10^{-18} = 10^{44}$, since $26 - (-18) = 44$.

Worked examples

Metres to kilometres Convert 80 m to kilometres.

We have metres and want kilometres.

The conversion factor is 1 km/1000 m.

$80 \text{ m} \times 1 \text{ km}/1000 \text{ m}$: the metres cancel, leaving km.

$80 \div 1000 = 0.080$.

Answer: 0.080 km

Practice

1. Convert 4.5 km to metres.

Hint: Multiply by 1000 m/1 km so that km cancels.

Answer: 4,500 m: $4.5 \text{ km} \times 1000 \text{ m}/1 \text{ km}$.

2. Convert 250 cm to metres.

Hint: Use 1 m/100 cm.

Answer: 2.5 m: $250 \text{ cm} \times 1 \text{ m}/100 \text{ cm}$.

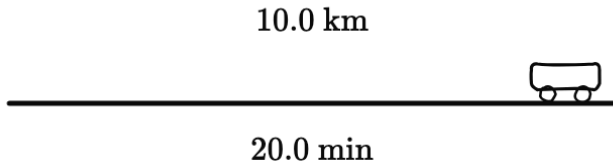
3. One day is 8.64×10^4 s. How many seconds are there in 10 days, as a power of ten?

Hint: Multiply by 10, which adds one to the exponent.

Answer: 8.64×10^5 s: $10 \times 8.64 \times 10^4$ s.

A Short Drive Home

You drive the 10.0 km from school to home, and the clock says the trip took 20.0 min. What was your average speed? The book asks for it twice: first in kilometres per hour, km/h, and then in metres per second, m/s. The plan is simple: calculate the speed in the units given, then convert by multiplying with the factor that cancels the unwanted unit and leaves the wanted one in its place.



$$v = \frac{d}{t} \quad \frac{10.0 \text{ km}}{20.0 \text{ min}} = 0.500 \frac{\text{km}}{\text{min}}$$

$$0.500 \frac{\text{km}}{\cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ h}} = 30.0 \frac{\text{km}}{\text{h}}$$

A drive home: 30.0 km/h

Step one is the definition: average speed = distance/time. In words: how far you went, shared out over the time it took. Step two: put in

the numbers. average speed = $10.0 \text{ km} / 20.0 \text{ min} = 0.500 \text{ km/min}$.
Step three: change minutes into hours. The factor that cancels minutes and leaves hours is $60 \text{ min}/1 \text{ h}$. $0.500 \text{ km/min} \times 60 \text{ min/h} = 30.0 \text{ km/h}$. The minutes on the bottom of the first fraction cancel the minutes on top of the second, and kilometres per hour is left.

Now the checks, which the book insists on. First, did the units cancel properly? Had you turned the factor upside down, $\text{km/min} \times 1 \text{ h}/60 \text{ min} = (1/60) \text{ km}\cdot\text{h}/\text{min}^2$: kilometre-hours per minute squared, which is obviously not a speed in kilometres per hour. Second, is the final unit the one asked for? Yes, kilometres per hour. Third, the digits: 10.0 and 20.0 each have three significant figures, so the answer, 30.0, has three as well. The 60 does not count, because an hour is defined to be exactly sixty minutes; that factor is perfectly precise. Fourth, is the answer reasonable? Twenty minutes is a third of an hour, so in a whole hour you would go three times ten kilometres, which is thirty. Thirty kilometres per hour is a speed you might see on your car's speedometer on a slow street in town, so it passes the common-sense test as well.

Part (b) asks for metres per second. Start with 30.0 km/h , which needs two factors: one changing hours into seconds, $1 \text{ h}/3,600 \text{ s}$, and one changing kilometres into metres, $1,000 \text{ m}/1 \text{ km}$. $30.0 \text{ km/h} \times 1 \text{ h}/3,600 \text{ s} \times 1,000 \text{ m}/1 \text{ km} = 8.33 \text{ m/s}$. Hours cancel hours, kilometres cancel kilometres, and metres per second is left. In plain numbers: thirty times a thousand is thirty thousand, divided by three thousand six hundred is about eight point three three.

$$30.0 \text{ km/h} \rightarrow \text{m/s}$$

$$\frac{1 \text{ h}}{3600 \text{ s}} \qquad \frac{1000 \text{ m}}{1 \text{ km}}$$

$$30.0 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1 \cancel{\text{h}}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} = 8.33 \frac{\text{m}}{\text{s}}$$

$$\frac{500 \text{ m}}{60 \text{ s}} \approx 8.33 \frac{\text{m}}{\text{s}}$$

OTHER ROUTE

$$\text{walking: } 1.4 \text{ m/s} \approx 5 \text{ km/h}$$

From km/h to m/s

There is more than one route. Starting from 0.500 km/min, you would use other factors, sixty seconds per minute and a thousand metres per kilometre, yet you would land on the same 8.33 m/s. Check it: half a kilometre is five hundred metres, and five hundred divided by sixty is about eight point three three. Said another way, as long as every unit you do not want is cancelled, any correct path leads to the same answer.

Try it yourself

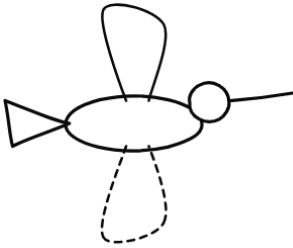
Next time you walk briskly, time yourself over a measured hundred metres with your phone's stopwatch. If it takes about seventy-two seconds, divide one hundred by seventy-two: about one point four metres per second. Then convert to kilometres per hour: multiply by three thousand six hundred seconds per hour, and divide by a thousand metres per kilometre. You should get about five kilometres per hour, a typical walking pace.

Why were all the answers given to exactly three digits? Why not write down everything the calculator shows? The book holds that question over for the section on accuracy, precision and significant figures.

A short note on curious units. Many units are obscure: the firkin, for instance, once measured beer, and one firkin is about thirty-four litres. The book suggests looking up old weights and measures, such as the barleycorn, and working out how each relates to SI units.



1 firkin $\approx$ 34 L



50 BEATS/S

$$1 \text{ ms} = 10^{-3} \text{ s}$$

$$\frac{1 \text{ s}}{50} = 0.02 \text{ s} = 20 \text{ ms}$$

A hummingbird's wingbeat

Finally, choosing a sensible unit for a quick event. Some hummingbirds beat their wings more than fifty times a second. A scientist timing one beat uses the fundamental unit of time, the second, but needs a smaller version of it: the millisecond, 10^{-3} s. In numbers: $1 \text{ s} / 50 = 0.02 \text{ s} = 20 \text{ ms}$ per beat.

Remember

Average speed is distance divided by time: 10.0 km in 20.0 min is 0.500 km/min. Multiplying by 60 min per hour cancels minutes and gives 30.0 km/h; then 1 h per 3,600 s and 1,000 m per km give 8.33 m/s. Check every conversion: the units cancel, the final unit is right, the significant figures match the data, and the answer is reasonable. Exact factors, such as sixty minutes in an hour, do not limit significant figures. For fast events choose a small unit: a hummingbird's wingbeat lasts about twenty milliseconds.

Questions

1. Why is 60 min/h the right factor to turn km/min into km/h?
 - a. Because it makes the number larger
 - b. Because its minutes cancel the minutes in km/min, leaving km/h
 - c. Because there are 60 seconds in a minute
 - d. Because 60 has three significant figures
2. What would show that you wrote the factor upside down, as 1 h/60 min?
 - a. The units come out as km·h/min², not km/h
 - b. The answer comes out negative
 - c. The answer has too many digits
 - d. The answer is exactly 30

3. Why does the answer 30.0 km/h have three significant figures?
- a. Because 60 has two digits
 - b. Because calculators give three digits
 - c. Because 10.0 and 20.0 each have three, and the exact 60 does not limit it
 - d. Because speeds are always given to three digits
4. Starting from 0.500 km/min instead of 30.0 km/h, what answer do you get in m/s?
- a. 8.33×60 m/s
 - b. 0.500 m/s
 - c. 30.0 m/s
 - d. The same 8.33 m/s, by a different route

Answers

1. **b)** The minutes on the bottom of km/min cancel the minutes on top of 60 min/h, leaving kilometres per hour.
2. **a)** Upside down, nothing cancels and you get kilometre-hours per minute squared, plainly not a speed in km/h.
3. **c)** The data each have three significant figures; the 60 is exact, since an hour is defined as sixty minutes.
4. **d)** Five hundred metres per sixty seconds is about 8.33 m/s; any correct route leads to the same result.

Worked examples

A short drive home You drive 10.0 km in 20.0 min. Find the average speed in km/h and in m/s.

$$\text{Average speed} = \text{distance/time} = 10.0 \text{ km} / 20.0 \text{ min} = 0.500 \text{ km/min.}$$

Multiply by 60 min/h: $0.500 \text{ km/min} \times 60 \text{ min/h} = 30.0 \text{ km/h}$.

Multiply by $1 \text{ h}/3,600 \text{ s}$ and $1,000 \text{ m}/1 \text{ km}$: $30.0 \times 1,000 \div 3,600 = 8.33$.

Check: units cancel, final units are right, three significant figures, and 30 km/h is reasonable.

Answer: 30.0 km/h and 8.33 m/s

Practice

1. A runner covers 1.50 km in 6.00 min. What is the average speed in km/h?

Hint: Find km/min first, then multiply by 60 min/h.

Answer: 15.0 km/h: $1.50 \div 6.00 = 0.250 \text{ km/min}$, and $0.250 \times 60 = 15.0 \text{ km/h}$.

2. Convert 90.0 km/h into m/s.

Hint: Multiply by $1,000 \text{ m}/\text{km}$ and by $1 \text{ h}/3,600 \text{ s}$.

Answer: 25.0 m/s: $90.0 \times 1,000 \div 3,600 = 25.0$.

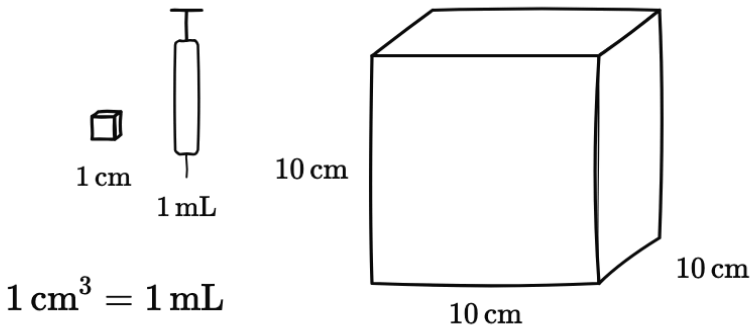
3. A fly beats its wings about 200 times per second. How long is one beat in milliseconds?

Hint: Divide one second by 200, then use $1 \text{ s} = 1,000 \text{ ms}$.

Answer: 5 ms: $1 \text{ s} \div 200 = 0.005 \text{ s}$, which is 5 ms.

Volume from Length, and Two Kinds of Good

Fill a small medicine syringe to the one millilitre mark, and squirt it into a hollow plastic cube one centimetre on each side. It fills the cube exactly. That is no coincidence, and the book uses it as a quick check of understanding.



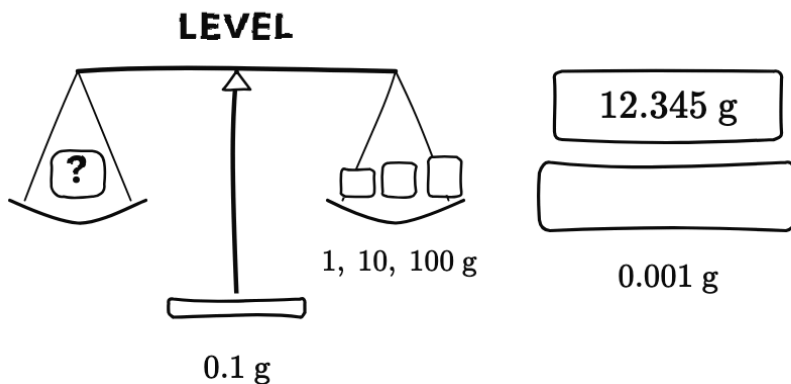
$$(10 \text{ cm})^3 = 1000 \text{ cm}^3 = 1 \text{ L}$$

A litre is a 10 cm cube

The fact is: $1 \text{ cm}^3 = 1 \text{ mL}$. What does it tell us about the metric units? That the unit of volume, the litre, is built from the unit of length, the metre; the size of a millilitre depends on the size of a centimetre. In

other words, volume is a derived unit. Here is the calculation behind it. A cube ten centimetres on a side has a volume of $(10\text{ cm})^3 = 1,000\text{ cm}^3$, and that is exactly one litre. So $1\text{ L} = 1,000\text{ cm}^3$, and dividing both sides by a thousand gives one millilitre per cubic centimetre. The little cup that comes with a bottle of cough syrup, marked in millilitres, is really measuring small cubes of centimetres.

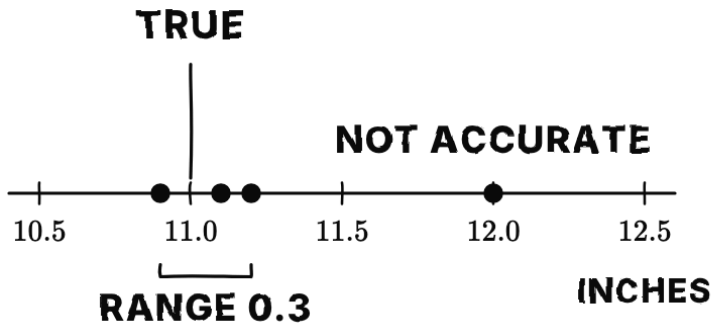
Now a new section: accuracy, precision and significant figures. It begins with two pictures of weighing. The first is a double-pan balance: the unknown object sits in one pan, and metal masses of one gram, ten grams, a hundred grams go in the other until the bar between them is level, which means the masses are equal. The second is a digital scale, which has replaced many such balances. A mechanical balance may read only to the nearest tenth of a gram, but many digital scales read to the nearest thousandth. Your kitchen scale, reading to the nearest gram, sits well below either in precision, yet it is quite good enough for flour.



Balance and digital scale

Science is built on observation and experiment, and so on measurements. Every measurement can be judged in two different ways.

The first is accuracy: how close a measurement is to the correct value. Take a sheet of standard computer paper, sold as 11.0 in. long. You measure it three times and get 11.1 in., 11.2 in. and 10.9 in. Each reading is close to the correct 11.0 in., so the measurements are quite accurate. A reading of 12 in. would not be accurate.



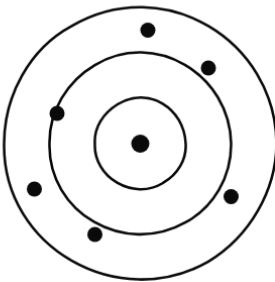
Accurate and precise

The second is precision: how closely repeated measurements, made under the same conditions, agree with each other. One way to judge precision is the range: $\text{range} = \text{highest} - \text{lowest}$. For the paper, $\text{range} = 11.2 \text{ in.} - 10.9 \text{ in.} = 0.3 \text{ in.}$ The readings differ by at most three tenths of an inch, so they are relatively precise. If instead you had measured 10.9, 11.1 and 11.9 in., the range would be $11.9 - 10.9 = 1.0 \text{ in.}$, more than three times as wide, and the measurements would not be very precise. Said another way, accuracy is about the target, precision is about the spread.

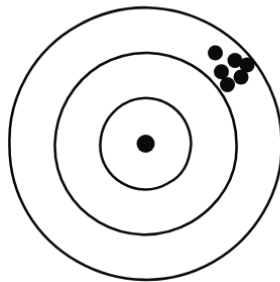
Try it yourself

Weigh the same apple five times on a kitchen scale, lifting it off and putting it back each time. Write down each reading and subtract the smallest from the largest: that range is your scale's precision for this job. If you have a second, better scale, weigh the apple on it too and see how far your first average lies from it: that is a rough check of accuracy.

● = RESTAURANT



**ACCURATE,
NOT PRECISE**



**PRECISE,
NOT ACCURATE**

Two GPS targets

The paper readings were both accurate and precise. But the two qualities can come apart, and the book shows how with a satellite navigation system hunting for a restaurant. Picture the restaurant at

the centre of a bull's-eye, and each attempt to locate it as a black dot. In the first picture, the dots are spread out far apart from one another, but they are centred close to the restaurant. That is low precision with high accuracy: each fix wanders, but on average they point the right way. In the second picture, the dots crowd tightly together, but far away from the restaurant. That is high precision with low accuracy: the system agrees with itself, but it agrees on the wrong place. It is the difference between a phone's map dot that jitters around your true position and one that sits calmly on the wrong side of the street.

Remember

One cubic centimetre equals one millilitre, because the litre is derived from the metre: a ten-centimetre cube holds exactly one litre. Accuracy is closeness to the correct value; precision is closeness of repeated measurements to each other. The range, highest minus lowest, is a simple measure of precision: for the paper, 11.2 minus 10.9 is 0.3 inches. Measurements can be accurate but not precise, like scattered dots around a target, or precise but not accurate, like a tight cluster off target. A digital scale reading to a thousandth of a gram is more precise than a balance reading to a tenth.

Questions

1. Why does one cubic centimetre equal one millilitre?
 - a. Because both are fundamental units

- b. Because the litre is derived from the metre: a 10 cm cube holds exactly 1 L
 - c. Because millilitres are measured with syringes
 - d. Because volume and length are the same quantity
2. For the paper readings 11.1, 11.2 and 10.9 in., what is the range?
- a. 0.3 in.
 - b. 0.1 in.
 - c. 11.0 in.
 - d. 1.0 in.
3. Navigation fixes crowd tightly together but far from the restaurant. What does that show?
- a. High accuracy, low precision
 - b. Low accuracy, low precision
 - c. High precision, low accuracy
 - d. High accuracy, high precision
4. Readings of 10.9, 11.1 and 11.9 in. for paper sold as 11.0 in. are judged how?
- a. Very precise, because one equals 10.9
 - b. Inaccurate, because none is exactly 11.0
 - c. Precise, because their average is near 11
 - d. Not very precise, because their range is 1.0 in.

Answers

1. **b)** A cube 10 cm on a side holds $1,000 \text{ cm}^3$, which is 1 L, so 1 mL is 1 cm^3 .
2. **a)** Range = highest – lowest = $11.2 \text{ in.} - 10.9 \text{ in.} = 0.3 \text{ in.}$

3. c) Tight clustering means high precision; being far from the target means low accuracy.

4. d) A range of 1.0 in. means the readings vary significantly from one to the next.

Worked examples

From litres to cubic centimetres Show that $1 \text{ mL} = 1 \text{ cm}^3$, starting from a cube 10 cm on each side holding 1 L.

Volume of the cube = $(10 \text{ cm})^3 = 1,000 \text{ cm}^3$.

That cube holds 1 L, so $1 \text{ L} = 1,000 \text{ cm}^3$.

A millilitre is a thousandth of a litre: divide both sides by 1,000.

$1 \text{ mL} = 1 \text{ cm}^3$.

Answer: $1 \text{ mL} = 1 \text{ cm}^3$

Practice

1. How many millilitres does a box $5.0 \text{ cm} \times 4.0 \text{ cm} \times 10.0 \text{ cm}$ hold?

Hint: Find its volume in cubic centimetres; each is one millilitre.

Answer: 200 mL: $5.0 \times 4.0 \times 10.0 = 200 \text{ cm}^3 = 200 \text{ mL}$.

2. Four readings of a pencil are 17.6, 17.8, 17.5 and 17.7 cm. What is the range?

Hint: Range = highest – lowest.

Answer: 0.3 cm: $17.8 - 17.5$.

3. How many cubic centimetres are in 2.5 L?

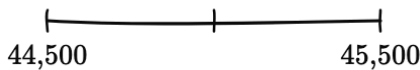
Hint: Use $1 \text{ L} = 1,000 \text{ cm}^3$.

Answer: $2,500 \text{ cm}^3$: $2.5 \times 1,000$.

Uncertainty: The Honest Disclaimer

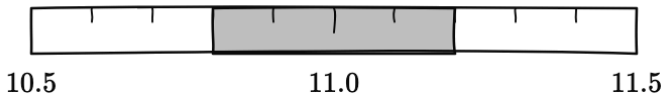
Ask a friend how far their car has gone, and they might say: "Forty-five thousand miles, give or take five hundred." That "give or take" is exactly what physicists call uncertainty.

$$\boxed{45,000} \pm 500 \text{ mi}$$



$$A \pm \delta A$$

$$11 \pm 0.2 \text{ in}$$



$$A \pm \delta A$$

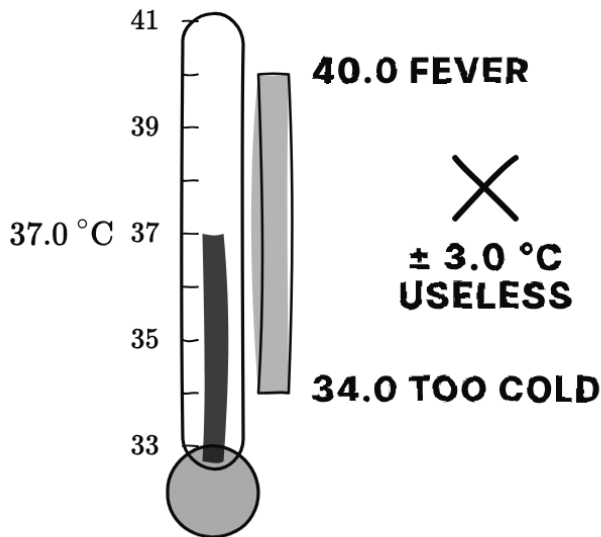
Accuracy and precision both feed into the uncertainty of a measurement. Uncertainty is a measure of how far your measured values may deviate from a standard or expected value. If the measurements are not very accurate or precise, the uncertainty is high. The book calls it a disclaimer for your measured values. For the

car, the uncertainty is the five hundred miles: the true reading may lie anywhere from $45,000 - 500 = 44,500$ to $45,000 + 500 = 45,500$ miles.

The notation is short. If a measured quantity is A , its uncertainty is written δA , and the result is recorded as $A \pm \delta A$. In words: the value, then the amount it might be off in either direction. For the sheet of paper from the last part, $11 \text{ in.} \pm 0.2 \text{ in.}$, so the true length lies between 10.8 in. and 11.2 in. . Every measurement carries some uncertainty; the only question is how much.

Where does it come from? The book names four sources. First, the limits of the measuring device: the smallest division on the ruler may be 0.1 in. . Second, the skill of the person: perhaps their eyesight is poor. Third, irregularities in the object: one side of the paper may be slightly longer than the other. Fourth, anything else that affects the outcome, which depends on the situation. An honest uncertainty must rest on careful thought about all of these.

Why is it so important? The book's real-world case is a sick child. You suspect a fever and take the child's temperature; the thermometer reads 37.0°C , normal body temperature. Now suppose the thermometer's uncertainty were 3.0°C . Then the true temperature lies between $37.0 - 3.0 = 34.0^\circ\text{C}$, hypothermic, and $37.0 + 3.0 = 40.0^\circ\text{C}$, a dangerously high fever. The reading tells you nothing you need to know, so the thermometer would be useless. A home thermometer is useful only because its uncertainty is far smaller than the one or two degrees that separate a normal temperature from a fever. Said another way, an instrument is only good enough when its uncertainty is small compared with the differences you care about.



A thermometer that is useless

Remember

Uncertainty is how far a measurement may deviate from the true or expected value; it is written $A \pm \delta A$. Forty-five thousand miles plus or minus five hundred means anywhere from forty-four thousand five hundred to forty-five thousand five hundred. It comes from the device, the person, the object and other factors. A thermometer with an uncertainty of three degrees would turn a normal reading into anything from hypothermia to high fever, so it would be useless.

Questions

1. What does writing a result as $A \pm \delta A$ tell you?
 - a. That A was measured twice
 - b. The value A and how far it might be off in either direction
 - c. That A is exactly correct
 - d. That δA is the true value
2. A thermometer reading $37.0\text{ }^{\circ}\text{C}$ has an uncertainty of $3.0\text{ }^{\circ}\text{C}$. What range could the true temperature be in?
 - a. $34.0\text{ }^{\circ}\text{C}$ to $40.0\text{ }^{\circ}\text{C}$
 - b. $36.0\text{ }^{\circ}\text{C}$ to $38.0\text{ }^{\circ}\text{C}$
 - c. $37.0\text{ }^{\circ}\text{C}$ to $40.0\text{ }^{\circ}\text{C}$
 - d. Exactly $37.0\text{ }^{\circ}\text{C}$
3. When is an instrument good enough for a job, according to the lecture?
 - a. When it gives many digits
 - b. When it is digital
 - c. When its uncertainty is small compared with the differences you care about
 - d. When it was expensive
4. Poor eyesight of the person reading a ruler is an example of which source of uncertainty?
 - a. Limits of the device
 - b. Irregularities in the object

- c. The situation
- d. The skill of the person making the measurement

Answers

- 1. **b)** The value comes first, then the amount it might deviate either way.
- 2. **a)** $37.0 - 3.0 = 34.0$ and $37.0 + 3.0 = 40.0$, from hypothermic to dangerously high.
- 3. **c)** A thermometer is useful only if its uncertainty is far smaller than the gap between normal and fever.
- 4. **d)** The book lists the person's skill, such as their eyesight, as a source of uncertainty.

Worked examples

Range of a measurement with uncertainty A car's mileage is 45,000 miles $\pm$ 500 miles. Between what values does the true mileage lie?

The measurement is $A \pm \delta A$ with $A = 45,000$ and $\delta A = 500$.

Lowest: $A - \delta A = 45,000 - 500 = 44,500$.

Highest: $A + \delta A = 45,000 + 500 = 45,500$.

Answer: between 44,500 and 45,500 miles

Practice

- 1. A sheet of paper is 11 in. $\pm$ 0.2 in. Between what lengths does it lie?

Hint: Subtract and add δA .

Answer: Between 10.8 in. and 11.2 in.: $11 - 0.2$ and $11 + 0.2$.

2. A thermometer has an uncertainty of $0.1\text{ }^{\circ}\text{C}$ and reads $38.5\text{ }^{\circ}\text{C}$. Can you tell this is above a normal $37.0\text{ }^{\circ}\text{C}$?

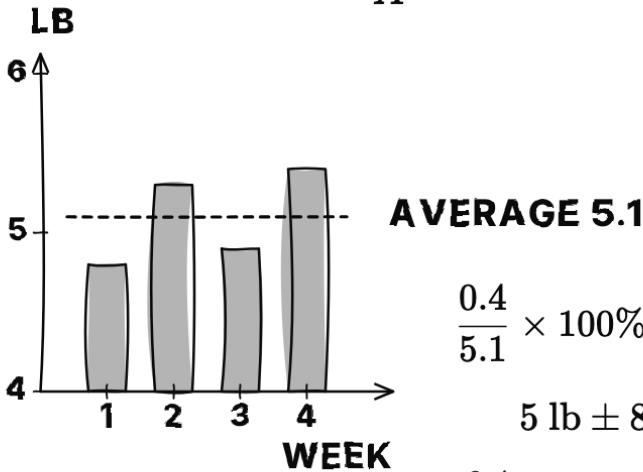
Hint: Find the range the true value could lie in.

Answer: Yes: the true value lies between $38.4\text{ }^{\circ}\text{C}$ and $38.6\text{ }^{\circ}\text{C}$, well above $37.0\text{ }^{\circ}\text{C}$, so the instrument is good enough for this purpose.

Percent Uncertainty

Is an uncertainty of half a pound large or small? For a sack of potatoes it is nothing; for a bag of sweets it is enormous. To judge an uncertainty, we need to compare it with the size of what is measured. That comparison is the percent uncertainty.

$$\% \text{ unc} = \frac{\delta A}{A} \times 100\%$$



$$\frac{0.4}{5.1} \times 100\% = 8\%$$

$$5 \text{ lb} \pm 8\%$$

$$\frac{0.4}{2.55} \times 100\% \approx 16\%$$

Percent uncertainty of a bag of apples

The book defines it this way: $\% \text{unc} = \delta A / A \times 100\%$. Here A is the measured value and δA its uncertainty. In words: divide the uncertainty by the value, and multiply by a hundred per cent. Why divide? Because it asks what fraction of the measurement the

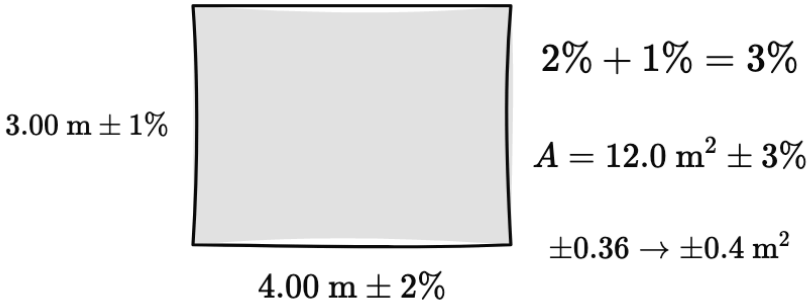
uncertainty is; half a pound out of fifty is one per cent, but half a pound out of one is fifty per cent.

The book's example is a five-pound bag of apples. You buy one each week for four weeks and weigh them: 4.8, 5.3, 4.9 and 5.4 lb. From other measurements, the uncertainty in the average weight is known to be 0.4 lb. First the average: $(4.8 + 5.3 + 4.9 + 5.4)/4 = 20.4/4 = 5.1$ lb. It agrees with the label's 5 lb within the 0.4 lb uncertainty. Then the percent uncertainty: $\%unc = 0.4 \text{ lb}/5.1 \text{ lb} \times 100\% = 8\%$. The pounds cancel, leaving a pure percentage. So the bag weighs $5 \text{ lb} \pm 8\%$. On your kitchen scale, eight per cent of a five-pound bag is about one small apple's worth.

The book asks you to consider a bag half as heavy with the same 0.4 lb uncertainty. Then $0.4/2.55 \times 100\% \approx 16\%$, twice as large. Said another way, the same absolute error weighs twice as much on a value half the size. And one practical hint: always multiply the fraction by one hundred per cent, or you will be left with a decimal, not a percentage.

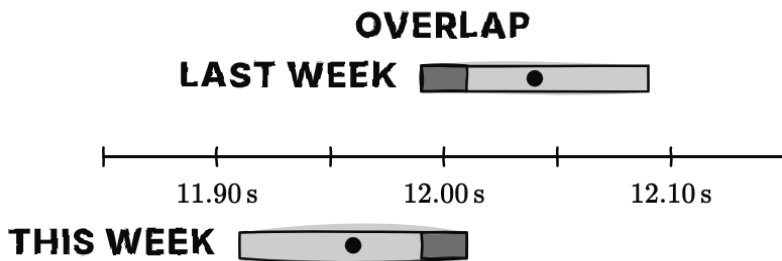
Uncertainty also travels into calculations. Anything calculated from measured quantities is uncertain too: the area of a floor, worked out from its length and width, inherits their uncertainties. For multiplication and division, when each uncertainty is small, a few per cent or less, the book gives a rule: add the percent uncertainties. For a floor 4.00 m long with 2% uncertainty and 3.00 m wide with 1%, the area is 12.0 m^2 with an uncertainty of $2\% + 1\% = 3\%$. As an area, three per cent of 12.0 is 0.36 m^2 , rounded to 0.4 m^2 , because the area is given to a tenth of a square metre.

multiply $\rightarrow$ add the percents



Adding percents for a floor

Finally, uncertainty decides whether two measurements can be told apart. A coach's new stopwatch has an uncertainty of $\pm 0.05 \text{ s}$. The best sprinter ran 100 m in 12.04 s last week and 11.96 s this week. The difference is $12.04 - 11.96 = 0.08 \text{ s}$. But last week's true time could be as low as 11.99 s, and this week's as high as 12.01 s, so the two ranges overlap. The book's verdict: we cannot conclude that this week was faster; the stopwatch is too uncertain to distinguish the times. It is the same reason a cheap bathroom scale cannot tell you whether you lost a hundred grams overnight.



$$12.04 - 11.96 = 0.08 \text{ s}$$

Faster or not?

Remember

Percent uncertainty is the uncertainty divided by the measured value, times one hundred per cent. Bags averaging 5.1 lb with 0.4 lb uncertainty give about eight per cent; a bag half as heavy would give about sixteen. When multiplying or dividing measurements with small uncertainties, add their percent uncertainties. A 4.00 m by 3.00 m floor with two and one per cent uncertainties has an area of 12.0 square metres, plus or minus about 0.4. Two measurements can be told apart only if their difference is larger than their uncertainties allow.

Questions

1. Why is an uncertainty of half a pound small for a sack of potatoes but large for a bag of sweets?
 - a. Because potatoes are heavier, so the same error is a smaller fraction of the value
 - b. Because sweets are measured less carefully
 - c. Because percent uncertainty ignores weight
 - d. Because potatoes are fruit
2. Why do the pounds disappear in $\%unc = 0.4 \text{ lb} / 5.1 \text{ lb} \times 100\%$?
 - a. Because pounds are not SI units
 - b. Because the pounds on top and bottom cancel, leaving a pure percentage
 - c. Because the answer is in kilograms
 - d. Because 0.4 is small
3. A floor's length has 2% uncertainty and its width 1%. What is the percent uncertainty of its area?
 - a. 1%
 - b. 2%
 - c. 3%
 - d. $2\% \times 1\% = 2\%$
4. Why can't we say 11.96 s is faster than 12.04 s with a ± 0.05 s stopwatch?
 - a. Because 0.08 s is too large
 - b. Because the sprinter was tired

- c. Because stopwatches are always slow
- d. Because the possible ranges of the two times overlap

Answers

- 1. **a)** Percent uncertainty divides the uncertainty by the value, so the same error is a smaller share of a larger value.
- 2. **b)** Dividing pounds by pounds leaves a pure number, expressed as a percentage.
- 3. **c)** For multiplication with small uncertainties, the percent uncertainties add: $2\% + 1\% = 3\%$.
- 4. **d)** Last week could be as low as 11.99 s and this week as high as 12.01 s, so the times cannot be distinguished.

Worked examples

Percent uncertainty of a bag of apples Four bags weigh 4.8, 5.3, 4.9 and 5.4 lb; the uncertainty in the average is 0.4 lb. Find the percent uncertainty.

Average: $(4.8 + 5.3 + 4.9 + 5.4)/4 = 20.4/4 = 5.1$ lb.

Use $\%unc = \delta A/A \times 100\%$.

$0.4 \text{ lb} / 5.1 \text{ lb} \approx 0.078$.

$\times 100\% \approx 8\%$.

Answer: about 8%

Practice

- 1. A rod measures $2.00 \text{ m} \pm 0.04 \text{ m}$. What is its percent uncertainty?

Hint: Divide δA by A and multiply by 100%.

Answer: 2%: $0.04/2.00 = 0.02$, times 100% = 2%.

2. A box is 0.50 m long (4% uncertainty) and 0.20 m wide (5% uncertainty). What is the percent uncertainty of its area?

Hint: Add the percent uncertainties for a product.

Answer: 9%: $4\% + 5\% = 9\%$, for an area of $0.10 \text{ m}^2 \pm$ about 0.009 m^2 .

3. Two runs are timed at 10.50 s and 10.30 s with a stopwatch uncertain by ± 0.05 s. Can you say the second was faster?

Hint: Find the range of each time and see whether they overlap.

Answer: Yes: the first lies between 10.45 and 10.55 s, the second between 10.25 and 10.35 s; the ranges do not overlap.

Significant Figures and the Puzzle of Zeros

Lay a stick against a ruler and look at its end. It sits somewhere between the 36.6 cm mark and the 36.7 cm mark. You squint, decide it is nearer 36.7, and write that down. Could you write 36.71 cm instead?

The book's answer starts with the tool. A precise measuring tool is one that measures in very small steps. A standard ruler reads to the nearest millimetre, 0.1 cm; a caliper, a sliding gauge with fine jaws, reads to the nearest 0.01 mm, ten times finer than even a tenth of a millimetre. The caliper is the more precise tool because it detects much smaller differences in length. And the more precise the tool, the more precise and accurate the measurements can be. Your kitchen scale reading to the nearest gram and a jeweller's scale reading to a thousandth of a gram differ in exactly this way.

When we write a measured value, we may give only as many digits as the tool measured. With a ruler, 36.7 cm is fine; 36.71 cm is not, because a ruler cannot measure a hundredth of a centimetre. Notice that the last digit, the 7, was already an estimate: you judged whether the end was nearer 36.6 or 36.7. That is the rule of significant figures: the last digit written is the first digit with some uncertainty. To count significant figures, start at the first measured digit on the left and count through the last digit written on the right. So 36.7 cm has three significant figures, and they tell you how precise the tool was.

Zeros need special care. In 0.053 the zeros only locate the decimal point; they are placekeepers, so 0.053 has two significant figures. In 10.053 the zeros sit between measured digits, so they count: five significant figures. In 1300 the zeros are ambiguous: they could mean the number is known to the last unit, or just hold places, so it might have two, three or four significant figures. Scientific notation settles it: 1.3×10^3 has two significant figures, 1.30×10^3 has three, and 1.300×10^3 has four. In one line: zeros are significant except when they serve only as placekeepers.

	SIG. FIGS
0.0009	1
15,450.0	6
6×10^3	1
87.990	5
30.42	4

Which zeros count

The book gives five numbers to try. 0.0009 has one significant figure: the zeros are placekeepers marking the decimal point. 15,450.0 has six: the zeros show the measurement was made to a tenth, so they

count. 6×10^3 has one: the power of ten only places the decimal point, it is not a measured digit. 87.990 has five: the final zero shows a measurement to a thousandth. 30.42 has four: a zero between significant figures always counts.

Try it yourself

Take a ruler and measure the width of your phone. Write the result with exactly as many digits as the ruler allows, ending with your estimated digit. Then count the significant figures, and ask whether a caliper would let you add one more.

Remember

A precise tool measures in small steps: a ruler to the millimetre, a caliper to a hundredth of a millimetre. Write only the digits your tool measured; the last one is the first uncertain digit. Count significant figures from the first measured digit to the last one written: 36.7 cm has three. Zeros that only place the decimal point are not significant; zeros between digits or measured at the end are. Scientific notation removes ambiguity: 1.30×10^3 has three significant figures.

Questions

1. Why can't a ruler measurement be written as 36.71 cm?
 - a. Because rulers are inaccurate

- b. Because a ruler cannot measure a hundredth of a centimetre
 - c. Because 36.71 has too few digits
 - d. Because the stick is shorter
2. What is special about the last digit written in a measurement?
- a. It is always zero
 - b. It is copied from the scale's label
 - c. It is the first digit with some uncertainty
 - d. It is always exact
3. How does scientific notation settle the number of significant figures in 1300?
- a. Writing 1.30×10^3 shows exactly three significant figures
 - b. It always gives four
 - c. It removes all the zeros as meaningless
 - d. It changes the value of the number
4. Why does 30.42 have four significant figures?
- a. Because it is more than 30
 - b. Because the zero is a placekeeper
 - c. Because it has a decimal point
 - d. Because a zero between significant figures always counts

Answers

1. **b)** You may give only as many digits as the tool measured, and a ruler reads to millimetres.
2. **c)** The last digit written is an estimate, the first one with some uncertainty.

3. a) 1.3×10^3 has two, 1.30×10^3 three and 1.300×10^3 four significant figures.

4. d) Zeros located between significant figures are themselves significant.

Worked examples

Counting significant figures How many significant figures are in 0.0009, 15,450.0, 6×10^3 , 87.990 and 30.42?

0.0009: the zeros only place the decimal point, so 1.

15,450.0: the zeros show a measurement to a tenth, so 6.

6×10^3 : the power of ten only places the decimal point, so 1.

87.990: the final zero is measured, so 5; 30.42: the zero between digits counts, so 4.

Answer: 1, 6, 1, 5 and 4

Practice

1. How many significant figures are in 0.00450, 2.050 and 7.0×10^4 ?

Hint: Leading zeros are placekeepers; trailing zeros after a decimal point are measured.

Answer: 3, 4 and 2: 0.00450 has 4, 5, 0; 2.050 has all four digits; 7.0×10^4 has 7 and 0.

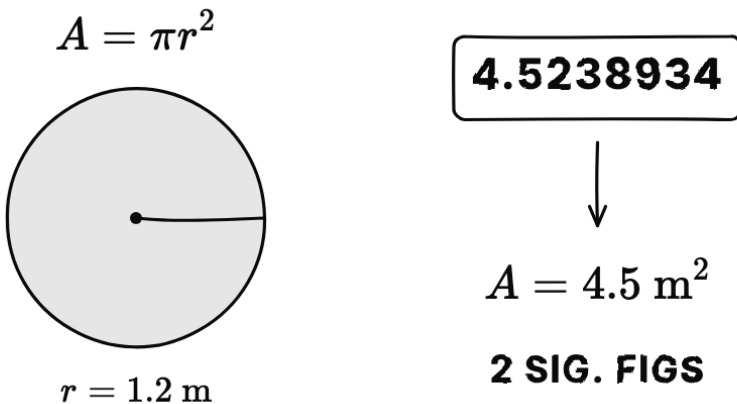
2. Write 52,000 in scientific notation with exactly three significant figures.

Hint: Move the decimal point after the first digit and keep three digits.

Answer: 5.20×10^4 .

Significant Figures in Calculations

Measure the radius of a round table as 1.2 m, then ask a calculator for its area. It shows 4.5238934 m². Eight digits, from a tape measure that gave you two. Which of them can you trust?



Rounding the area of a circle

The book's principle is simple: when measurements of different precision are combined, the answer can have no more significant digits than the least precise measurement allows. There are two rules, one for multiplying and dividing, and one for adding and subtracting.

Rule one, for multiplication and division: the result keeps as many significant figures as the input with the fewest. The area of a circle is $A = \pi r^2$: pi times the radius times the radius. With $r = 1.2$ m, the calculator gives $A = 3.1415927... \times (1.2 \text{ m})^2 = 4.5238934 \text{ m}^2$. The radius has only two significant figures, so the area must have two: $A = 4.5 \text{ m}^2$. It does not matter that pi is known to at least eight digits; the radius is the weak link. Why? Because the radius could easily be 1.15 or 1.25 m, and then the area would lie anywhere between about 4.2 and 4.9 square metres; the digits after 4.5 are pure decoration.

Rule two, for addition and subtraction: the answer can have no more decimal places than the least precise measurement. The book's example follows some potatoes. You buy 7.56 kg in a shop, on a scale reading to 0.01 kg. You leave 6.052 kg at your laboratory, weighed on a scale reading to 0.001 kg. At home you add 13.7 kg, weighed on a bathroom scale reading only to 0.1 kg. The total is $7.56 - 6.052 + 13.7 = 15.208$ kg. The least precise reading, 13.7 kg, goes only to tenths, so the answer is rounded to tenths: 15.2 kg. Said another way, a sum is only as fine as its coarsest scale. It is like adding a spoonful weighed on a jeweller's scale to a sack weighed on your bathroom scale: the total is only known to the bathroom scale's step.

POTATOES, KG

$$\begin{array}{r} 7.56 \\ - 6.052 \\ + 13.7 \\ \hline 15.208 \end{array}$$

0.01 kg
0.001 kg
0.1 kg

→ 15.2 kg

Adding potatoes

The book then states its own conventions. Most numbers in the text are taken to have three significant figures, and the worked examples keep that consistent. An answer to three digits rests on input good to at least three digits; with fewer digits in, there are fewer out. In some topics, especially optics, more accuracy is needed and more than three digits are used. And exact numbers do not limit a result: the two in the circumference of a circle, $c = 2\pi r$, is exactly two, not a measurement.

Two exercises close the section. First: a woman has three bags; two weigh 13.5 pounds each and one weighs 10.2 pounds. The total is $2 \times 13.5 + 10.2 = 37.2$ lb. The 2 is a count of bags, an exact number, so it does not limit the significant figures. Second: force equals mass times

acceleration, $F = ma$. A wagon of 55 kg accelerates at 0.0255 m/s^2 . $F = 55 \text{ kg} \times 0.0255 \text{ m/s}^2 = 1.4025 \text{ N}$, and since 55 kg has two significant figures, the answer is 1.4 N.

Remember

In multiplication and division, keep as many significant figures as the least precise input: a 1.2 m radius gives an area of 4.5 square metres. In addition and subtraction, keep as many decimal places as the least precise input: the potatoes total 15.2 kg. This book usually works to three significant figures. Exact numbers, such as the 2 in $2\pi r$ or a count of bags, never limit the result. A 55 kg wagon at 0.0255 m/s^2 feels a force of 1.4 N, two significant figures.

Questions

1. Why must the area of a circle of radius 1.2 m be given as 4.5 m^2 ?
 - a. Because pi has only two digits
 - b. Because the radius has two significant figures, and the product keeps the fewest
 - c. Because areas are always rounded to tenths
 - d. Because the calculator rounds
2. Why is the potato total rounded to 15.2 kg?
 - a. Because the least precise reading, 13.7 kg, goes only to tenths

- b. Because 15.208 has too many digits for a sum
 - c. Because kilograms must be rounded to tenths
 - d. Because the shop scale reads to hundredths
3. Why does the 2 in $c = 2\pi r$ not limit significant figures?
- a. Because it is small
 - b. Because it is even
 - c. Because it is exact, not a measurement
 - d. Because it multiplies pi
4. A wagon of 55 kg accelerates at 0.0255 m/s^2 . Why is the force 1.4 N and not 1.4025 N?
- a. Because force is measured in whole newtons
 - b. Because 0.0255 has five digits
 - c. Because acceleration is uncertain
 - d. Because 55 kg has only two significant figures

Answers

- 1. **b)** In multiplication the result keeps as many significant figures as the least precise input, the radius.
- 2. **a)** In addition and subtraction, the answer keeps no more decimal places than the least precise measurement.
- 3. **c)** Exact numbers do not limit the precision of a result.
- 4. **d)** The product keeps as many significant figures as the least precise input, 55 kg with two.

Worked examples

Area of a round table A table has radius $r = 1.2$ m. Find its area $A = \pi r^2$ to the correct significant figures.

The calculator gives $3.1415927... \times (1.2 \text{ m})^2 = 4.5238934 \text{ m}^2$.

For multiplication, keep the fewest significant figures among the inputs.

The radius 1.2 m has two; pi has many more.

Round to two significant figures.

Answer: 4.5 m^2

Practice

1. Add 2.45 m, 0.3 m and 1.234 m and give the result to the correct decimal places.

Hint: Keep as many decimal places as the least precise value.

Answer: 4.0 m: the sum is 3.984 m, and 0.3 m goes only to tenths.

2. A box has a mass of 4.0 kg and accelerates at 1.25 m/s^2 . Using $F = ma$, what is the force to the right significant figures?

Hint: Multiply, then keep the fewest significant figures.

Answer: 5.0 N: $4.0 \times 1.25 = 5.0$, and 4.0 kg has two significant figures.

3. A circle has radius 2.00 m. What is its circumference, $c = 2\pi r$, to the right significant figures?

Hint: The 2 is exact; the radius has three significant figures.

Answer: 12.6 m: $2 \times 3.14159 \times 2.00 = 12.566$, rounded to three significant figures.

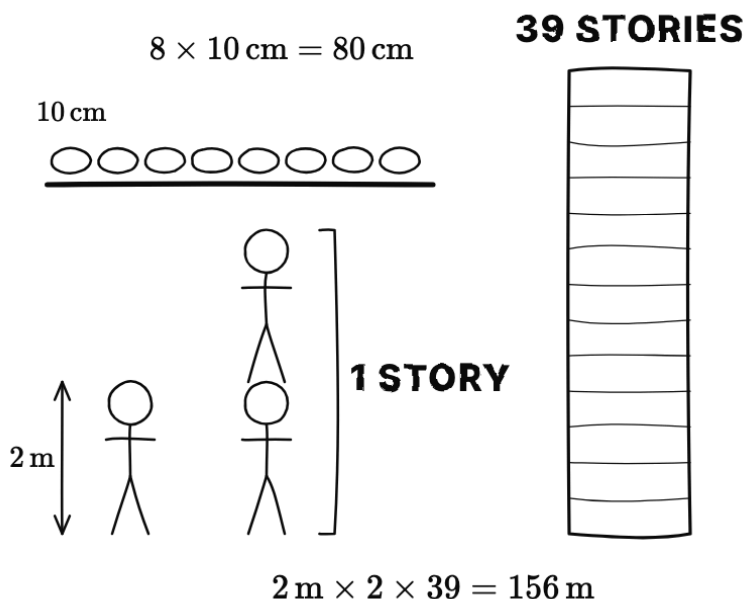
Approximation: How High Is a Trillion Dollars?

In 2021 the United States federal debt was a little more than twenty-eight trillion dollars. Can you picture even one trillion? Almost nobody can. The last section of the chapter shows how a few rough numbers, careful units and a willingness to guess can make such a quantity real.

Physicists, scientists and engineers constantly need approximations, or "guesstimates". How far away is a certain place? Roughly how dense is some object? About how large a current will flow in a circuit? Many approximate numbers come from formulas whose inputs are known only to limited accuracy. The book says this skill grows alongside problem-solving: by thinking in numbers, by being willing to take risks, and with experience and a good feel for units.

Approximations let us rule out impossible scenarios and unrealistic numbers, challenge other people's claims, and steer our own approach to a problem.

The first example: estimate the height of a 39-storey building. The strategy is to scale up from something familiar, the height of a person. An adult is about 2 m tall, and one storey is about the height of two adults. So the estimate is $2 \text{ m/person} \times 2 \text{ persons/storey} \times 39 \text{ storeys} = 156 \text{ m}$. Look at the units: persons cancel persons, storeys cancel storeys, and metres remain. Dimensional analysis, which we met with conversion factors, works just as well for estimates.



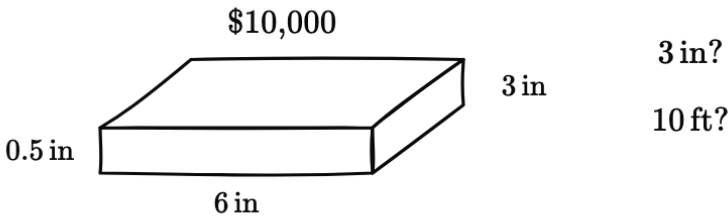
A 39-story building in people

Try it yourself

Measure the width of your hand once with a ruler; say it is 10 cm. Now walk your hand across your desk, counting hand-widths. If you count eight, the desk is about $8 \times 10 \text{ cm} = 80 \text{ cm}$. Then ask yourself, as the book does, what else besides length you could estimate this way.

Now the trillion dollars. Suppose you were given a trillion dollars in \$100 bills, bundled into bank stacks of one hundred bills, each stack worth \$10,000. You lay the stacks evenly over an American football field, between the end zones. How high is the pile? One friend guesses

3 in.; another says 10 ft. The book works in inches and feet here, because American football fields are measured in yards.



$$6 \times 3 \times 0.5 = 9 \text{ in}^3$$

$$\frac{10^{12}}{10^4} = 10^8 \text{ stacks}$$

A trillion dollars in stacks

The strategy: start with something easy to picture, one stack; find its volume; find how many stacks make a trillion; then spread the total volume over the field's area to find the height.

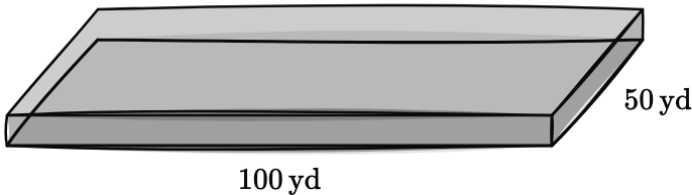
Step one, the volume of one stack. A bill is about 3 in. by 6 in., and a stack of one hundred is about 0.5 in. thick. volume of stack = 6 in. $\times$ 3 in. $\times$ 0.5 in. = 9 in.³. That is roughly the size of a chunky smartphone.

Step two, the number of stacks. A trillion dollars is $\$1 \times 10^{12}$, and one stack is $\$1 \times 10^4$. $\$1 \times 10^{12} / \$1 \times 10^4$ per stack = 1×10^8 stacks: a

hundred million stacks. Dividing powers of ten subtracts exponents, twelve minus four, as we saw with prefixes.

Step three, the area of the field in square inches. The field is $100 \text{ yd} \times 50 \text{ yd} = 5,000 \text{ yd}^2$. To convert, use the factors three feet per yard and twelve inches per foot, each twice, because the area has two dimensions: $5,000 \text{ yd}^2 \times 3 \text{ ft/yd} \times 3 \text{ ft/yd} \times 12 \text{ in./ft} \times 12 \text{ in./ft} = 6,480,000 \text{ in.}^2$. For a rough estimate we keep one significant figure: about $6 \times 10^6 \text{ in.}^2$.

$$V = 9 \times 10^8 \text{ in}^3$$



$$A \approx 6 \times 10^6 \text{ in}^2$$

$$h = \frac{9 \times 10^8}{6 \times 10^6} = 150 \text{ in} \qquad \approx 12 \text{ ft}$$

How high the money piles

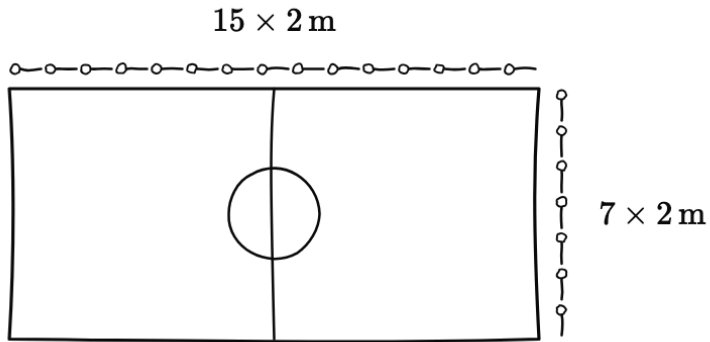
Step four, the total volume: $9 \text{ in.}^3/\text{stack} \times 10^8 \text{ stacks} = 9 \times 10^8 \text{ in.}^3$.

Step five, the height. Volume equals area times height, so height equals volume divided by area. $\text{height} = 9 \times 10^8 \text{ in.}^3 / 6 \times 10^6 \text{ in.}^2 =$

$1.5 \times 10^2 \text{ in.} = 150 \text{ in.}$ Cubic inches divided by square inches leaves inches, exactly the unit a height should have. In feet: $150 \text{ in.} \times 1 \text{ ft}/12 \text{ in.} = 12.5 \text{ ft} \approx 12 \text{ ft.}$ Picture a whole football field covered with money piled about one and a half times as high as the ceiling of your living room.

So the guess of 3 in. was far too low, while the guess of 10 ft, which is 120 in., was roughly right. The book asks how your own first guess compared, and what this says about quick guesses versus careful approximations. Said another way, a few honest steps with units turn a wild guess into a number you can defend.

One last check of understanding: estimate the area of a regulation basketball court. Use an average man, about 2 m tall, as your ruler. About fifteen men end to end cover the length, about $15 \times 2 \text{ m} = 30 \text{ m}$, and about seven cover the width, $7 \times 2 \text{ m} = 14 \text{ m}$. The area is about $30 \text{ m} \times 14 \text{ m} = 420 \text{ m}^2$.



A basketball court in people

With that the chapter closes: a few unifying laws, shared units and standards, honest uncertainty, and finally the confidence to estimate the world in your head.

Remember

Approximations rule out unrealistic numbers, test claims and guide problem-solving. A 39-storey building is about 2 m per person times 2 persons per storey times 39 storeys, about 156 m. A trillion dollars in \$100 bills is 10^8 stacks of 9 cubic inches, 9×10^8 cubic inches in all. Spread over a football field of about 6×10^6 square inches, the pile is about 150 inches, roughly 12 feet, high. A basketball court is about 15 by 7 two-metre people, roughly 420 square metres.

Questions

1. In $2 \text{ m/person} \times 2 \text{ persons/storey} \times 39 \text{ storeys}$, why is the answer in metres?
 - a. Because 39 is a count
 - b. Because persons and storeys cancel, leaving metres
 - c. Because buildings are always measured in metres
 - d. Because 156 is a large number
2. Why does dividing 10^{12} by 10^4 give 10^8 ?
 - a. Because dividing powers of ten subtracts exponents: $12 - 4 = 8$
 - b. Because 12 divided by 4 is 3
 - c. Because powers of ten add when divided
 - d. Because a stack holds 10^8 dollars

3. Why does the area conversion use 3 ft/yd and 12 in./ft twice each?
 - a. Because the field is 100 yd long
 - b. Because one significant figure is kept
 - c. Because area has two dimensions, so each length factor appears twice
 - d. Because inches are smaller than feet
4. Why does volume divided by area give a height in inches?
 - a. Because the field is measured in yards
 - b. Because the stacks are rectangular
 - c. Because one significant figure is kept
 - d. Because cubic inches divided by square inches leaves inches

Answers

1. **b)** Persons cancel persons and storeys cancel storeys, so dimensional analysis leaves metres.
2. **a)** Dividing powers of ten subtracts their exponents, as seen with prefixes.
3. **c)** Square yards must be converted in both length and width, so each factor is used twice.
4. **d)** $\text{in.}^3/\text{in.}^2 = \text{in.}$, exactly the unit a height should have.

Worked examples

A trillion dollars on a football field A trillion dollars in \$100 bills, in stacks of 6 in. $\times$ 3 in. $\times$ 0.5 in. worth \$10,000 each, is spread over a 100 yd $\times$ 50 yd field. How high is the pile?

One stack: $6 \times 3 \times 0.5 = 9 \text{ in.}^3$.

Stacks: $\$10^{12} / \$10^4 = 10^8$, so total volume $9 \times 10^8 \text{ in.}^3$.

Field: $5,000 \text{ yd}^2 \times (3 \text{ ft/yd})^2 \times (12 \text{ in./ft})^2 = 6,480,000 \text{ in.}^2 \approx 6 \times 10^6 \text{ in.}^2$.

Height = $9 \times 10^8 / 6 \times 10^6 = 1.5 \times 10^2 \text{ in.} = 150 \text{ in.}$, about 12 ft.

Answer: about 150 in., roughly 12 ft

Practice

- 1.** Estimate the height of a 25-storey building using 2 m per person and 2 persons per storey.

Hint: Multiply the factors so that persons and storeys cancel.

Answer: 100 m: $2 \text{ m/person} \times 2 \text{ persons/storey} \times 25 \text{ storeys}$.

- 2.** How many square inches are in one square yard?

Hint: Use 3 ft/yd and 12 in./ft, each twice.

Answer: 1,296 in.²: $3 \times 3 \times 12 \times 12$.

- 3.** A billion dollars (10^9) in the same bank stacks: how many stacks is that, and what volume?

Hint: Divide by $\$10^4$ per stack, then multiply by 9 in.^3 per stack.

Answer: 10^5 stacks, a hundred thousand, with a volume of $9 \times 10^5 \text{ in.}^3$.