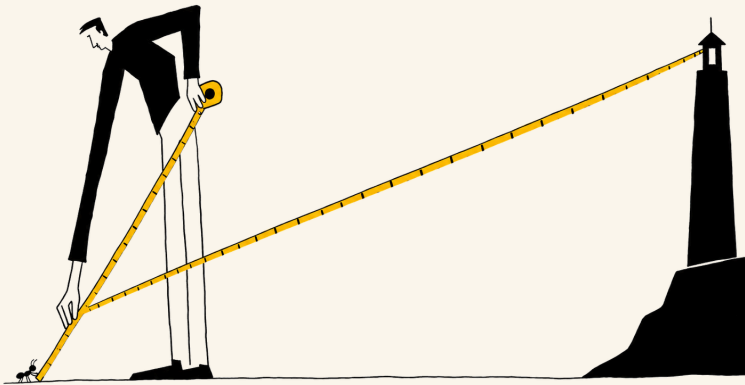


College Physics



Paul Peter Urone, Roger Hinrichs · 2022

Illustrated audio lectures

Lecturized

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Contents

The Same Forces, Out There and Here

A Few Laws for a Crowded World

Physics Without Being a Physicist

Models, Theories and Laws

Guess, Test, Guess Again

Old Physics, New Physics

Numbers Need Units

A Second, a Metre, a Kilogram

Powers of Ten

From Plankton to Galaxies, and Back to Kilometres

Finishing the Drive Home

Close to the Truth, Close to Each Other

Plus or Minus

Uncertainty Travels, and Digits Tell

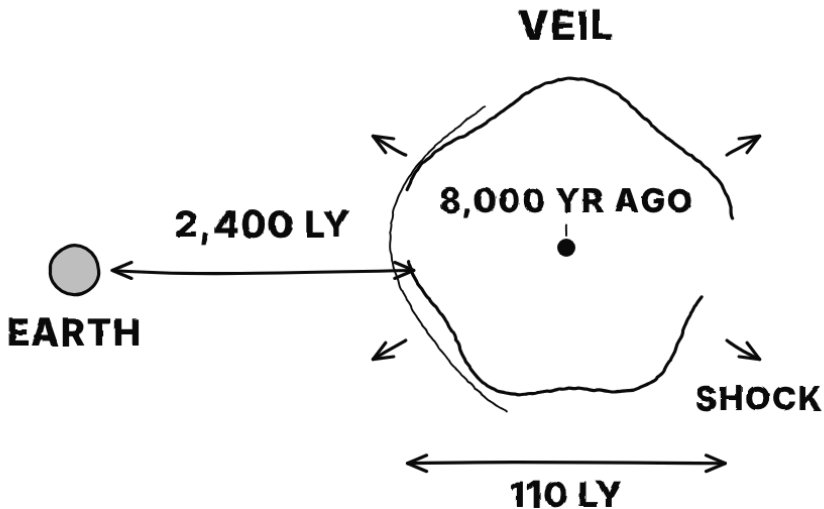
Zeros and Honest Answers

Three Digits, and a Guess at a Trillion

How High Is a Trillion?

The Same Forces, Out There and Here

Tonight, when it is dark, go outside and look up for a minute. Somewhere in the patch of sky above the constellation of the Swan there is a faint, torn veil of glowing gas that your eyes cannot see, but a large telescope can. It is called the Veil Nebula, and the book opens with a photograph of it: thin, twisted ropes and wide sheets of light in reds, greens and blues, stretched across the dark. What made those ropes, and why should anyone who is not an astronomer care? The answer the book gives is surprising: the forces that shaped that cloud are the very same forces at work in your kitchen.

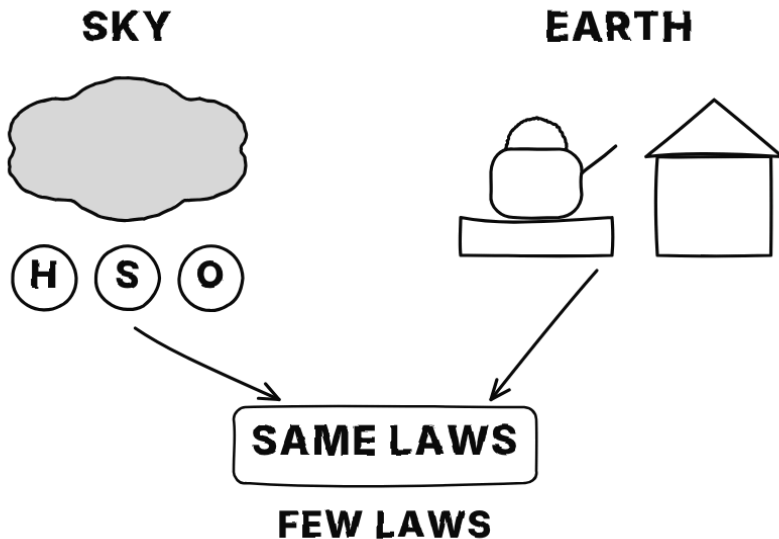


A torn cloud, far away

Here is what the picture shows. About eight thousand years ago a star, roughly twenty times larger than our Sun, blew itself apart in an explosion called a supernova. The blast sent a shock wave racing outward, and that wave is still moving today, ploughing into clouds of dust and gas and heating them until they glow. The whole glowing wreck now stretches more than a hundred and ten light years from one side to the other. A light year sounds like a time, but it is a distance: how far light travels in one year, which is about nine and a half trillion kilometres. The nebula itself sits about two thousand four hundred light years away from us.

How do scientists learn anything from something so far away? They watch it change. The Hubble Space Telescope took detailed pictures of the Veil in 1997, and newer pictures can be laid side by side with the old ones. Comparing them shows how far the shock wave has moved, how the filaments have stretched, and so how such clouds grow and spread. It is a little like comparing two photos of the same street taken years apart on your phone: what has shifted tells you what has been going on in between.

And now the surprising part. The colours in the picture are not paint added for beauty. They are the glows of hydrogen, sulfur, oxygen and similar elements, the same elements that make up the air you breathe and the water you drink. The gas out there pushes, heats and mixes by the same rules as gas here on Earth, only on a vastly larger scale. Whether engineers plan to send a rocket into space or simply work out how to heat a new home, they deal with the same forces that are tearing through the Veil. The book puts it plainly: the forces out there are the same as the ones here on Earth. Said another way, there is not one physics for the sky and another for the kitchen; there is one set of rules, and it works everywhere.

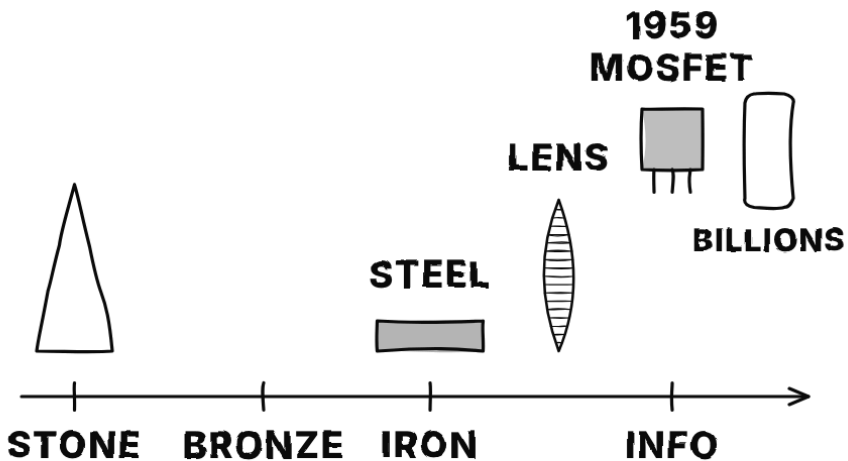


One set of rules

That idea is the heart of the whole chapter. The caption under the photograph adds something remarkable: the laws of physics are surprisingly few in number. Behind the dazzling variety of the world, from exploding stars to boiling kettles, there seems to be an underlying simplicity.

Many people meet physics with a little fear. They picture difficult equations and formulas to be memorised for an exam and then forgotten, with no use outside the classroom. The book admits this openly, and then argues the opposite: physics plays a much larger role in your life than you might think, whatever you do for a living.

Think of the long story of human tools. We name whole ages of history after materials: the Stone Age, the Bronze Age, the Iron Age. Each age was marked by what people knew about the physical properties of some material and how well they could work it. Knowing how a stone flakes when you strike it made a sharp spear point. Knowing what happens when carbon is worked into iron, in furnaces in South India and Sri Lanka, gave the world its earliest high-quality steel. Knowing how to grind and polish pieces of glass, and how to put them together, gave us lenses and the instruments built from them. All of this, the book says, is physics, even when nobody called it that.



Ages named after materials

Our own age is called the Information Age, and it too rests on a physical invention. In 1959, in a laboratory in New Jersey, two

scientists, Mohamed Atalla, born in Egypt, and Dawon Kahng, born in Korea, built a tiny switch called the MOSFET, short for metal-oxide-semiconductor field-effect transistor. Most people have never heard of it, yet more of these switches have been made than any other object in human history. There are billions of them inside the phone in your pocket, switching on and off to run every app, and more in the chip that drives your car's speedometer. They sit in computers, microwave ovens, medical instruments and almost every electronic device you own. It is a good reminder that the big inventions of our time came from people of many backgrounds working together.

The news is full of technologies that sound like magic: trains that float above their tracks, cloaks that bend light around themselves, tiny robots that hunt cancer cells inside the body. Every one of them, ordinary or astonishing, depends on the principles of physics. And not only inventors use physics. Engineers, pilots, doctors, physical therapists, electricians and programmers use it in their daily work. A pilot must understand how the wind pushes the plane off its path, and a physical therapist must understand the forces muscles feel as the body bends and moves. Even the map app that guides you through town is quietly doing physics, working out where you are from signals that arrive from satellites high above.

Finally, the book tells you what lies ahead. First comes a short history of how people came to study nature carefully, from the natural philosophers of the ancient Middle East and Mediterranean up to Isaac Newton and the laws that carry his name. Then come the agreed standards that scientists use to measure things, so that everyone speaks a single language of numbers and units. And last comes an honest look at the limits of measurement: how close a measurement is

to the truth, how well repeated measurements agree, and why scientists take great pains to say clearly what they cannot know.

Remember

The Veil Nebula is the glowing wreck of a star that exploded about eight thousand years ago, and its shock wave is still spreading. The forces shaping it are the same forces at work on Earth, in rockets and in the heating of a house. The laws of physics are few, which hints at a deep simplicity behind the world's variety. Every age of human technology, from stone tools to the transistors in your phone, has rested on knowing the physical properties of materials. Ahead lie the history of physics, the standards of measurement, and the honest limits of how precisely anything can be measured.

Questions

1. Why do the shapes in the Veil Nebula keep changing, so that pictures from 1997 and today differ?
 - a. Because the telescope cameras have improved
 - b. Because the shock wave from the explosion is still moving into gas and dust
 - c. Because the star is still exploding again and again
 - d. Because the Earth has moved closer to the cloud

2. A light year is mentioned in the lecture. What kind of quantity is it?
 - a. A time: the length of one year
 - b. A speed: how fast light moves
 - c. A distance: how far light travels in one year
 - d. An age: how old a star is
3. What follows if the forces in the Veil Nebula are the same as those on Earth?
 - a. The rules learned on Earth, for rockets or heating a house, also describe the distant cloud
 - b. The nebula must be much closer than it looks
 - c. Physics on Earth is only a small copy of astronomy
 - d. Gases on Earth will one day glow like the nebula
4. Why does the lecture name whole historical ages after materials like stone, bronze and iron?
 - a. Because those were the only materials people had
 - b. Because each age was marked by knowing how a material behaves and how to work it
 - c. Because those materials were discovered by physicists
 - d. Because the ages ended when the materials ran out

Answers

1. **b)** The blast's shock wave is still racing outward, ploughing into clouds of gas and dust, so comparing pictures shows how far it has moved.
2. **c)** It sounds like a time, but it is a distance, about nine and a half trillion kilometres.

3. a) There is not one physics for the sky and another for the kitchen; one set of rules works everywhere.

4. b) Each age was marked by what people knew about the physical properties of a material and how well they could work it.

Worked examples

How far is the Veil Nebula? The Veil Nebula is about 2,400 light years away, and one light year is about 9.5 trillion kilometres. About how many kilometres away is it?

A light year is a distance, so 2,400 light years means 2,400 equal steps, each 9.5 trillion kilometres long.

To add up equal steps we multiply: 2,400 times 9.5 trillion kilometres. 2,400 times 9.5 is 22,800, so the distance is about 22,800 trillion kilometres.

That is about twenty-three thousand trillion kilometres, a number far too large to picture, which is why astronomers prefer light years.

Answer: about 22,800 trillion kilometres

Practice

1. The whole Veil Nebula stretches about 110 light years across. About how many kilometres is that, using 9.5 trillion kilometres for one light year?

Hint: Each light year is one equal step of 9.5 trillion kilometres; count how many steps there are.

Answer: About 1,045 trillion kilometres, roughly a thousand trillion: 110 times 9.5 trillion kilometres.

2. Explain in your own words why a pilot and a physical therapist both need physics, though their jobs look nothing alike.

Hint: Ask what forces each of them deals with.

Answer: A pilot must understand how the wind's forces push the plane off its path, and a physical therapist must understand the forces muscles feel as the body bends and moves. Both are dealing with forces, so the same physics serves both jobs.

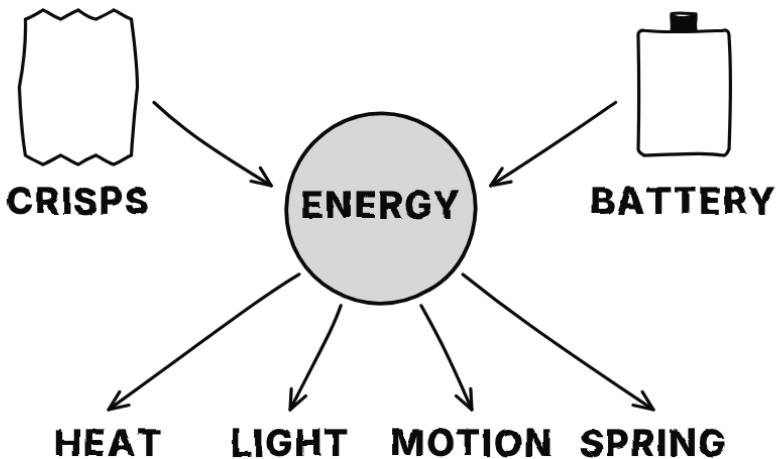
3. Most people have never heard of the MOSFET, yet the lecture calls it the basis of modern electronics. How can both be true?

Hint: Think about where the device sits and how many there are.

Answer: It is a tiny switch hidden inside phones, computers, microwave ovens and medical instruments, so nobody sees it, yet more of them have been made than any other object in history and almost every electronic device depends on them.

A Few Laws for a Crowded World

Here is a riddle: what does a bag of crisps have in common with a car battery? One you eat, the other sits under the bonnet of a car, and they seem to belong to different worlds. Yet the book's answer is that they are, in a deep sense, the same kind of thing. Both hold energy, and that energy can be turned into other forms: the crisps into the warmth and movement of your body, the battery into the spark that starts the engine.



One law, many forms of energy

Before we see why that matters, look at how crowded the world is. Every day you meet an endless variety of things: birds in flight, the colours of flowers, lightning, the pull that makes things fall. Over centuries people have gone further, from the tiniest particles inside atoms, called quarks, to huge clusters of galaxies, and on to the flow of time and the question of how the universe began. We have piled up an enormous heap of facts about all of it. And here is the surprise the book stresses: in the face of all that detail, a surprisingly small and unified set of laws explains what we see. We look for order, and nature, as the book says, turns out to be "remarkably cooperative".

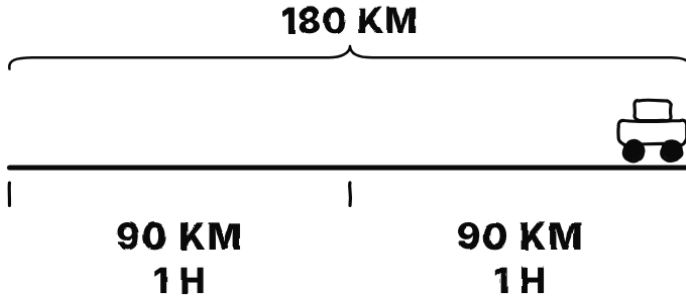
The crisps and the battery show how this works. One law, called the conservation of energy, says that energy can change its form but is never lost. That single sentence ties together food calories, batteries, heat, light and even the coiled spring inside an old wind-up watch. Once you know it, these are no longer five separate topics to memorise but five faces of one idea. Said another way, a good law is a key that opens many doors at once, so you do not have to carry a separate key for each. That, the book says, is what makes physics enjoyable to study: understanding instead of lists.

So what exactly is physics? Science as a whole is made of the theories and laws that state general truths about nature, together with all the knowledge they hold. Physics is the part of it that describes how energy, matter, space and time act on each other, and it always asks what basic mechanism lies underneath a happening. Its reach is wide: from the movement of tiny charged particles to the motion of people, cars and spaceships. Almost everything around you can be described quite accurately by its laws.

Take the smartphone in your hand. Inside your phone, electricity flows through a maze of tiny circuits, and physics describes exactly how it moves through them. Engineers use that knowledge to choose the right materials and to lay out the circuits when they build the phone.

Now open the map on that phone. The navigation system, called GPS, leans on one of the simplest relations in physics: the link between speed, distance and time. In words: the distance you cover equals your speed multiplied by the time you keep it up. Where does that come from? If you go at the same pace every hour, each hour adds the same stretch of road, so the stretches simply pile up, hour after hour. Suppose your car's speedometer holds steady at ninety kilometres an hour for two hours; that is ninety plus ninety, or one hundred and eighty kilometres. The same relation works backwards: one hundred and eighty kilometres at ninety kilometres an hour takes two hours, which is how the map estimates your driving time.

$$d = v t$$



Distance is speed times time

Of course real navigation is far more demanding. It has to account for the shape of the land, the exact distances to satellites circling far above, and even a strange effect called time dilation, in which clocks moving fast or sitting in a different pull of gravity tick at slightly different rates. Most of these calculations rest on methods worked out by Gladys West, a mathematician and computer scientist who programmed the first computers able to locate places on Earth with great accuracy from far away. Each time your phone shows the blue dot of where you are, it is using her methods to work out your position and how it relates to everything around you.

The book closes this opening by telling what the reader will gain. Of course you will learn the main topics of physics. But more

importantly, you will learn ways of reasoning that let you apply these few laws far beyond what any single book could cover. Those habits of careful thought, the book argues, help not only in exams but in any career you choose.

Remember

The world is full of different happenings, yet a small and unified set of laws explains them. The conservation of energy says energy changes form but is never lost, so food, batteries, heat, light and springs all follow one rule. Physics describes how energy, matter, space and time act on each other, and it looks for the basic mechanism underneath. Your phone's circuits and its navigation both run on physics; distance equals speed multiplied by time, so ninety kilometres an hour for two hours covers one hundred and eighty kilometres. Learning a few laws well, and how to apply them, is worth more than memorising long lists of facts.

Questions

1. Why are a bag of crisps and a car battery examples of the same law?
 - a. Both are made in factories
 - b. Both hold energy that can change into other forms without any being lost
 - c. Both are used up completely and leave nothing

- d. Both contain electricity
2. What is the advantage of knowing a broad law like the conservation of energy?
 - a. You no longer need to measure anything
 - b. It replaces the need for experiments
 - c. It links many apparently separate topics, so you understand instead of memorising lists
 - d. It explains only food and batteries
 3. A car travels at a steady speed. If it keeps that speed for twice as long, what happens to the distance?
 - a. It doubles
 - b. It stays the same
 - c. It halves
 - d. It grows four times
 4. Which of these does real satellite navigation have to account for, according to the lecture?
 - a. Only the speed of the car
 - b. The colour of the satellites
 - c. The weather on the day
 - d. The shape of the land, the distances to satellites and time dilation

Answers

1. **b)** Both hold energy that turns into other forms, and the conservation of energy says energy changes form but is never lost.

- 2. c)** One law ties together food calories, batteries, heat, light and springs, so they become faces of one idea.
- 3. a)** Each hour at the same pace adds the same stretch of road, so twice the time gives twice the distance.
- 4. d)** Real navigation has to account for the land's shape, exact distances to satellites and even time dilation.

Worked examples

Driving time from distance and speed A route is 180 kilometres long and you drive at a steady 90 kilometres an hour. How long does the trip take?

Distance equals speed multiplied by time, so time is distance divided by speed.

Each hour covers 90 kilometres, so we ask how many 90-kilometre stretches fit into 180 kilometres.

180 divided by 90 is 2.

Answer: 2 hours

Practice

1. Your speedometer stays at 60 kilometres an hour for three hours. How far do you travel?

Hint: Each hour adds the same stretch of road; add up three of them.

Answer: 180 kilometres: distance is speed multiplied by time, 60 times 3.

2. A trip of 240 kilometres takes 3 hours at a steady speed. What is the speed?

Hint: Share the distance equally among the hours.

Answer: 80 kilometres an hour: speed is distance divided by time, 240 divided by 3.

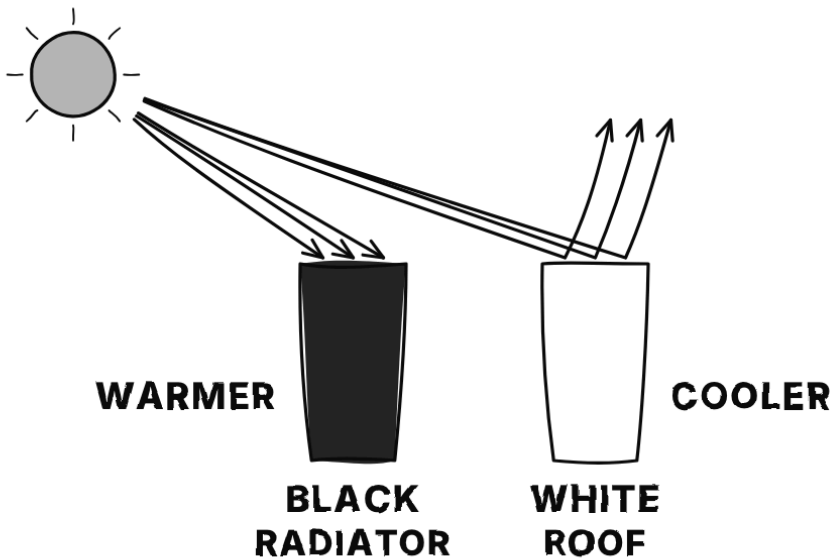
3. In your own words, explain what physics is and how it differs from just collecting facts.

Hint: Think about energy, matter, space and time, and about mechanisms.

Answer: Physics describes how energy, matter, space and time act on each other and looks for the basic mechanism underneath each happening. Instead of piling up facts, it finds a few laws that explain many of them at once.

Physics Without Being a Physicist

Why does the label on a microwave oven warn you never to put a metal spoon inside? And why do houses in hot countries so often have white roofs? You do not need to be a scientist to ask such questions, and the book's point is that you do not need to be one to answer them either. Physics, it says, is useful in everyday life and in jobs that have nothing to do with science.



Black takes in, white sends back

Try it yourself

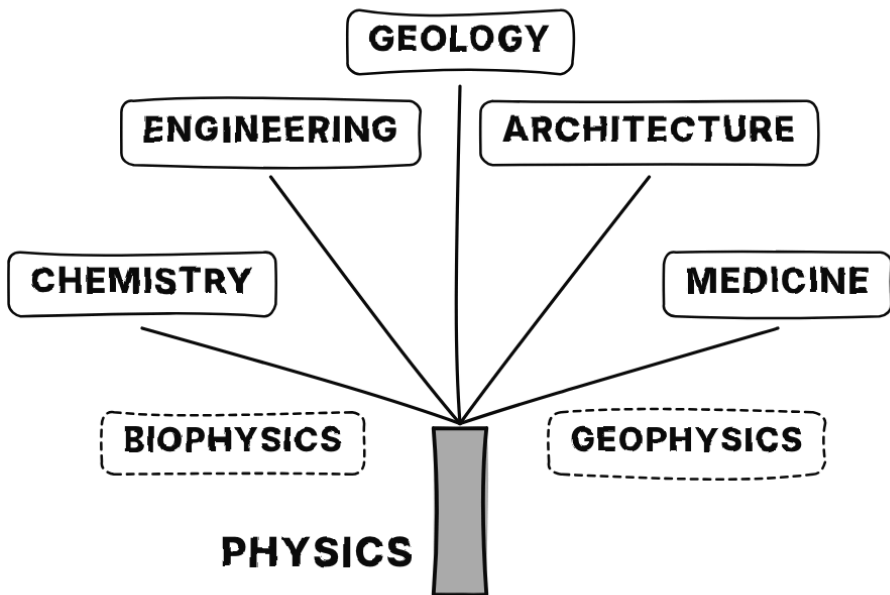
On a sunny day, take two identical glasses of water from the same tap. Wrap one in black paper and the other in white paper, and leave both in the sun for an hour. Then dip a finger in each. The water in the black glass is clearly warmer. A dark surface takes in more of the sunlight that falls on it, and a white one sends more of it away.

That little test explains two of the book's examples at once. A white roof keeps the inside of a house cooler because it sends much of the sunlight back. A black radiator in a car engine helps the engine get rid of heat, because a dark surface is good at giving off heat as well as taking it in. The same physics, run in opposite directions, serves two quite different purposes.

The book lists more of these everyday puzzles. Physics explains how a microwave oven heats food, why metal should stay out of it, and why such ovens might disturb a heart pacemaker. It helps you understand the dangers of radiation and judge them calmly rather than fearfully. It explains how a car's ignition sends its spark, and how electrical signals travel along the nerves of your own body. Even reading the dose on a medicine box and measuring it out correctly rests on the physics of careful measurement, which this chapter will come to.

Physics also sits underneath whole other fields. Chemistry deals with how atoms and molecules interact, so it is rooted in the physics of atoms and molecules. Most branches of engineering are, in the book's words, applied physics. In architecture, physics decides whether a building stands up, and it shapes the sound, heating, lighting and

cooling inside it. Parts of geology lean heavily on it too: dating rocks by their natural radioactivity, analysing earthquakes, and following how heat moves through the Earth. Some fields are simply hybrids, such as biophysics and geophysics.



Physics under other fields

Life itself is full of physics. At the smallest scale, it helps describe the walls of plant cells and the thin membranes that surround animal cells, where things as basic as the firing of a nerve take place. At the scale of a whole person, it explains the heat our bodies give off and the work and power of our muscles. In medicine it gives us x-rays, the scanners called MRI, and ultrasound measurements of blood flow; and some treatments use it directly, as cancer radiotherapy does with radiation strong enough to knock electrons out of atoms, called

ionizing radiation. A caption in the book points out a surprising family resemblance: a microwave oven and an MRI scanner both use electromagnetic waves, one to heat food and the other to make a picture of the brain precise enough to locate a tumour. Physics even explains our senses: how instruments make music, how the eye sees colour, and how lasers carry information.

Does that mean you must study every one of these applications? No, says the book, and this is perhaps its most useful message. What matters most is knowing the basic laws and having the skill to apply them; with that, the applications follow. Studying physics also sharpens your problem-solving, and because physics keeps the most basic parts of science, every other science uses it, so learning physics makes them easier too. Said another way: learn the roots well, and the branches become easy to climb.

Remember

You do not have to be a scientist to use physics: it explains microwave ovens, white roofs, car radiators and the nerves in your body. Dark surfaces take in and give off heat well, while white ones send sunlight back; that one fact explains both the radiator and the cool roof. Chemistry, engineering, architecture and geology all rest on physics, and fields like biophysics and geophysics blend it with others. In living things and in medicine, from cell membranes to MRI scans and radiotherapy, physics is at work. What matters most is not memorising applications but knowing the basic laws and how to apply them.

Questions

1. In the two-glasses test, why does the water in the black-wrapped glass get warmer?
 - a. Because black paper is thicker
 - b. Because a dark surface takes in more of the sunlight falling on it
 - c. Because white paper makes water colder
 - d. Because black paper produces its own heat
2. Why is a car radiator black while roofs in hot countries are white?
 - a. Both choices are only about appearance
 - b. Black is cheaper to paint
 - c. The radiator must give off heat, which a dark surface does well; the roof must send sunlight back, which a white surface does well
 - d. White roofs absorb more heat than black radiators
3. Why is chemistry said to be rooted in physics?
 - a. Because it deals with how atoms and molecules interact, which is the physics of atoms and molecules
 - b. Because chemists use microwave ovens
 - c. Because chemistry is a branch of engineering
 - d. Because chemistry was discovered by physicists
4. What does the book say matters most in learning physics?
 - a. Studying every application one by one

- b. Memorising the names of fields like biophysics
- c. Learning medical uses first
- d. Knowing the basic laws and having the skill to apply them

Answers

- 1. **b)** A dark surface takes in more of the sunlight that falls on it, while a white one sends more of it away.
- 2. **c)** The same physics runs in opposite directions: dark surfaces give off heat well, white ones send sunlight back.
- 3. **a)** Chemistry deals with how atoms and molecules interact, so it rests on the physics of atoms and molecules.
- 4. **d)** You need not study every application; the basic laws and the skill to apply them are what count.

Practice

- 1. A friend is choosing a colour for a garden shed in a hot, sunny place. Using the lecture's ideas, which colour would you suggest, and why?

Hint: Think of what dark and white surfaces do with sunlight.

Answer: White, or a very light colour. A white surface sends much of the sunlight back, so less of it warms the shed, just as a white roof keeps a house cooler.

- 2. Name two ways physics appears in medicine, as the lecture describes, and say what each is for.

Hint: Think of looking inside the body and of treating a disease.

Answer: Scanners such as x-rays and MRI let doctors see inside the body, for example to find a tumour, and ultrasound measures blood flow.

Radiotherapy uses ionizing radiation directly to treat cancer.

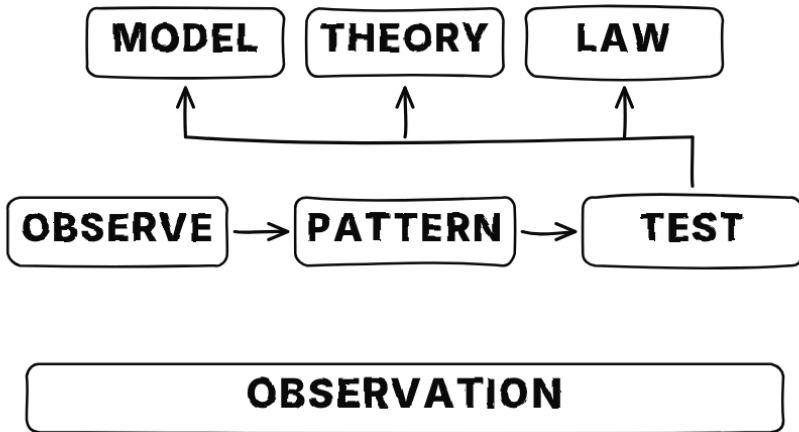
3. Explain in your own words why learning the basic laws is worth more than memorising many applications.

Hint: Think of the roots and the branches.

Answer: Each application, from microwave ovens to medical scanners, comes from a few basic laws. If you know those laws and how to apply them, you can understand applications you have never studied, while a memorised list covers only what is on it.

Models, Theories and Laws

Look out of the window at the clouds this afternoon. Some are white and puffy, some flat and grey, some tall and dark at the bottom. Have you ever caught yourself thinking that a certain kind of cloud means rain is on the way? If so, you have already done the first step of science: you looked, you noticed a pattern, and you made a guess about what it means.



From looking to laws

The book starts from exactly that everyday curiosity. We all look around, generalise, and try to understand what we see. When we

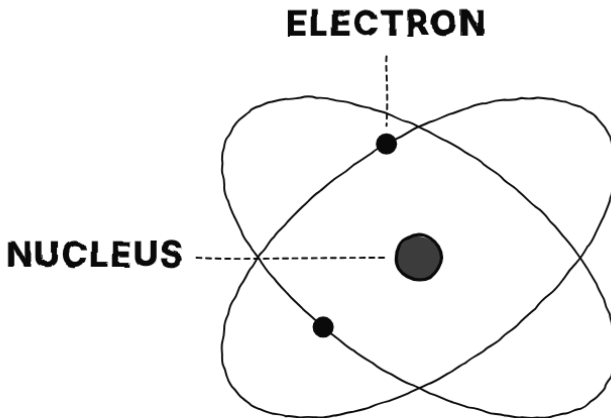
become serious about it, we become more organised: we collect data carefully, try to be more precise, run controlled experiments where we can, and write down ideas about how the data might fit together. From all of that we build three kinds of statements, called models, theories and laws, to sum up and share what the experiments showed. And underneath them all lies one rule the book states firmly: the cornerstone is observation, because science must describe the universe as it is, not as we might imagine it to be.

First, what is a law of nature? The book calls laws concise descriptions of the universe: human statements of the underlying rules that every natural process follows. The rules themselves are built into the universe. People did not make them and cannot change them; we can only discover and understand them. That discovery, the book adds, is a deeply human adventure, full of mystery, imagination, struggle, triumph and disappointment.

Two short stories from the book's pictures show how human it is. Isaac Newton, whose laws of motion we still use, was so reluctant to publish his revolutionary work that friends had to persuade him. Later in life he ran the Royal Mint, and he took the job so seriously that he introduced ridges on the edges of coins, so that cheats could no longer shave silver off them unnoticed. Marie Curie spent her own money to fund her early research and damaged her health through exposure to radiation; she remains the only person to have won Nobel Prizes in both physics and chemistry.

Now the first of the three words: a model. A model is a picture of something that is too hard, or impossible, to show directly. The classic example is the planetary model of the atom, in which tiny electrons circle a central nucleus the way planets circle the Sun. Nobody can

watch an electron go round, but this mental image helps explain what we can observe, such as the particular colours of light that hot gases give off. A model is backed by experiment, but it is accurate only in limited situations. Your phone's map is a good comparison: it is not the city itself, but a simplified picture that is reliable for finding your way, though useless for telling you how tall the buildings are. Physicists use models to analyse a situation, to do a calculation, or to build a computer simulation.



A PICTURE, NOT A PHOTO

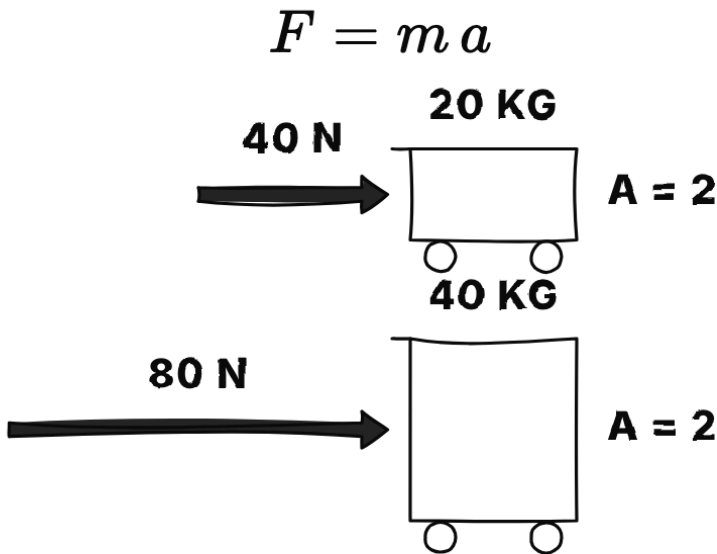
The planetary model of the atom

Next, a theory. Here the everyday word misleads us. In ordinary talk, a theory is a guess or a hunch, and people say "it's just a theory". To a scientist that phrase sounds silly, because science is built on theories. A scientific theory is a testable explanation of patterns in nature,

supported by evidence and checked many times by different groups of researchers. A model describes only some aspects of a system, but a theory should describe every aspect of anything inside its domain, the range of cases it covers. Every consequence of a theory that can be tested should be tested, and if an experiment ever shows one consequence to be false, the theory is either thrown out or changed, for example by narrowing its domain.

Some theories need a model and some do not. Newton's theory of gravity needs no mental picture, because the falling apples and orbiting planets it describes can be seen directly. The theory of gases is different: it pictures a gas as countless atoms and molecules, far too small to see, and we imagine them so that we can make sense of what our instruments tell us about how gases behave.

Finally, a law. A law describes a general pattern of nature in concise language, backed by evidence and repeated experiments, and often it fits into a single equation. The book's two examples are the law that energy is conserved in every process, and Newton's second law of motion, which links force, mass and acceleration. In words, that second law says the force needed to speed something up equals its mass multiplied by its acceleration, that is, by how quickly its speed changes. The idea is easy to feel: a heavier object, or a quicker change of speed, needs a harder push. Here is a calculation aloud. A shopping trolley of twenty kilograms that gains two metres per second of speed every second needs a force of twenty times two, or forty units of force called newtons. Double the load to forty kilograms and keep the same speeding-up, and the force doubles to eighty newtons.



Force is mass times acceleration

So how do laws and theories differ? Both are scientific statements that grew from tested hypotheses and are backed by evidence. But a law is short and very general, and describes a single kind of action, while a theory is bigger, more complex and more changeable, and explains a whole family of related events. The theory of evolution and the theory of relativity, for example, cannot be squeezed into one short statement, so they are not called laws. The book puts it neatly: a law is a starting point on which the scientific method builds, and a theory is the end result of that method. Said another way, a law tells you what happens; a theory tells you the larger story of why a whole group of things happens.

Remember

Science rests on observation: it must describe the world as it is, not as we imagine it. A model is a simplified picture of something we cannot show directly, like the planetary atom, and it works only in limited situations. A scientific theory is not a guess but a well-tested explanation of a whole range of related events, and it must change if an experiment contradicts it. A law is a short, general statement of a pattern, often a single equation, such as Newton's second law: force equals mass times acceleration. A trolley of twenty kilograms speeding up by two metres per second each second needs forty newtons.

Questions

1. Why can people only discover laws of nature, not make them?
 - a. Because only scientists are allowed to write laws
 - b. Because the rules are built into the universe and cannot be changed by us
 - c. Because laws are agreed on by vote
 - d. Because laws are too complicated to invent
2. Why is a phone's map a good comparison for a model?
 - a. Because it shows the whole city exactly as it is
 - b. Because it is drawn from photographs
 - c. Because it is a simplified picture, reliable for some questions and useless for others

- d. Because it never needs to be updated
3. A trolley needs 40 newtons to speed up at a certain rate. If its mass is doubled and the speeding-up stays the same, what force is needed?
- a. 80 newtons
 - b. 40 newtons
 - c. 20 newtons
 - d. 160 newtons
4. Why are relativity and evolution called theories, not laws?
- a. Because they have not been tested
 - b. Because they are guesses
 - c. Because they are newer than the laws
 - d. Because they explain whole families of events and cannot be put into one short statement

Answers

1. **b)** The rules are built into the universe; people did not make them and cannot change them, only discover and understand them.
2. **c)** Like a model, the map is backed by reality yet accurate only for limited uses, such as finding your way.
3. **a)** Force equals mass times acceleration, so doubling the mass at the same acceleration doubles the force.
4. **d)** A law is short and general; these explain a whole family of related events and cannot be squeezed into one statement.

Worked examples

Pushing a shopping trolley A shopping trolley of 20 kilograms gains 2 metres per second of speed every second. Using Newton's second law, what force does it need?

Newton's second law says force equals mass multiplied by acceleration.

The mass is 20 kilograms and the acceleration is 2 metres per second gained every second.

Multiply: 20 times 2 is 40.

Force is measured in newtons, so the push is 40 newtons.

Answer: 40 newtons

Practice

1. A cart of 50 kilograms speeds up by 3 metres per second every second. What force is needed?

Hint: Force equals mass multiplied by acceleration.

Answer: 150 newtons: 50 times 3.

2. A force of 60 newtons acts on a box of 15 kilograms. How quickly does the box speed up?

Hint: Acceleration is force divided by mass.

Answer: 4 metres per second gained every second: 60 divided by 15, since force equals mass times acceleration.

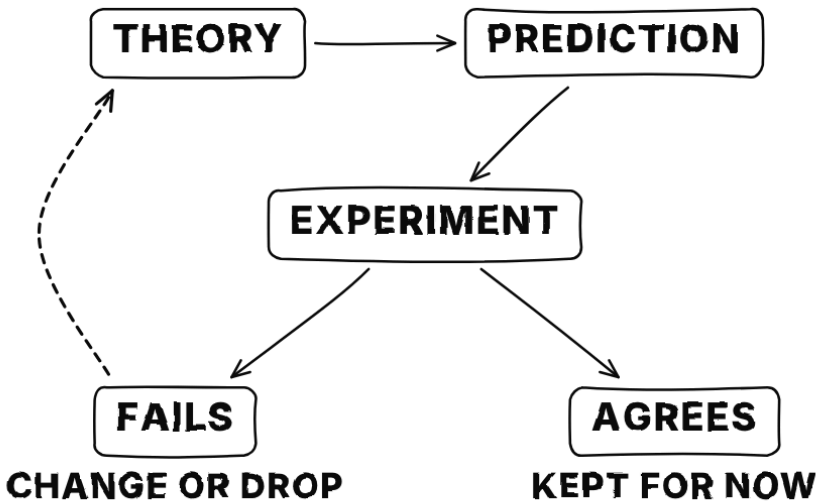
3. Explain in your own words the difference between a model and a scientific theory.

Hint: Compare how much of a system each must describe.

Answer: A model is a simplified picture of something we cannot show directly and is accurate only in limited situations. A theory is a well-tested explanation that should describe every aspect of anything in its domain, and it must be dropped or changed if an experiment contradicts it.

Guess, Test, Guess Again

It is a cold morning, you turn the key, and the car will not start. What do you do next? Most people do something rather scientific without noticing: they guess a cause, check it, and if the guess was wrong, they guess again. The book uses exactly this scene to show how science works.



A prediction is a test

Before the car, one small point of naming. Besides laws and theories, there are statements that apply more narrowly, which are usually called principles. Pascal's principle, for example, holds only in fluids.

The book admits, though, that people do not always keep laws and principles carefully apart.

Models, theories and laws first help scientists make sense of data they have already gathered. But often, once one is in place, it does something more exciting: it points scientists toward discoveries they would not otherwise have made. Sometimes it implies that some object or happening exists which nobody has yet seen. When the search then finds it, that is a remarkable triumph, and the book credits the underlying order of the universe for making such spectacular predictions possible.

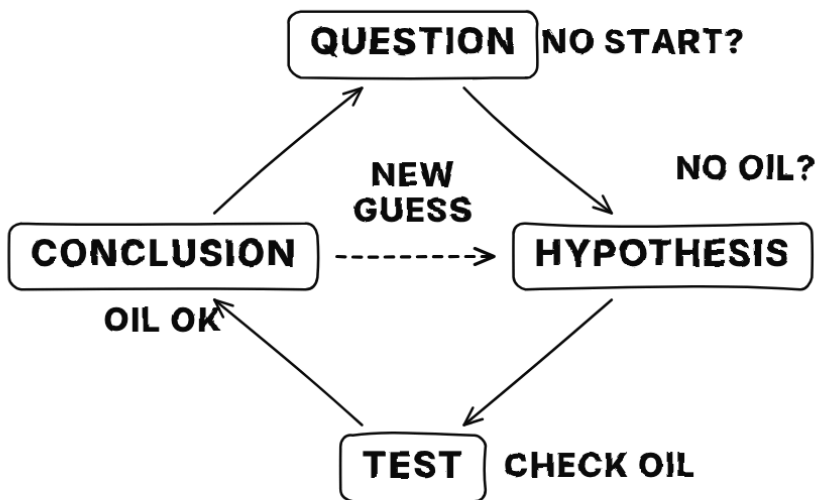
The same rule that makes predictions powerful also makes them dangerous. If an experiment does not confirm a prediction, then the theory or law is wrong, however elegant or convenient it is. And no law can ever be known with absolute certainty, because nobody can carry out every imaginable experiment in every possible situation. So physicists treat a law or theory as valid until a counterexample turns up. If a good, checkable experiment contradicts even a well-established law, that law must be changed or thrown out completely. Said another way, a law is trusted only for as long as careful experiments keep agreeing with it.

The book calls this whole effort an adventure, like sailing into an uncharted ocean: discoveries are made, models, theories and laws are formed, and the universe looks all the more beautiful for what we come to understand.

The habits behind this adventure are old. Ibn al-Haytham, also called Alhazen, worked in Cairo around a thousand years ago and greatly advanced the understanding of light and vision. He taught that a scientist must be ready to reject what everyone believes, to be "the

enemy" of everything they read, and to trust only objective evidence. He insisted on repeating experiments and checking results, and he warned that our senses and expectations can lead us astray. His work was a forerunner of what we now call the scientific method.

That method usually runs like this. It starts with an observation and a question. Then comes some research, and from it a hypothesis, a possible answer that can be tested. Then an experiment tests the hypothesis. Finally the results are analysed and a conclusion is drawn. The book stresses that the method is not only for scientists; it can be used in many situations and adjusted to fit them.



The method, run on a car

Back to the car that will not start. The question is plain: why? You look into the reasons a car might fail to start, and you form a hypothesis: perhaps it has no engine oil. To test it, you open the bonnet and check the oil level. The oil is at a proper level, so you conclude that oil is not the problem. That is not a failure; it is a result. You form a new hypothesis and run through the steps again. It works the same way when your phone will not connect: you suspect the network, you check it on another device, and if that is fine you move on to the next suspect. Said another way, each test either finds the cause or crosses one suspect off the list, and both bring you closer to the answer.

Remember

Principles are statements that apply more narrowly than laws, such as Pascal's principle for fluids. Models, theories and laws not only organise data but also predict things not yet seen, and a successful prediction is a triumph of science. A failed prediction shows the theory or law is wrong, and no law is ever certain, because not every experiment can be done. The scientific method runs from observation and question, through research and hypothesis, to experiment, analysis and conclusion. When a hypothesis fails, as with checking the oil in a car that will not start, you form a new one and test again.

Questions

1. Why is Pascal's principle called a principle rather than a law?

- a. Because it was discovered by one person
 - b. Because it applies more narrowly, only in fluids
 - c. Because it has not been tested
 - d. Because it cannot be written as an equation
2. What must follow if a careful experiment fails to confirm a theory's prediction?
- a. The theory or law is wrong, however elegant it is
 - b. The experiment should be ignored
 - c. The prediction becomes a principle
 - d. The theory is kept if it is convenient
3. You check the oil of a car that will not start and find it at a proper level. What does the scientific method tell you to do next?
- a. Conclude that nothing is wrong with the car
 - b. Repeat the same oil check until it fails
 - c. Form a new hypothesis and test it
 - d. Give up, since the test failed
4. Why is no law ever known with absolute certainty?
- a. Because laws are written by people
 - b. Because laws change every century
 - c. Because scientists disagree
 - d. Because no one can carry out every imaginable experiment in every situation

Answers

1. **b)** Statements that apply more narrowly are usually called principles; Pascal's holds only in fluids.
2. **a)** If an experiment does not confirm a prediction, the theory or law is wrong, however elegant or convenient.
3. **c)** A failed hypothesis is a result: it crosses one suspect off, and you form a new hypothesis and test again.
4. **d)** Since not every experiment can be done, laws are treated as valid until a counterexample turns up.

Worked examples

The car that will not start Your car will not start. Walk through the scientific method as the book does, testing the idea that it has no engine oil.

Observation and question: the car will not start, so why?

Research: look into the reasons a car might fail to start.

Hypothesis: perhaps the car has no engine oil.

Experiment: open the bonnet and check the oil level.

Conclusion: the oil is at a proper level, so oil is not the cause; form a new hypothesis and repeat.

Answer: The oil hypothesis is rejected, and the search moves on to the next possible cause.

Practice

1. Your phone will not connect to the internet. Describe how you would use the scientific method to find the cause, with at least two hypotheses.

Hint: Start with a question, then propose a cause you can check, test it, and decide.

Answer: Question: why will it not connect? Hypothesis one: the home network is down; test it on another device. If the other device connects, reject it. Hypothesis two: the phone's Wi-Fi is switched off; check the setting. Each test either finds the cause or removes one suspect.

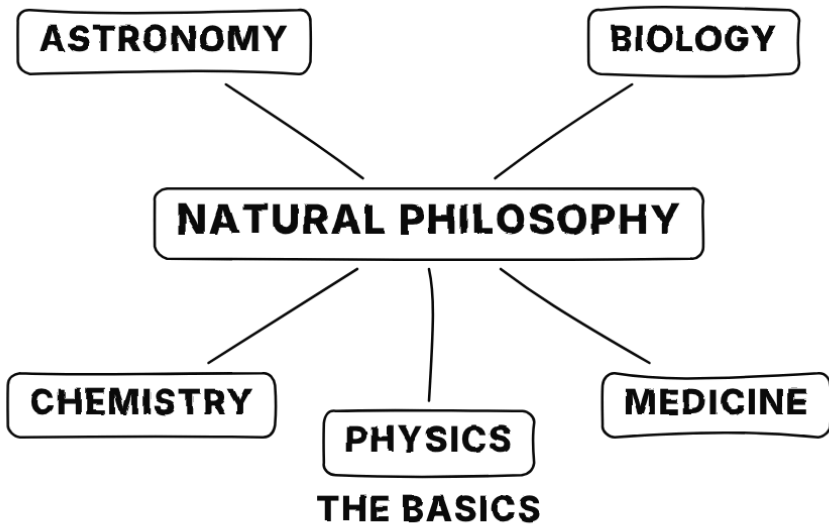
2. In your own words, explain what Ibn al-Haytham contributed to the way science is done.

Hint: Think about his attitude toward what he read and toward evidence.

Answer: He taught that a scientist must be ready to reject accepted knowledge and trust only objective evidence, that experiments should be repeated and results checked, and that our senses and expectations can mislead us. This made him a forerunner of the scientific method.

Old Physics, New Physics

Throw a ball across the room and watch it curve down into your hand. Everything about it feels obvious: it moves at a gentle speed, you can see it plainly, and the Earth's gravity tugs it down in a familiar way. The physics that describes this ball is centuries old, and it still works beautifully. Yet it is not the whole truth about the universe, and this part is about where it stops working, and why we still learn it first.

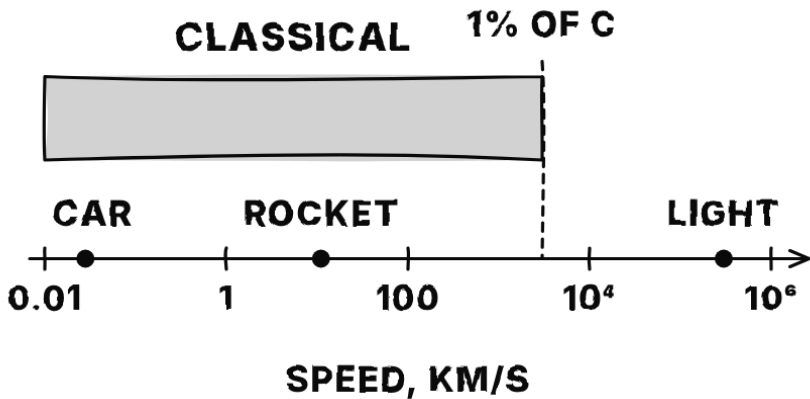


Natural philosophy splits up

Start with history. Physics was not always a subject of its own, and it is still tied to the other sciences today. Its name comes from the Greek

word for nature, and the study of nature was long called natural philosophy. From ancient times through the Renaissance, natural philosophy took in astronomy, biology, chemistry, physics, mathematics and medicine together. The Greek philosopher Aristotle, more than two thousand years ago, wrote about physics, animals, the soul, politics and poetry, all as one study. As knowledge grew over the last few centuries, natural philosophy split into more and more specialised fields, and physics kept the most basic parts. The book's pictures name a few of the great minds along the way: Ibn al-Haytham and Galileo Galilei, so important to the practice of science that a later scientist put them both on the cover of his major work, and Niels Bohr, who helped found quantum mechanics.

Physics as it grew from the Renaissance to the end of the nineteenth century is called classical physics. At the start of the twentieth century, revolutionary discoveries turned it into modern physics.



Where classical physics holds

Classical physics is not an exact description of the universe, but it is an excellent approximation when three conditions hold. Things must move at less than about one per cent of the speed of light. They must be large enough to be seen with a microscope. And only weak gravity may be involved, like the gravity of the Earth.

How fast is one per cent of the speed of light? Light covers about three hundred thousand kilometres every second, and one hundredth of that is three thousand kilometres every second. Your car on the motorway, at about a hundred kilometres an hour, covers less than three hundredths of a kilometre in a second, about a hundred thousand times below that limit. So the thrown ball, the car and even a rocket all live comfortably inside classical physics.

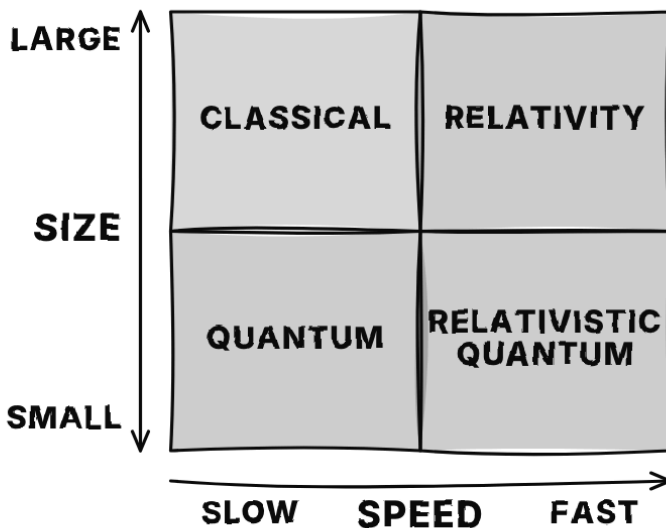
That is exactly why classical physics feels like common sense. We live among slow speeds, visible things and weak gravity, so classical physics seems intuitively right, while much of modern physics seems bizarre. This is where models become so valuable. They let us picture things we never experience: what happens at enormous speeds, or what things too small to see might be like. We can understand the properties of an atom because we can picture one, though nobody has ever seen an atom with the eye. New instruments help too; a scanning tunnelling microscope now shows the individual atoms of a sheet of gold.

Modern physics has brought some of science's most spectacular advances. Many laws of classical physics have been changed or abandoned, and with them have come revolutions in technology, in society and in our picture of the universe. Like science fiction, modern physics is full of strange objects beyond everyday experience, but unlike science fiction, they are real.

So why does most of the book still teach classical physics? For two reasons. It describes the universe extremely accurately across a wide range of everyday situations. And you need it in order to understand modern physics at all. Said another way, classical physics is both the everyday tool and the ground floor on which the newer physics is built.

Modern physics itself rests on two revolutionary theories: relativity, for the very fast, and quantum mechanics, for the very small. Relativity is needed when something moves faster than about one per cent of the speed of light, or feels strong gravity, such as near the Sun. Quantum mechanics is needed for things too small to see with a microscope. Joined together, they make relativistic quantum

mechanics, which handles small things that move very fast or feel strong gravity. It is the best theory we have that applies everywhere, but it is mathematically so demanding that it is used only when necessary; simpler theories are used whenever they are accurate enough. The book adds a hopeful note: a great deal of modern physics can still be done with ordinary algebra and trigonometry.



Which physics, for which world

Notice the words: relativity and quantum mechanics are called theories, not laws. The book closes the section with a small puzzle about exactly that. A friend tells you they have learned a new law of nature. Even before hearing it, you know something: it will be a concise description of the universe, a statement of a rule that natural processes follow. Had your friend learned a theory instead, you would

know it was a large-scale, broadly applicable explanation. Said another way, a law is short and states what happens; a theory is broad and explains a whole range of things.

Remember

Physics grew out of natural philosophy, which once included all the sciences, and kept the most basic parts as the fields separated. Classical physics works well for speeds below about one per cent of the speed of light, for things visible under a microscope, and for weak gravity. One per cent of the speed of light is about three thousand kilometres a second, far above anything in everyday life. Modern physics rests on relativity for the very fast and quantum mechanics for the very small; combined, they form relativistic quantum mechanics. Classical physics is still taught first because it is very accurate for daily life and is needed to understand modern physics.

Questions

1. A spacecraft moves at a tenth of the speed of light. Which theory is needed to describe its motion?
 - a. Classical physics
 - b. Relativity
 - c. Quantum mechanics
 - d. Natural philosophy

2. Why does classical physics feel like common sense?
 - a. Because we live among slow speeds, visible things and weak gravity, where it works
 - b. Because it is the oldest physics
 - c. Because it is an exact description of nature
 - d. Because modern physics has been abandoned
3. Why is relativistic quantum mechanics not used for every problem, though it is the best general theory?
 - a. Because it has been disproved
 - b. Because it applies only to the Sun
 - c. Because it is mathematically so demanding that simpler theories are used when accurate enough
 - d. Because it covers only slow objects
4. A friend says they learned a new theory, not a law. What can you expect?
 - a. A short statement of a single rule
 - b. A guess that has not been tested
 - c. A list of measurements
 - d. A large-scale, broadly applicable explanation

Answers

1. **b)** Relativity is needed when something moves faster than about one per cent of the speed of light.
2. **a)** We live where the three conditions of classical physics hold, so it seems intuitively right.

3. c) It is used only when necessary; simpler theories are used whenever they are accurate enough.

4. d) A theory is a large-scale, broadly applicable explanation, while a law is a concise statement of a rule.

Worked examples

One per cent of the speed of light Light covers about 300,000 kilometres each second. Below what speed does classical physics work well, and how does a car at 100 kilometres an hour compare?

Classical physics needs speeds below about one per cent, one hundredth, of the speed of light.

One hundredth of 300,000 kilometres per second is 3,000 kilometres per second.

A car at 100 kilometres an hour covers 100 kilometres in 3,600 seconds, about 0.028 kilometres each second.

3,000 divided by about 0.028 is roughly 100,000, so the car is about a hundred thousand times below the limit.

Answer: About 3,000 kilometres per second; the car is roughly 100,000 times slower

Practice

1. A rocket leaves Earth at about 11 kilometres per second. Is it below the classical limit of about 3,000 kilometres per second? By roughly how many times?

Hint: Divide 3,000 by 11.

Answer: Yes, by roughly 270 times: 3,000 divided by 11 is about 273, so classical physics describes it well.

2. For each case, say whether classical physics, relativity or quantum mechanics is needed: an electron inside an atom; a planet orbiting the Sun at ordinary speed; light passing very close to the Sun.

Hint: Check the three conditions: speed, size and strength of gravity.

Answer: The electron needs quantum mechanics, being too small to see with a microscope. The planet is described well by classical physics. Light near the Sun involves both the speed of light and strong gravity, so relativity is needed.

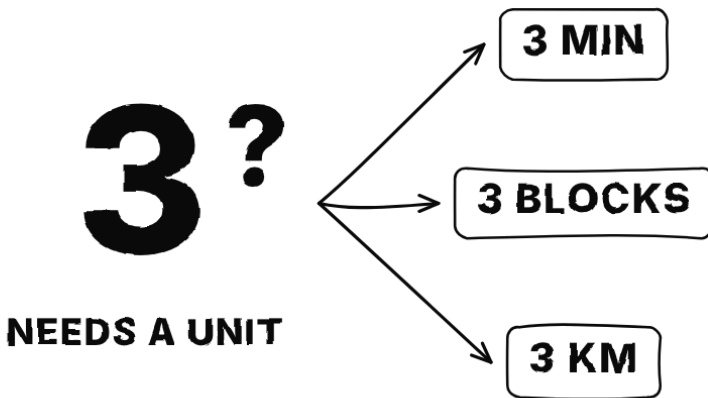
3. Give the two reasons the book offers for teaching mostly classical physics.

Hint: One is about daily life, one is about modern physics.

Answer: Classical physics describes the universe extremely accurately in a wide range of everyday situations, and it is needed in order to understand modern physics.

Numbers Need Units

A friend texts you: "The pharmacy is three away." Three what: three minutes, three blocks, three kilometres? The number by itself tells you nothing. The book makes the same point with a picture captioned, rather wonderfully, "Distances given in unknown units are maddeningly useless."



Three what?

Physics covers a staggering range. It deals with the incredibly short life of an atomic nucleus and the age of the Earth, with particles smaller than a nucleus and the distance to the edge of the known

universe, with the push of a jumping flea and the pull between the Earth and the Sun. The book opens the section with a picture of the Earth and the Moon, and notes that this distance, huge as it seems, is a tiny fraction of the distances to other bodies in the sky. Describing such things with numbers, and with equations for the principles behind them, lets us understand nature far more deeply than words alone. But to handle such ranges we need agreed units.

How is a quantity like length or speed defined in the first place? In one of two ways. Some quantities are defined by saying how to measure them: distance and time are like that. Others are defined by saying how to calculate them from other measurements. Average speed is the book's example: it is the distance travelled divided by the time taken. If your car's trip computer shows one hundred and twenty kilometres covered in one and a half hours, your average speed was one hundred and twenty divided by one and a half, which is eighty kilometres an hour.

$$v = \frac{d}{t}$$

SPEED = DISTANCE ÷ TIME

$$\frac{120 \text{ km}}{1.5 \text{ h}} = 80 \text{ km/h}$$



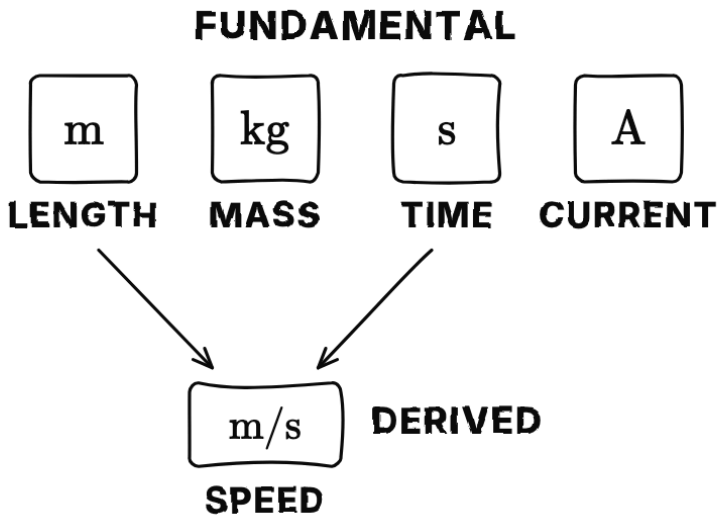
Average speed, calculated

Every measurement is given in units, which are agreed, standard amounts. The length of a race, for example, is stated in metres for sprinters and in kilometres for long-distance runners. Without such standards, scientists could not report or compare their results in any meaningful way.

Two big systems of units are in use. One is the International System, known by its French initials SI, also called the metric system. The other is the system of English units, also called customary or imperial units, used historically in lands once ruled by the British Empire and still common in the United States. Almost every other country uses the metric system, and it is the standard agreed upon by scientists and mathematicians. It is the reason a gram on your kitchen scale and a

milligram on a medicine label mean the same thing anywhere in the world.

Now the surprising part. Some quantities are more basic than others. The most basic can be defined only by the procedure used to measure them, and their units are called fundamental units. This book takes four quantities as fundamental: length, measured in metres; mass, in kilograms; time, in seconds; and electric current, in amperes, which will come much later. Every other quantity, such as force or electric charge, can be built from these four by multiplying and dividing, and its unit is called a derived unit. Speed, for instance, is a length divided by a time, so its unit, metres per second, is derived. Said another way, four building blocks are enough to put together almost all of the measurements in physics, which is another sign of nature's underlying simplicity.



Four building blocks

The book adds one practical note. In a few places it uses units outside the metric system because they are so common, such as millimetres of mercury for blood pressure. Whenever it does, it ties them back to metric units by conversion.

Remember

A number without a unit is useless; agreed units let scientists report and compare measurements. A quantity is defined either by how it is measured, like distance and time, or by how it is calculated, like average speed, distance divided by time. The metric system, or SI, is the standard for scientists and most of the world; English units survive mainly in the United States. The four fundamental quantities in this book are length, mass, time and electric current, measured in metres, kilograms, seconds and amperes. All other quantities, such as speed and force, are combinations of these, and their units are called derived units.

Questions

1. How is average speed defined, in contrast with distance and time?
 - a. By a measuring procedure, like a ruler or clock
 - b. By a calculation: distance travelled divided by time taken
 - c. As one of the four fundamental quantities
 - d. By its unit alone
2. Which list gives the four fundamental quantities this book uses?
 - a. Length, mass, time and electric current
 - b. Length, speed, force and time
 - c. Mass, weight, volume and temperature
 - d. Metre, kilogram, second and newton

3. Why is the unit of speed, metres per second, called a derived unit?
- a. Because speed is measured with a speedometer
 - b. Because it is used only in science
 - c. Because it is built from fundamental units, a length divided by a time
 - d. Because it was invented later
4. Why are agreed units essential for scientists?
- a. Because they make numbers smaller
 - b. Because they replace experiments
 - c. Because each country needs its own
 - d. Because without them results could not be reported or compared meaningfully

Answers

1. **b)** Distance and time are defined by how to measure them; average speed by how to calculate it from them.
2. **a)** The book takes length, mass, time and electric current as fundamental.
3. **c)** Speed is a length divided by a time, so its unit is built from fundamental units.
4. **d)** Without standard units, scientists could not report or compare their results in any meaningful way.

Worked examples

Average speed from a trip computer A car covers 120 kilometres in one and a half hours. What is its average speed?

Average speed is the distance travelled divided by the time taken.

The distance is 120 kilometres and the time is 1.5 hours.

120 divided by 1.5 is 80.

Answer: 80 kilometres an hour

Practice

1. A runner covers 400 metres in 80 seconds. What is the runner's average speed, and is its unit fundamental or derived?

Hint: Divide distance by time; then ask what the unit is built from.

Answer: 5 metres per second: 400 divided by 80. The unit is derived, because it is a length divided by a time.

2. Sort these into fundamental and derived quantities as the book does: time, speed, mass, force, electric current.

Hint: The book names exactly four fundamental quantities.

Answer: Fundamental: time, mass and electric current (with length, the fourth). Derived: speed and force, which are built from the fundamental ones.

A Second, a Metre, a Kilogram

Imagine buying a kilogram of apples in two shops, and finding that one shop's kilogram is a little heavier than the other's. Or two rulers that disagree about how long a metre is. Everything we measure would quietly fall apart. That is why the book spends time on how the basic units themselves are fixed, and how those definitions have changed.



$$24 \times 60 \times 60 = 86\,400$$



SLOWING



CESIUM



$$9\,192\,631\,770$$

$$= 1 \text{ s}$$

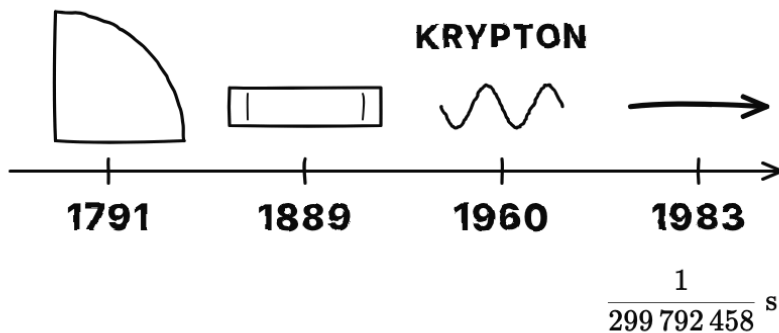
From the day to cesium

Start with time. For many years the second was defined as one part in eighty-six thousand four hundred of an average day. The number

comes straight from the clock: twenty-four hours, times sixty minutes, times sixty seconds, makes eighty-six thousand four hundred seconds in a day. But the Earth's spin is very gradually slowing, so the day is slowly getting longer, and a second tied to it would slowly change too. So scientists turned to something that does not change. Atoms of cesium can be made to vibrate in an extremely steady way, and their vibrations can be counted. Since 1967, one second has been the time it takes for a little over nine billion one hundred and ninety-two million of these vibrations. The atomic clocks built on this keep time to better than a millionth of a second per year. The time on your phone is regularly set from signals that trace back to such atomic clocks.

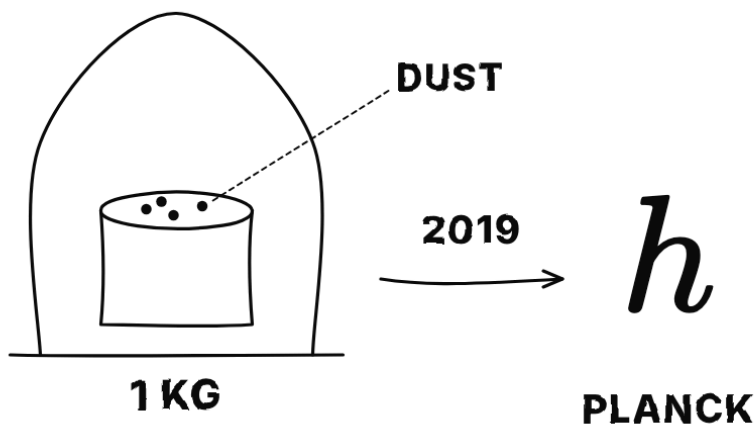
Next, length. In 1791 the metre was defined as one ten-millionth of the distance from the equator to the North Pole. In 1889 it became the distance between two fine lines engraved on a bar of platinum and iridium, kept near Paris. By 1960 it could be fixed more precisely by light, as a certain number of wavelengths of the orange light given off by krypton atoms. Since 1983 the metre has been the distance light travels in empty space in one part in two hundred and ninety-nine million, seven hundred and ninety-two thousand, four hundred and fifty-eight of a second. That choice makes the speed of light exactly that many metres per second, by definition. A caption in the book adds the reasoning: distance travelled is speed multiplied by time. Light covers almost three hundred thousand kilometres in one second, so in that tiny fraction of a second it covers exactly one metre. So a better measurement can no longer change that number; it only lets laboratories mark out the metre more precisely.

THE METRE



Four definitions of the metre

Then mass. For a long time the kilogram was simply the mass of one particular cylinder of platinum and iridium, kept with the old metre bar near Paris, with exact copies in other places, including the National Institute of Standards and Technology in Maryland. Every other mass in the world could in the end be traced back to a comparison with that cylinder. But although the metal resisted corrosion, tiny particles from the air stuck to its surface and slowly changed its mass. So in May 2019 scientists adopted a steadier definition, tying the kilogram to the second, the metre and a constant of quantum physics called Planck's constant, which links the energy of a particle of light to its frequency. The kilogram on your kitchen scale now rests, through a long chain of comparisons, on that unchanging constant rather than on a lump of metal.



The kilogram leaves its cylinder

Why all this care? Because every measurement is finally expressed in fundamental units, and no measurement can be more accurate than those units. Said another way, a ruler is only as good as the metre it copies, and a clock only as good as the second it counts. The fourth fundamental unit, the ampere for electric current, will come much later, with electricity and magnetism. For mechanics, fluids, heat and waves, which the book treats first, length, mass and time are all that is needed.

Remember

The second was once a fraction of the day, but the day slowly lengthens; since 1967 it is a fixed count of cesium vibrations. The metre has gone from a fraction of the Earth to an engraved bar to the distance light travels in a tiny, exact fraction of a second. That definition makes the speed of light exactly two hundred and ninety-nine million, seven hundred and ninety-two thousand, four hundred and fifty-eight metres per second. The kilogram, once a metal cylinder that slowly gathered dirt, has since 2019 been tied to the second, the metre and Planck's constant. Every measurement rests on these units, so it can be no more accurate than they are.

Questions

1. Why was the old metal kilogram replaced in 2019?
 - a. It was stolen
 - b. Tiny particles from the air stuck to it and slowly changed its mass
 - c. It corroded away
 - d. It was too large to copy
2. Since the speed of light is fixed by definition, what would a more accurate measurement of it change?
 - a. The length of the metre
 - b. The number 299,792,458

- c. Only how precisely labs can mark out a metre
 - d. The mass of the kilogram
3. Why do the definitions of fundamental units matter for every measurement?
- a. Because they are written in French
 - b. Because they are used only in laboratories
 - c. Because every measurement is expressed in them and can be no more accurate than they are
 - d. Because they change every year
4. Which fundamental quantities are enough for mechanics, fluids, heat and waves?
- a. Length, mass, time and electric current
 - b. Only time
 - c. Length and electric current
 - d. Length, mass and time

Answers

1. **b)** Although it resisted corrosion, particles from the air settled on its surface and slowly changed its mass.
2. **c)** The number is exact by definition and the metre is defined from it; a better measurement only improves how precisely a metre can be realised.
3. **c)** A ruler is only as good as the metre it copies, and a clock only as good as the second it counts.
4. **d)** Electric current comes later; for the first subjects, length, mass and time are all that is needed.

Worked examples

How far light goes in a moment Light travels almost 300,000 kilometres in one second. About how far does it travel in one thousandth of a second?

Distance travelled is speed multiplied by time.

In one second light covers about 300,000 kilometres.

In a thousandth of a second it covers a thousandth of that.

300,000 divided by 1,000 is 300.

Answer: about 300 kilometres

Practice

1. In 1791 the metre was one ten-millionth of the distance from the equator to the North Pole. How many kilometres is that distance by this definition?

Hint: Ten million metres; a kilometre is a thousand metres.

Answer: 10,000 kilometres: ten million metres divided by a thousand.

2. How many seconds are in a day, and why did the old definition of the second slowly become unreliable?

Hint: Multiply hours, minutes and seconds; then think of the Earth's spin.

Answer: 86,400 seconds: 24 times 60 times 60. Because the Earth's spin is gradually slowing, the day lengthens, so a second defined as a fraction of the day would slowly change.

Powers of Ten

Look at the label on a medicine box: the dose might say two hundred and fifty milligrams. Look at the map on your phone: the next town is twelve kilometres away. Milli, kilo: these little front pieces, called prefixes, quietly do a great deal of work. Each one says "multiply by a certain power of ten", and that is the whole secret of the metric system.

METRIC

km

×1000

m

×100

cm

×10

mm

CUSTOMARY

mi

×5280

ft

×12

in

Steps of ten against odd steps

The metric system, of which the international units are part, is convenient for science and engineering because its units are grouped

by factors of ten. Converting from one unit to another means only moving by powers of ten. There are a hundred centimetres in a metre and a thousand metres in a kilometre. Compare that with the customary units still used in the United States: twelve inches in a foot, five thousand two hundred and eighty feet in a mile. Those odd numbers make every conversion a small chore.

Try it yourself

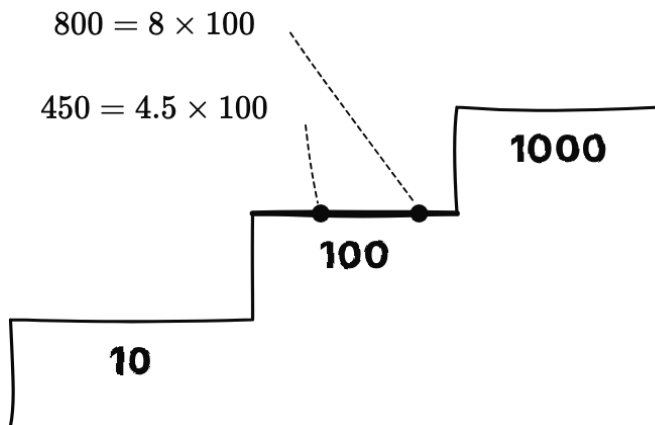
Take a ruler marked in centimetres and millimetres. Count the small millimetre spaces between two centimetre marks: there are ten. Now picture a metre stick: a hundred centimetres, or a thousand millimetres. Every step is a factor of ten, so converting is just a matter of moving the decimal point.

The second advantage is that one unit stretches across huge ranges just by changing its prefix. Metres suit a building site, kilometres suit air travel, and the tiny nanometre suits the design of lenses and optical devices. With the metric system there is no need to invent a new unit for each job.

The book's table lists the prefixes from the very large to the very small, each with an example. Kilo means a thousand, and a kilometre is about six tenths of a mile. Mega means a million; giga, a thousand million, and a gigahertz is a typical microwave frequency. Going down, centi means a hundredth, and a centimetre is about the thickness of a fingertip. Milli means a thousandth, and a millimetre is the height of a flea at its shoulders. Micro means a millionth, nano a thousand-millionth, and a femtometre, a thousand-million-millionth of a metre, is about the size of a proton. At the extremes sit exa, a one

followed by eighteen zeros, where an exametre is roughly the distance light travels in a century, and atto, its tiny mirror image, where an attosecond is about the time light takes to cross an atom. The two hundred and fifty milligrams on the medicine label is a quarter of a gram, and the twelve kilometres on your map is twelve thousand metres.

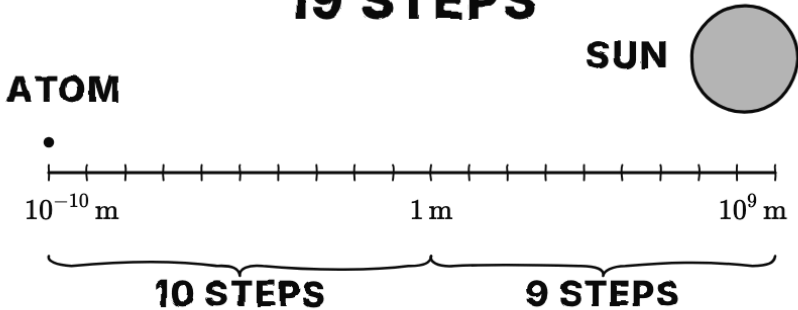
Powers of ten also give a way to judge the scale of a number, called its order of magnitude. Every number can be written as some number times a power of ten, and that power tells its order of magnitude. Eight hundred is eight times ten times ten, that is, eight times a hundred. Four hundred and fifty is four and a half times a hundred. Both sit on the "hundreds" step of the ladder, so they share the same order of magnitude. Said another way, order of magnitude is a ballpark: it tells you which step you are on, not exactly where on the step.



Same step, same order

Here is a striking comparison from the book. An atom is about one ten-thousand-millionth of a metre across: a one divided by ten, ten times over. The Sun is about a thousand million metres across: a one followed by nine zeros. How many steps of ten separate them? Ten steps take you from the atom up to one metre, and nine more take you up to the Sun, so nineteen steps in all. The Sun is about ten million million million times wider than an atom, a number we can only grasp by counting the steps.

19 STEPS



Nineteen steps from atom to Sun

The book ends the section with a note on where measurement is heading. Because matter is built from atoms, physicists would most like to base the units on tiny, unchanging things in nature. For time that is already done: the second rests on the vibrations of the cesium atom. Length was once tied to the wavelength of light from a certain atom, but has since moved to the even more precise speed of light. If the mass of atoms, or of a carefully made sphere of silicon atoms, can be measured more precisely than the kilogram standard, mass might one day rest on the small scale too. And someday a unit of electric charge might be based on the charges of electrons and protons, though for now current and charge are tied to large currents and the forces between wires.

Remember

Metric prefixes each mean a power of ten: kilo a thousand, milli a thousandth, micro a millionth, and so on. That makes metric conversions simple, unlike twelve inches to a foot or five thousand two hundred and eighty feet to a mile. The order of magnitude of a number is the power of ten that sets its scale; eight hundred and four hundred and fifty share the same one, the hundreds. An atom and the Sun differ by about nineteen powers of ten in size. Physicists hope to base all the units on tiny, unchanging things; the second already rests on the cesium atom.

Questions

1. Why are conversions easier in the metric system than in U.S. customary units?
 - a. Because metric units are smaller
 - b. Because every metric step is a power of ten
 - c. Because metric units are newer
 - d. Because customary units cannot be converted
2. Why do eight hundred and four hundred and fifty have the same order of magnitude?
 - a. Because both are times a hundred, on the same step of the ladder of tens
 - b. Because both are even
 - c. Because both are less than a thousand and end in zero

- d. Because their difference is small
3. An atom is about one ten-thousand-millionth of a metre across and the Sun about a thousand million metres. How many powers of ten separate them?
- a. Nine
 - b. Ten
 - c. Nineteen
 - d. Ninety
4. Which basic unit already rests on a tiny, unchanging thing in nature?
- a. The kilogram, on a silicon sphere
 - b. The ampere, on electrons
 - c. The metre, on a metal bar
 - d. The second, on the cesium atom

Answers

- 1. b)** Metric units are grouped by factors of ten, while customary units use steps like twelve and five thousand two hundred and eighty.
- 2. a)** Eight hundred is eight times a hundred and four hundred and fifty is four and a half times a hundred; both sit on the hundreds step.
- 3. c)** Ten steps take you from the atom up to one metre and nine more up to the Sun, nineteen in all.
- 4. d)** For time it is already done: the second rests on the vibrations of the cesium atom.

Worked examples

From atom to Sun in steps of ten An atom is about one ten-thousand-millionth of a metre across (ten to the minus ten metres), and the Sun about a thousand million metres (ten to the ninth). How many orders of magnitude apart are they?

Each order of magnitude is one step of ten.

From the atom up to one metre takes ten steps of ten.

From one metre up to a thousand million metres takes nine more.

Ten plus nine makes nineteen.

Answer: nineteen orders of magnitude

Practice

1. How many grams is a dose of two hundred and fifty milligrams?

Hint: Milli means a thousandth, so divide by a thousand.

Answer: A quarter of a gram, 0.25 grams: two hundred and fifty divided by a thousand.

2. A town is twelve kilometres away. How many metres is that, and how many centimetres?

Hint: Kilo means a thousand; a metre holds a hundred centimetres.

Answer: Twelve thousand metres, and one million two hundred thousand centimetres: twelve times a thousand, then times a hundred.

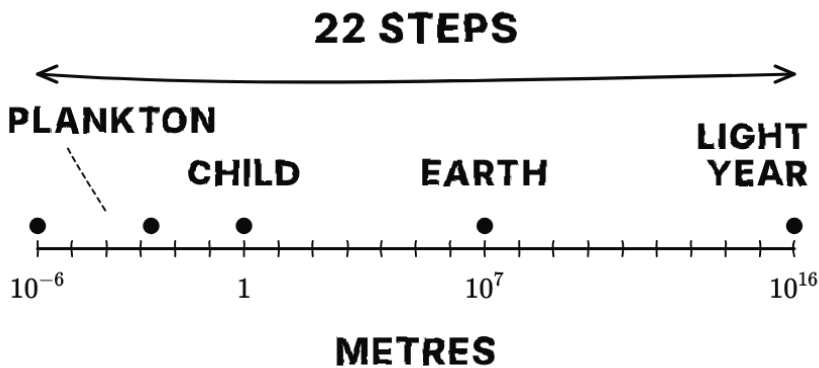
3. Do thirty and seventy have the same order of magnitude? What about thirty and three hundred?

Hint: Write each as a number times a power of ten.

Answer: Thirty and seventy both sit on the tens step, so yes. Three hundred is on the hundreds step, one order of magnitude higher than thirty.

From Plankton to Galaxies, and Back to Kilometres

Picture two photographs side by side. In one, tiny creatures called phytoplankton drift among ice crystals in the Antarctic sea; the largest are about two millimetres long, the smallest just a few thousandths of a millimetre. In the other, two clusters of galaxies collide, so far away that their light took two point four billion years to reach us. Physics deals with both, and with everything in between, and the book uses these pictures to make you feel just how wide that range is.



Dozens of powers of ten

To give a sense of it, the book has a table of known lengths, masses and times, each given as a power of ten. On the small side, the finest detail we can currently observe is about a billionth of a billionth of a metre, an electron has a mass with thirty zeros after the decimal point before its first digit, and light crosses a proton in about a hundred-thousandth of a billionth of a billionth of a second. In the middle sit familiar things: a person has a mass of about a hundred kilograms in the table's rounding, a car about a thousand, a heartbeat takes about one second, and a four-year-old child stands about one metre tall. On the large side, the Earth is about ten million metres across, light travels about ten thousand million million metres in a year, and the edge of the known universe lies about a hundred million million million million metres away, a one followed by twenty-six zeros. Said another way, the numbers in physics stretch across dozens of powers of ten, and simply looking through the table gives a feeling for the range of topics ahead.

With such a range, we constantly need to switch from one unit to another. You may follow a recipe from a European cookbook that measures milk in litres while your measuring jug is marked in cups. Or walking directions may give distances in feet when you want to know how many miles you will walk. The book shows a simple, reliable way to do any such conversion.

have: m

want: km

$$80 \cancel{\text{m}} \times \frac{1 \text{ km}}{1000 \cancel{\text{m}}} \\ = 0.080 \text{ km}$$

Units cancel like numbers

Take the plain example of turning eighty metres into kilometres. First, write down the unit you have, metres, and the unit you want, kilometres. Next, find a conversion factor: a ratio that says how many of one unit equal another. Familiar ones are twelve inches in a foot, a hundred centimetres in a metre, sixty seconds in a minute; here, a thousand metres in a kilometre. Then multiply what you have by that ratio, written so that the unwanted unit cancels. Eighty metres times one kilometre per thousand metres: the metres on top and the metres below cancel each other, just as equal numbers cancel in a fraction, and what remains is kilometres. Eighty divided by a thousand is eight hundredths, so eighty metres is zero point zero eight zero kilometres.

Try it yourself

Write "eighty metres" on a slip of paper. Beside it write the fraction "one kilometre over one thousand metres". Cross out the word "metres" above and below the line. Only "kilometres" is left, and the arithmetic, eighty divided by a thousand, tells you how many.

Why does this work? Because the ratio of one kilometre to a thousand metres is really just one: the top and the bottom are the same length. Multiplying by one never changes a quantity; it only changes the way the quantity is written. And the check is built in: if the units do not cancel the way you want, you have set up the ratio wrongly. The same method works for any units at all, and the book points to an appendix with a long list of conversion factors. It is exactly what a phone's unit-converter app does for you behind the scenes.

The book then begins a worked example. You drive the ten kilometres from school to home in twenty minutes. What was your average speed, first in kilometres per hour and then in metres per second? The plan is to calculate the speed in the units given, and then convert by picking the conversion factor that cancels the unwanted unit and leaves the wanted one. Average speed, the book reminds us, is the distance travelled divided by the time taken. Ten kilometres divided by twenty minutes gives half a kilometre per minute. That is a perfectly good speed, but in an awkward unit, and turning minutes into hours is the next step.

SCHOOL

10 km



20 min

$$v = \frac{d}{t}$$

$$\frac{10 \text{ km}}{20 \text{ min}} = 0.5 \text{ km/min} \rightarrow \text{NEXT: HOURS}$$

Ten kilometres in twenty minutes

Remember

Physics spans dozens of powers of ten, from the finest observable detail to the edge of the known universe, as the book's table shows. A conversion factor is a ratio of equal amounts in different units, such as a thousand metres in a kilometre. Multiply by the factor so that the unwanted unit cancels: eighty metres times one kilometre per thousand metres gives zero point zero eight zero kilometres. The factor is really equal to one, so the quantity stays the same; only its unit changes. Driving ten kilometres in twenty minutes is an average speed of half a kilometre per minute.

Questions

1. Why does multiplying by a conversion factor not change the amount being converted?
 - a. Because the factor is always very small
 - b. Because the factor's top and bottom are equal amounts, so it equals one
 - c. Because the units are ignored
 - d. Because conversion factors only change the number, not the unit
2. How can you tell that a conversion factor has been set up wrongly?
 - a. The units do not cancel the way you want
 - b. The answer is a whole number
 - c. The answer is smaller than the starting number
 - d. The factor contains a thousand
3. Driving ten kilometres in twenty minutes gives what average speed in the units given?
 - a. Two kilometres per minute
 - b. Twenty kilometres per hour
 - c. Half a kilometre per minute
 - d. Thirty kilometres per minute
4. What does the book's table of lengths, masses and times show?
 - a. That most quantities are about one
 - b. That physics deals only with large things

- c. That masses are always larger than lengths
- d. That physical quantities stretch across dozens of powers of ten

Answers

- 1. **b)** One kilometre and a thousand metres are the same length, so the ratio is one; multiplying by one changes only how the quantity is written.
- 2. **a)** The check is built in: if the units do not cancel as intended, the ratio is set up wrongly.
- 3. **c)** Average speed is distance divided by time: ten divided by twenty is half a kilometre per minute.
- 4. **d)** From the finest observable detail to the edge of the universe, the numbers span dozens of powers of ten.

Worked examples

Eighty metres in kilometres Convert eighty metres into kilometres using a conversion factor.

We have metres and want kilometres.

The conversion factor is one kilometre per thousand metres.

Multiply eighty metres by one kilometre per thousand metres; the metres cancel, leaving kilometres.

Eighty divided by a thousand is eight hundredths.

Answer: 0.080 kilometres

Practice

- 1. Convert two and a half kilometres into metres.

Hint: Use a thousand metres per kilometre, so that kilometres cancel.

Answer: Two thousand five hundred metres: two and a half times a thousand.

2. How many seconds are there in three and a half minutes?

Hint: Use sixty seconds per minute.

Answer: Two hundred and ten seconds: three and a half times sixty.

3. A walk is thirty-six inches long. How many feet is that?

Hint: Use one foot per twelve inches, so that inches cancel.

Answer: Three feet: thirty-six divided by twelve.

Finishing the Drive Home

We left the drive home with an awkward answer: ten kilometres in twenty minutes is half a kilometre per minute. Nobody's speedometer reads in kilometres per minute. So how do we turn it into kilometres per hour, and then into metres per second, without getting lost?

$$0.5 \frac{\text{km}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ h}}$$

$$0.5 \frac{\text{km}}{\cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ h}} = 30 \frac{\text{km}}{\text{h}}$$

Minutes cancel into hours

The trick is the same as before: multiply by a conversion factor that cancels the unit you do not want. Minutes are unwanted, and there are sixty minutes in an hour. So multiply half a kilometre per minute by sixty minutes per hour. The minutes cancel, leaving kilometres per

hour, and half times sixty is thirty. The average speed is thirty kilometres per hour.

The book then shows how to check such an answer, in four steps. First, make sure the units really cancelled. If you had turned the factor upside down, multiplying by one hour per sixty minutes, nothing would cancel; you would end up with kilometres times hours divided by minutes squared, which is plainly not a speed in kilometres per hour. Second, check that the final unit is the one you were asked for: kilometres per hour, and it is. Third, check the digits: the distance, ten point zero, and the time, twenty point zero, each have three meaningful digits, so the answer, thirty point zero, should have three as well. The sixty does not limit this, because an hour is defined to be exactly sixty minutes, so that factor is perfectly precise. Fourth, ask whether the answer is reasonable. Twenty minutes is a third of an hour; in a full hour you would go three times as far, three times ten kilometres, which is thirty. The answer makes sense. It is the kind of speed your car's speedometer might show on a slow road through town.

Now the second request: metres per second. Start from thirty kilometres per hour and use two conversion factors, one turning hours into seconds and one turning kilometres into metres. There are three thousand six hundred seconds in an hour and a thousand metres in a kilometre. Multiply thirty by a thousand and divide by three thousand six hundred, and you get about eight point three three metres per second. Had you started from half a kilometre per minute instead, you would have needed different factors, sixty seconds per minute and a thousand metres per kilometre, but you would reach the very same answer. Said another way, the route can differ, but correct conversions always arrive at the same place.

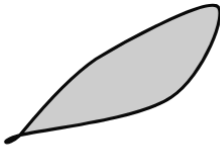
$$30 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \times \frac{1 \cancel{\text{h}}}{3600 \text{ s}}$$

$$\approx 8.33 \frac{\text{m}}{\text{s}}$$

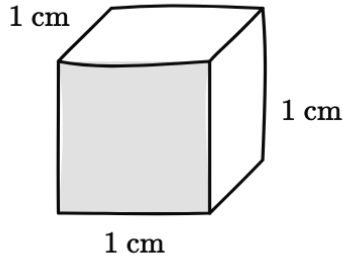
Kilometres per hour into metres per second

Why give the answers to three digits, and not all the digits a calculator shows? The book raises that question here and promises to answer it in the next section, on accuracy, precision and significant figures.

A note on curious units follows. Besides the familiar ones, there are obscure units such as the firkin, once used to measure beer; one firkin is about thirty-four litres. The book invites you to look up old weights and measures, such as the barleycorn, and to work out how each relates to the metric units.



$$\frac{1}{50} \text{ s} = 20 \text{ ms}$$



$$1 \text{ cm}^3 = 1 \text{ mL}$$

A millilitre is a little cube

Two quick questions close the section. Some hummingbirds beat their wings more than fifty times a second. To time a single beat, a scientist would use the fundamental unit of time, the second, but in a suitably small size: the millisecond, a thousandth of a second. Fifty beats in one second means each beat lasts a fiftieth of a second, which is twenty milliseconds. The second question: one cubic centimetre equals one millilitre. What does that tell us? That the unit of volume, the litre, is built from the unit of length; a millilitre is simply a small cube one centimetre on each side. The dose cup that comes with a bottle of cough syrup, marked in millilitres, is really measuring little cubes of centimetres.

Remember

Half a kilometre per minute times sixty minutes per hour is thirty kilometres per hour; the minutes cancel. Check every conversion: units cancel, the final unit is right, the digits match the data, and the answer is reasonable. Thirty kilometres per hour is about eight point three three metres per second, using three thousand six hundred seconds per hour and a thousand metres per kilometre. A hummingbird's wingbeat, a fiftieth of a second, is best timed in milliseconds: twenty of them. One cubic centimetre equals one millilitre, because volume is a unit derived from length.

Questions

1. Why does the factor of sixty minutes per hour not limit the number of meaningful digits in the answer?
 - a. Because sixty is a round number
 - b. Because an hour is defined to be exactly sixty minutes, so the factor is perfectly precise
 - c. Because minutes cancel out
 - d. Because the answer is in hours
2. How does the lecture check that thirty kilometres per hour is reasonable?
 - a. Twenty minutes is a third of an hour, so in an hour you would go three times ten kilometres

- b. By comparing with the speed of light
 - c. By counting the digits
 - d. By converting it to metres per second
3. Starting from half a kilometre per minute instead of thirty kilometres per hour, what happens to the answer in metres per second?
- a. It comes out sixty times larger
 - b. It comes out in different units
 - c. It is the same, about eight point three three metres per second
 - d. It cannot be found
4. Why would a scientist time one hummingbird wingbeat in milliseconds?
- a. Because milliseconds are the fundamental unit of time
 - b. Because a wingbeat lasts several seconds
 - c. Because milliseconds are more accurate than seconds
 - d. Because each beat lasts only a fiftieth of a second, twenty thousandths

Answers

- 1. b)** An hour is exactly sixty minutes by definition, so the factor carries no measurement uncertainty.
- 2. a)** In a full hour you would go three times as far as in twenty minutes: thirty kilometres.
- 3. c)** Different factors are needed, but correct conversions always arrive at the same answer.

4. d) At fifty beats a second, each beat lasts twenty milliseconds, a convenient size for such a quick event.

Worked examples

The drive home in kilometres per hour and metres per second You drive ten kilometres in twenty minutes. What is your average speed in kilometres per hour, and in metres per second?

Distance divided by time: ten kilometres in twenty minutes is half a kilometre per minute.

Multiply by sixty minutes per hour; the minutes cancel, giving thirty kilometres per hour.

For metres per second, multiply thirty by a thousand metres per kilometre and divide by three thousand six hundred seconds per hour.

Thirty thousand divided by three thousand six hundred is about eight point three three.

Answer: thirty kilometres per hour, about eight point three three metres per second

Practice

1. You cycle six kilometres in fifteen minutes. What is your average speed in kilometres per hour?

Hint: First find kilometres per minute, then multiply by sixty minutes per hour.

Answer: Twenty-four kilometres per hour: six divided by fifteen is zero point four kilometres per minute, times sixty is twenty-four.

2. Convert seventy-two kilometres per hour into metres per second.

Hint: Multiply by a thousand metres per kilometre and divide by three thousand six hundred seconds per hour.

Answer: Twenty metres per second: seventy-two thousand divided by three thousand six hundred.

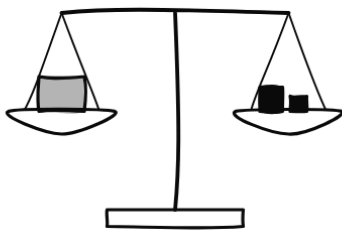
3. How many millilitres are in a cube five centimetres on each side?

Hint: One cubic centimetre is one millilitre; find the volume in cubic centimetres.

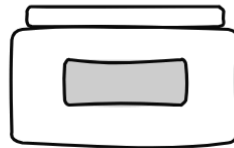
Answer: One hundred and twenty-five millilitres: five times five times five cubic centimetres.

Close to the Truth, Close to Each Other

Step on your bathroom scale three times in a row. Does it show exactly the same number each time? And even if it does, is that number your true weight? Those are two different questions, and this part is about the difference between them.



0.1 g



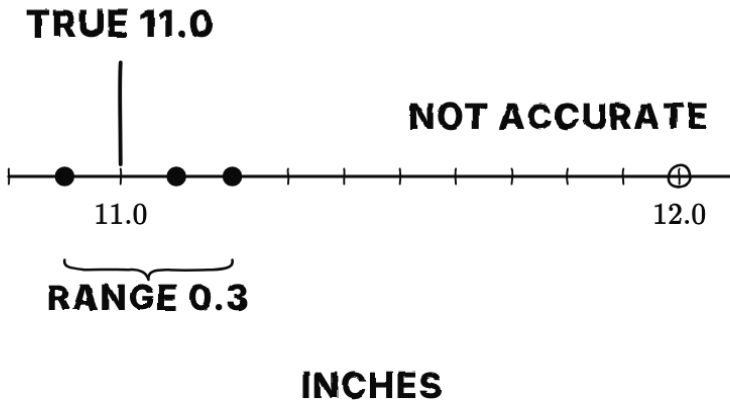
0.001 g

Balance against digital scale

The book opens with two pictures. The first is an old double-pan balance: the unknown object goes in one pan and metal weights of known mass, such as one gram, ten grams and a hundred grams, go in the other, until the bar between the pans hangs level and the masses

are equal. The second is a digital scale, which has replaced many such balances. A mechanical balance may read only to the nearest tenth of a gram, while many digital scales read to the nearest thousandth of a gram. Your kitchen scale probably reads to the nearest gram, good for flour but too coarse for weighing a medicine dose.

Science rests on observation and experiment, which means on measurements, and every measurement can be judged in two ways. The first is accuracy: how close a measurement is to the correct value. The book's example is a sheet of standard computer paper, sold as eleven point zero inches long. You measure it three times and get eleven point one, eleven point two and ten point nine inches. All three are close to eleven point zero, so the measurements are quite accurate. A reading of twelve inches would not be.



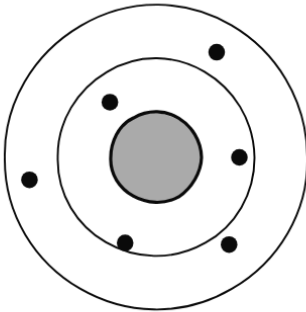
Three readings of 11 inches

The second is precision: how closely repeated measurements, made the same way, agree with each other. One simple way to judge it is the range, the gap between the lowest and the highest reading. For the paper, the lowest was ten point nine inches and the highest eleven point two, so the readings differ by at most three tenths of an inch. They are fairly precise, because they do not wander much. Had you measured ten point nine, eleven point one and eleven point nine, the range would be a whole inch, and the measurements would not be very precise. Said another way, accuracy asks "how near the target?", while precision asks "how near to each other?"

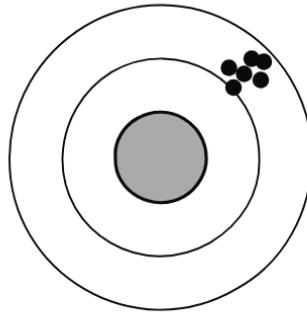
Try it yourself

Measure the width of a table five times with a tape measure, writing down each reading to the nearest millimetre. Subtract the smallest reading from the largest: that range shows your precision. Then compare your average with the width printed in the table's instructions, if you have them: the closeness shows your accuracy.

The two qualities need not come together. The book uses a satellite navigation system trying to find a restaurant. Picture the restaurant as the bull's-eye at the centre of a target, and each attempt to locate it as a black dot. In the first picture, the dots are scattered far apart but spread evenly around the centre. That is low precision but high accuracy: no single fix is reliable, yet on average they point to the right place. In the second picture, the dots huddle tightly together, but in a cluster far off to one side of the bull's-eye. That is high precision but low accuracy: the system repeats itself faithfully, but it is faithfully wrong. When the blue dot on your phone's map jumps about the street, its precision is low; when it sits steadily on the wrong block, its accuracy is low.



**ACCURATE,
NOT PRECISE**



**PRECISE,
NOT ACCURATE**

Accurate or precise

The paper measurements, by contrast, were both accurate and precise: close to eleven inches, and close to each other. That is what every good measurement aims for.

Remember

Accuracy is how close a measurement comes to the correct value; precision is how closely repeated measurements agree. Paper readings of eleven point one, eleven point two and ten point nine inches are accurate, since they are near eleven, and precise, since their range is only three tenths of an inch. The range between the lowest and highest reading is a simple measure of precision.

Navigation fixes scattered around the target are accurate but not precise; fixes clustered away from it are precise but not accurate. A digital scale reading to a thousandth of a gram is more precise than a balance reading to a tenth.

Questions

1. A bathroom scale shows the same wrong weight every time. How would you describe it?
 - a. Accurate but not precise
 - b. Precise but not accurate
 - c. Both accurate and precise
 - d. Neither, because it repeats itself
2. Paper sold as eleven inches long measures eleven point one, eleven point two and ten point nine. Why are these accurate?
 - a. Because all three are close to the correct eleven inches
 - b. Because they are all different
 - c. Because there are three of them

- d. Because they were measured with a ruler
- 3. Navigation fixes are scattered widely, but evenly around the restaurant. What does that show?
 - a. High precision and high accuracy
 - b. High precision but low accuracy
 - c. Low precision but high accuracy
 - d. Neither precision nor accuracy

Answers

- 1. **b)** Readings that agree with each other are precise; being far from the true value makes them inaccurate.
- 2. **a)** Accuracy is closeness to the correct value, and all three readings are near eleven point zero inches.
- 3. **c)** Scattered dots mean low precision; their being centred on the target means high accuracy.

Worked examples

Judging precision by the range Three measurements of a sheet of paper are eleven point one, eleven point two and ten point nine inches. What is their range, and are they precise?

The range is the highest reading minus the lowest.

The highest is eleven point two inches and the lowest ten point nine.

Eleven point two minus ten point nine is three tenths of an inch.

The readings wander by at most three tenths of an inch, so they are fairly precise.

Answer: three tenths of an inch; fairly precise

Practice

1. Measurements of the same sheet come out ten point nine, eleven point one and eleven point nine inches. What is the range, and how does it compare with the first set?

Hint: Subtract the lowest from the highest.

Answer: One inch: eleven point nine minus ten point nine. That is more than three times the first range, so these readings are much less precise.

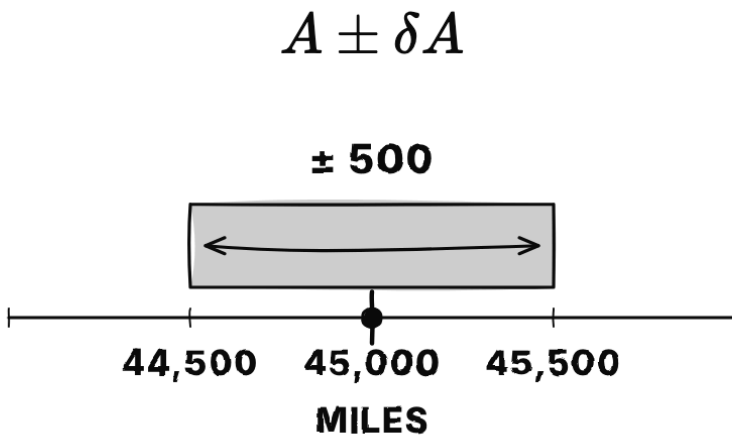
2. A kitchen scale weighs a bag of sugar sold as one kilogram five times: one thousand and four, one thousand and five, one thousand and four, one thousand and five, one thousand and four grams. Are the readings accurate, precise, or both?

Hint: Check the range, then how far the readings are from one thousand grams.

Answer: Both: the range is only one gram, so they are precise, and every reading is within five grams of one thousand, so they are close to the expected value.

Plus or Minus

Someone asks how far your car has been driven. You glance at the odometer and say: "About forty-five thousand miles, give or take five hundred." Without noticing, you have just given a measurement together with its uncertainty.



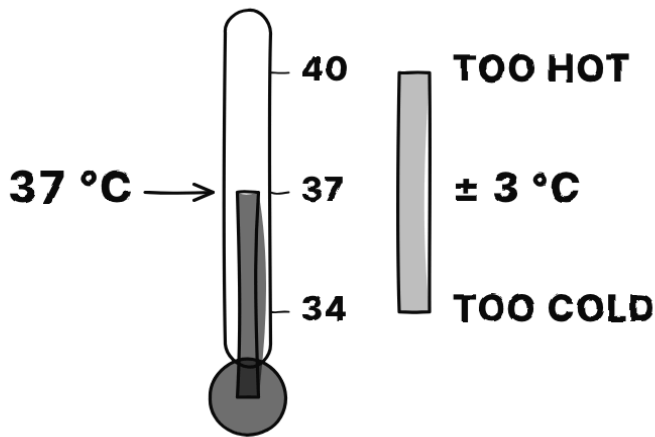
Plus or minus five hundred

Accuracy and precision, the book says, both show up in one number called the uncertainty. Uncertainty is a measure of how far your measured values may stray from a standard or expected value. If the measurements are not very accurate or precise, the uncertainty is

large. Put more simply, the uncertainty is a disclaimer attached to a measurement. "Forty-five thousand miles, plus or minus five hundred" means the true figure could be as low as forty-four thousand five hundred or as high as forty-five thousand five hundred, or anywhere in between. Every measurement has some uncertainty; for the sheet of paper from before, we might say eleven inches, plus or minus two tenths of an inch. Scientists often write the uncertainty with the Greek letter delta in front of the quantity, read as "delta A", and record the result as the value plus or minus that delta.

Where does uncertainty come from? The book lists four sources. The measuring tool has limits: the smallest marking on the ruler may be a tenth of an inch. The person measuring has limited skill: perhaps they have poor eyesight. The object itself may be irregular: one side of the paper may be slightly longer than the other. And there are other factors, which depend on the situation. An honest uncertainty comes from thinking carefully about all of them and how much each might matter.

Why does it matter? The book's example is a feverish child. You take the child's temperature, and the thermometer reads thirty-seven degrees Celsius, normal body temperature. But suppose the thermometer's uncertainty were three degrees. Then the true temperature could lie anywhere from thirty-four degrees, dangerously cold, to forty degrees, dangerously hot. Such a thermometer would be useless. That is why a medicine dose or a fever reading needs an instrument whose uncertainty is small compared with the differences that matter.



A useless thermometer

There is a neat way to compare uncertainties of very different measurements: the percent uncertainty. In words: divide the uncertainty by the measured value, then multiply by a hundred per cent. It tells you how big the uncertainty is compared with the thing being measured.

$$\% \text{ unc} = \frac{\delta A}{A} \times 100\%$$

4.8

5.3

4.9

5.4

average = 5.1 lb

$$\frac{0.4}{5.1} \times 100\% \approx 8\%$$

**HALF BAG:
ABOUT 16%**

Percent uncertainty of apples

The book works an example with apples. A shop sells five-pound bags. Over four weeks you buy one bag a week and weigh it: four point eight, five point three, four point nine and five point four pounds. From other measurements you already know the uncertainty in the average weight is four tenths of a pound. First, the average: add the four weights to get twenty point four, and divide by four, which gives five point one pounds. That agrees with the label's five pounds, within the four-tenths uncertainty. Now the percent uncertainty: four tenths divided by five point one is about eight hundredths, and times a hundred per cent that is about eight per cent. So the bag weighs five pounds, plus or minus eight per cent. On your kitchen scale, eight per cent of a five-pound bag is roughly the weight of one small apple.

The book adds a question to think about: what if the bag were only half as heavy, but the uncertainty stayed four tenths of a pound? The same uncertainty, divided by a value half as large, makes the percent uncertainty about twice as big. Said another way, the same small error matters more when the thing being measured is small. And a practical warning: always remember to multiply the fraction by a hundred per cent; otherwise you have a decimal, not a percentage.

Remember

Uncertainty says how far a measurement may stray from the true or expected value; it is written as plus or minus an amount. It comes from the limits of the tool, the skill of the person, irregularities in the object and other factors. A thermometer with an uncertainty of three degrees would be useless for checking a fever. Percent uncertainty is the uncertainty divided by the measured value, times a hundred per cent. Four bags averaging five point one pounds with four tenths of a pound uncertainty give about eight per cent.

Questions

1. Why would a thermometer with an uncertainty of three degrees be useless for checking a fever?
 - a. Because it reads in Celsius
 - b. Because a normal reading could mean anything from dangerously cold to dangerously hot

- c. Because three degrees is too precise
 - d. Because body temperature never changes
2. What does percent uncertainty tell you that the plain uncertainty does not?
- a. How big the uncertainty is compared with the thing being measured
 - b. The exact true value
 - c. Which tool was used
 - d. How many times the measurement was repeated
3. If a bag were half as heavy but its uncertainty stayed the same, what would happen to the percent uncertainty?
- a. It would halve
 - b. It would stay the same
 - c. It would roughly double
 - d. It would become zero
4. Which of these is one of the book's four sources of uncertainty?
- a. The colour of the object
 - b. The price of the instrument
 - c. The time of day
 - d. Irregularities in the object being measured

Answers

- 1. b)** A reading of thirty-seven degrees could mean anywhere from thirty-four to forty, so it tells you nothing useful.
- 2. a)** Dividing the uncertainty by the measured value shows its size relative to the measurement.

3. **c)** The same uncertainty divided by a value half as large gives a fraction about twice as big.
4. **d)** The four sources are the tool's limits, the person's skill, the object's irregularities and other situational factors.

Worked examples

A bag of apples Four five-pound bags of apples weigh four point eight, five point three, four point nine and five point four pounds, and the uncertainty in the average is four tenths of a pound. What are the average weight and the percent uncertainty?

Add the weights: four point eight plus five point three plus four point nine plus five point four is twenty point four pounds.

Divide by four: the average is five point one pounds.

Divide the uncertainty by the average: four tenths divided by five point one is about eight hundredths.

Multiply by a hundred per cent: about eight per cent.

Answer: five point one pounds, about eight per cent

Practice

1. A car's mileage is given as forty-five thousand miles, plus or minus five hundred. What is the lowest and highest the true mileage could be, and what is the percent uncertainty?

Hint: Subtract and add the uncertainty; then divide it by the value and multiply by a hundred per cent.

Answer: Between forty-four thousand five hundred and forty-five thousand five hundred miles. Five hundred divided by forty-five thousand is about one hundredth, so about one per cent.

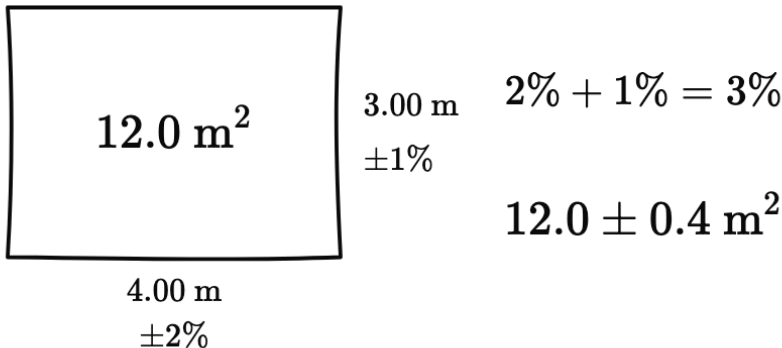
2. A length is measured as twenty centimetres, plus or minus half a centimetre. What is its percent uncertainty?

Hint: Divide half a centimetre by twenty centimetres, then multiply by a hundred per cent.

Answer: Two and a half per cent: half divided by twenty is one fortieth, which is two and a half per cent.

Uncertainty Travels, and Digits Tell

You measure a floor to buy carpet: four metres long and three metres wide. Neither measurement is perfect. So how sure can you be of the area you calculate from them?



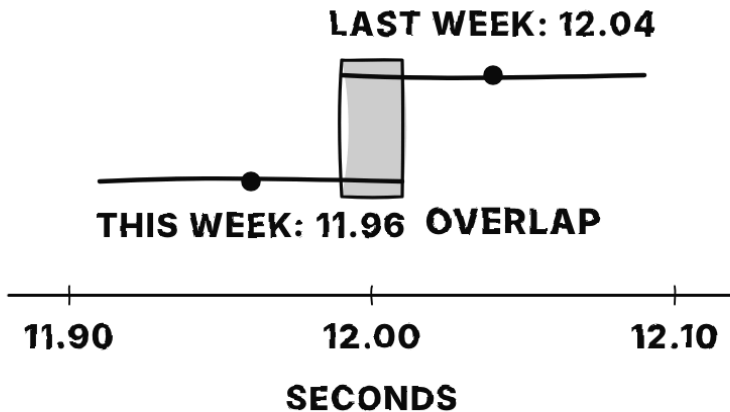
Uncertainties add up

Anything calculated from measured quantities inherits their uncertainty. The area of a floor, worked out from its length and width, is uncertain because the length and width are. The book gives a simple rule for multiplying or dividing, as long as each measurement's uncertainty is small, a few per cent or less. In words: the percent

uncertainty of the result is the sum of the percent uncertainties of the measurements that went into it.

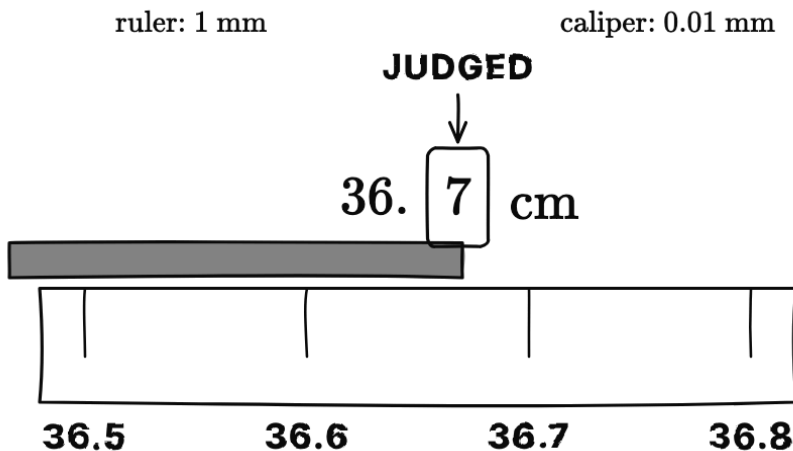
Here is the floor. The length is four point zero zero metres with an uncertainty of two per cent, and the width three point zero zero metres with one per cent. The area is four times three, twelve point zero square metres. Its percent uncertainty is two per cent plus one per cent, which is three per cent. Three per cent of twelve square metres is three hundredths of twelve, thirty-six hundredths of a square metre. Since the area is given to a tenth of a square metre, that is rounded to four tenths. Why add? Because each error stretches or shrinks the result by its own share, and in the worst case both pull the same way. When you buy carpet, those four tenths of a square metre are a reason to order a little extra.

The book then asks a sharp question. A track coach buys a new stopwatch whose manual says it is uncertain by five hundredths of a second. The team's best sprinter ran a hundred metres in twelve point zero four seconds last week, and eleven point nine six this week. Was this week's run faster? The times differ by eight hundredths of a second. But each reading could be off by five hundredths, so last week's time might really be as low as eleven point nine nine, and this week's as high as twelve point zero one. The ranges overlap. The book's answer: no, the stopwatch is too uncertain to tell the two times apart. Said another way, a difference only means something if it is bigger than the uncertainty.



Two times that cannot be told apart

That leads to the precision of the tool itself. A precise tool measures in very small steps. A standard ruler reads to the nearest millimetre, while a caliper, a sliding gauge with fine jaws, reads to the nearest hundredth of a millimetre. The caliper is the more precise tool because it can detect much smaller differences in length. The more precise the tool, the more precise and accurate the measurements can be.



3 SIGNIFICANT FIGURES

The last digit is a guess

Try it yourself

Lay a stick along a ruler and look closely at its end. It probably falls between two millimetre marks, say between thirty-six point six and thirty-six point seven centimetres. You have to judge which is nearer, and you might write thirty-six point seven. That last digit is your best estimate, not a certainty.

This is the heart of significant figures. You may write down only as many digits as your tool actually measured. With a ruler you can write thirty-six point seven centimetres, but not thirty-six point seven one, because a ruler cannot measure hundredths of a centimetre. The last

digit you write is always the first one with some doubt in it. To count significant figures, start from the first measured digit on the left and count through the last digit written on the right. Thirty-six point seven has three significant figures. So the number of significant figures tells you how precise the measuring tool was.

Remember

Anything calculated from measurements carries their uncertainty. When multiplying or dividing, add the small percent uncertainties: two per cent and one per cent give three per cent. A floor four by three metres has an area of twelve square metres, uncertain by about four tenths. A stopwatch uncertain by five hundredths of a second cannot tell twelve point zero four from eleven point nine six. Write only the digits your tool measured; the last one is the first uncertain digit, and the count of digits shows the tool's precision.

Questions

1. When you multiply two measurements with small uncertainties, how do you find the percent uncertainty of the result?
 - a. Multiply their percent uncertainties
 - b. Add their percent uncertainties
 - c. Take the larger one only
 - d. Ignore them, since multiplying removes uncertainty

2. Why can't the coach say twelve point zero four seconds and eleven point nine six seconds are different?
 - a. Because each reading could be off by five hundredths, so the possible ranges overlap
 - b. Because the sprinter ran different distances
 - c. Because stopwatches only read whole seconds
 - d. Because eight hundredths is a large difference
3. Why can a ruler reading not be written as thirty-six point seven one centimetres?
 - a. Because rulers are inaccurate
 - b. Because two decimals are not allowed
 - c. Because a ruler cannot measure hundredths of a centimetre
 - d. Because the stick is too short
4. What does the last digit written in a measurement represent?
 - a. A digit known for certain
 - b. A digit copied from the label
 - c. A rounding error to be ignored
 - d. The first digit with some doubt in it

Answers

1. **b)** The percent uncertainty of a product or quotient is the sum of the inputs' percent uncertainties.
2. **a)** Last week's time could be as low as eleven point nine nine and this week's as high as twelve point zero one.
3. **c)** You may write only as many digits as your tool measured, and a ruler reads to millimetres.

4. d) The last digit you write is always the first one with some uncertainty: your best estimate.

Worked examples

Uncertainty in the area of a floor A floor is four point zero zero metres long with two per cent uncertainty and three point zero zero metres wide with one per cent. Find its area and the uncertainty in square metres.

Area is length times width: four times three is twelve point zero square metres.

For multiplication, add the percent uncertainties: two per cent plus one per cent is three per cent.

Three per cent of twelve is thirty-six hundredths of a square metre.

The area is given to tenths, so round to four tenths.

Answer: twelve point zero square metres, plus or minus about four tenths

Practice

1. A rectangle is measured as five metres by two metres, with uncertainties of three per cent and two per cent. What is the area and its percent uncertainty?

Hint: Multiply the sides, then add the percents.

Answer: Ten square metres with five per cent uncertainty, that is, plus or minus half a square metre.

2. How many significant figures are in thirty-six point seven centimetres, and what does that tell you about the tool?

Hint: Count from the first measured digit to the last digit written.

Answer: Three; the last digit is in tenths of a centimetre, so the tool, a ruler, reads to the nearest millimetre.

Zeros and Honest Answers

Type one point two times one point two times pi into a calculator, and it proudly shows four point five two three eight nine three four. Eight digits. But if you measured the one point two with a simple tape, are all those digits real? The book's answer is no, and this part shows how to know which digits to keep.

SIGNIFICANT FIGURES

0.0009	1
--------	----------

15 450.0	6
----------	----------

6×10^3	1
-----------------	----------

87.990	5
--------	----------

30.42	4
-------	----------

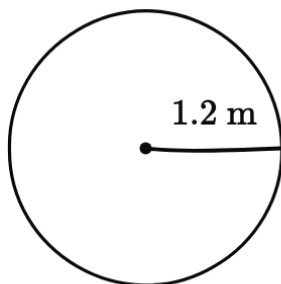
Which zeros count

First, the tricky digit: zero. In zero point zero five three, the zeros only mark where the decimal point goes; they are placekeepers, not measurements. So zero point zero five three has just two significant

figures, the five and the three. In ten point zero five three, the zeros sit between measured digits, so they count: five significant figures. A number like one thousand three hundred is ambiguous. Its zeros might mean it was measured to the last unit, or they might just hold places, so it could have two, three or four significant figures. To remove the doubt, write it in scientific notation, as a number times a power of ten. The rule in short: zeros are significant except when they only hold places.

The book offers five to practise on. Zero point zero zero zero nine has one significant figure, since its zeros just place the decimal point. Fifteen thousand four hundred and fifty point zero has six, because the zeros show the measurement was made to a tenth. Six times ten cubed has one; the power of ten only places the decimal point. Eighty-seven point nine nine zero has five, since the final zero shows a measurement to a thousandth. And thirty point four two has four, since a zero between significant figures counts.

Now combining measurements. A result cannot be more precise than the least precise measurement that went into it. There are two rules.



WEAK LINK

4.523893



4.5 m²

2 SIG. FIGS

The weak link: the radius

For multiplying and dividing, keep as many significant figures as the input with the fewest. Take the area of a circle, pi times the radius times the radius. If the radius is one point two metres, two significant figures, the calculator's four point five two three eight nine three four square metres must be cut to four point five square metres. Pi is known to many more digits, but that does not help: the radius is the weak link. It is like a recipe that says "about a cup" of flour; weighing the sugar to the milligram will not make the cake any more exact.

For adding and subtracting, the rule is about decimal places, not significant figures. The answer may have no more decimal places than the least precise measurement. The book's example: you buy seven point five six kilograms of potatoes on a shop scale that reads to

hundredths. You leave six point zero five two kilograms at your laboratory, weighed on a scale that reads to thousandths. Then at home you add thirteen point seven kilograms, weighed on a bathroom scale that reads only to tenths. Seven point five six minus six point zero five two plus thirteen point seven is fifteen point two zero eight kilograms. The least precise measurement, thirteen point seven, goes only to tenths, so the answer is rounded to tenths: fifteen point two kilograms. Said another way, a chain of weighings is only as fine as its coarsest scale.

$$\begin{array}{r} 7.56 \\ - 6.052 \\ + 13.7 \\ \hline 15.208 \end{array} \quad \begin{array}{l} \text{TENTHS ONLY} \\ \longrightarrow 15.2 \text{ kg} \end{array}$$

The coarsest scale decides

Remember

Zeros that only place the decimal point are not significant; zeros between digits, or measured at the end after a decimal point, are. A number like one thousand three hundred is ambiguous; scientific notation removes the doubt. When multiplying or dividing, keep as many significant figures as the least precise input: a radius of one point two metres gives an area of four point five square metres. When adding or subtracting, keep as many decimal places as the least precise input: the potatoes total fifteen point two kilograms. An answer must never claim more precision than its measurements.

Questions

1. Why does zero point zero five three have only two significant figures?
 - a. Because it is less than one
 - b. Because its zeros only mark where the decimal point goes
 - c. Because the five is uncertain
 - d. Because zeros never count
2. A circle's radius is one point two metres. Why is its area given as four point five square metres?
 - a. Because the radius has two significant figures, so the result keeps two
 - b. Because pi is only known to two digits

- c. Because areas are always rounded to one decimal
 - d. Because the calculator is wrong
3. Why are the potatoes reported as fifteen point two kilograms, not fifteen point two zero eight?
- a. Because the shop scale reads to hundredths
 - b. Because subtraction always loses digits
 - c. Because the bathroom scale, the least precise, reads only to tenths
 - d. Because kilograms are rounded to tenths by law
4. How many significant figures does eighty-seven point nine nine zero have?
- a. Three
 - b. Four
 - c. Six
 - d. Five

Answers

1. **b)** The zeros are placekeepers, not measurements, so only the five and the three count.
2. **a)** When multiplying, the result keeps as many significant figures as the least precise input, the radius.
3. **c)** When adding or subtracting, the answer keeps no more decimal places than the least precise measurement.
4. **d)** The final zero shows a measurement to a thousandth, so it counts: five significant figures.

Worked examples

Adding potato weighings You buy seven point five six kilograms of potatoes, leave six point zero five two kilograms at a laboratory, then add thirteen point seven kilograms at home. How much do you have, to the right precision?

Seven point five six minus six point zero five two is one point five zero eight.

Add thirteen point seven: fifteen point two zero eight kilograms.

The least precise measurement, thirteen point seven, goes only to tenths.

Round to tenths: fifteen point two kilograms.

Answer: fifteen point two kilograms

Practice

1. How many significant figures are there in zero point zero zero zero nine, in fifteen thousand four hundred and fifty point zero, and in thirty point four two?

Hint: Placekeeping zeros do not count; zeros between digits or measured at the end after a decimal point do.

Answer: One, six and four.

2. A rectangle measures two point five metres by one point two three metres. What is its area, to the right number of significant figures?

Hint: Multiply, then keep as many significant figures as the input with the fewest.

Answer: Three point one square metres: two point five times one point two three is three point zero seven five, and two point five has only two significant figures.

Three Digits, and a Guess at a Trillion

How tall is the block of flats down your street? You probably cannot measure it, but you can count its floors. With one familiar length and a little multiplication, you can make a surprisingly good guess, and this part is about exactly that skill. First, though, a word on how this book handles its digits.

HOW MANY DIGITS?

THIS BOOK: 3 SIGNIFICANT FIGURES

$$2 \times 13.5 + 10.2 = 37.2 \text{ lb}$$

EXACT

$$F = 55 \times 0.0255 \approx 1.4 \text{ N}$$

2 SIG. FIGS

Exact numbers do not count

In this book, most numbers are taken to have three significant figures, and the worked examples keep that count consistent. An answer given to three digits rests on input good to at least three digits; if the input

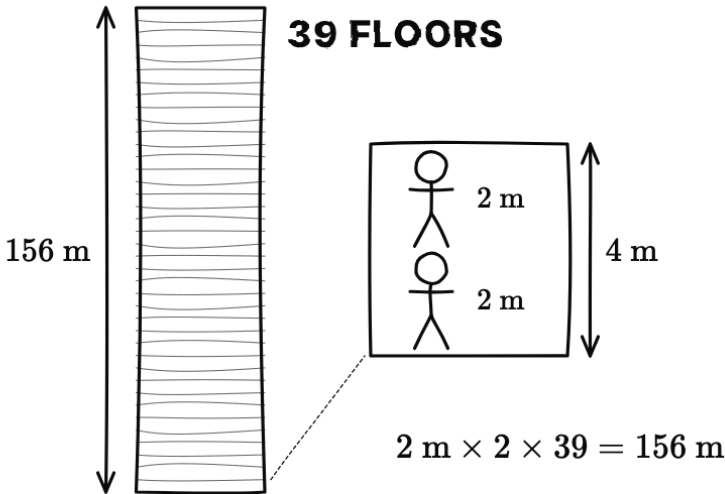
has fewer, so does the answer. The book also takes care that the number of digits makes sense for the situation, and in some topics, such as optics, it uses more than three because more accuracy is needed. And one important exception: an exact number does not limit the digits of a result. The two in the rule for the distance around a circle, two times π times the radius, is exactly two, not a measurement, so it does not count.

The book tests this with two quick calculations. A woman has three bags: two weigh thirteen point five pounds each, and one weighs ten point two. The total is two times thirteen point five, plus ten point two, which is thirty-seven point two pounds. The "two bags" is a count, an exact number, so it does not cut down the digits. Second, force equals mass times acceleration. A wagon of fifty-five kilograms speeds up by two hundred and fifty-five ten-thousandths of a metre per second every second. Fifty-five times that is about one point four zero, and since fifty-five kilograms has only two significant figures, the force is one point four newtons.

Now a new section: approximation. Physicists, scientists and engineers often need a quick "guesstimate". How far is a certain place? Roughly how dense is some object? About how large a current will flow in a circuit? Many approximate answers come from formulas whose inputs are only roughly known. The skill grows with practice: thinking in numbers, being willing to take a risk, and knowing your units well. Approximations let us rule out impossible scenarios and unrealistic numbers, challenge other people's claims, and steer our own approach to a problem.

The first example is a building of thirty-nine floors. The trick is to scale up from something familiar: a person. An adult is about two

metres tall, and one floor is roughly the height of two adults, about four metres. So the building is about two metres per person, times two people per floor, times thirty-nine floors, which is one hundred and fifty-six metres. That is a rough figure, but a useful one.



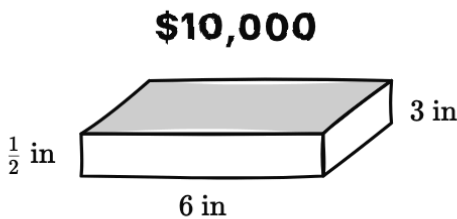
A building measured in people

Try it yourself

Spread your hand and measure it once with a ruler, say ten centimetres across. Now walk your hand across your desk and count the hand-widths. Eight hand-widths make roughly eighty centimetres. You have measured the desk with your own body as the ruler.

The book asks what else you could estimate besides length, and then sets a much bigger challenge. In 2021, the United States federal debt was a little over twenty-eight trillion dollars. Most of us have no feeling for even one trillion. So imagine a trillion dollars in hundred-dollar bills, bundled into bank stacks of one hundred bills each, worth ten thousand dollars a stack. Lay the stacks evenly over an American football field, between the end zones. How high would the pile be? One friend guesses three inches; another says ten feet. (The book works in inches and feet here, because American football fields are measured in yards.)

one trillion: $\$10^{12}$



$$\frac{10^{12}}{10^4} = 10^8$$

STACKS

$$6 \times 3 \times \frac{1}{2} = 9 \text{ in}^3$$

A trillion in bank stacks

The plan is to start from something easy to picture: one bank stack. Find its volume, find how many stacks make a trillion, and then

spread their total volume over the area of the field to get the height. A single bill is roughly three inches by six inches, and a stack of a hundred is about half an inch thick. So one stack takes up six times three times one half, which is nine cubic inches. That is roughly the size of a chunky smartphone, small enough to slip into a coat pocket. How many stacks? A trillion is a one followed by twelve zeros, and each stack holds ten thousand dollars, a one followed by four zeros. Dividing, we subtract the zeros: twelve minus four leaves eight, so there are a hundred million stacks.

Remember

This book usually works to three significant figures and keeps that count consistent in its examples. Exact numbers, such as the two in two pi r or a count of bags, do not limit the digits of a result. Approximations let us rule out unrealistic numbers, test claims and guide our approach. A thirty-nine-floor building is about one hundred and fifty-six metres tall, at two people, four metres, per floor. A trillion dollars in hundred-dollar bills makes a hundred million bank stacks, each about nine cubic inches.

Questions

1. Why does the count of two bags not limit the significant figures of the total weight?
 - a. Because two is a small number
 - b. Because a count is an exact number, not a measurement

- c. Because the bags weigh the same
 - d. Because pounds are not metric
2. Why is the wagon's force given as one point four newtons, not one point four zero?
- a. Because the mass, fifty-five kilograms, has only two significant figures
 - b. Because force is always rounded to tenths
 - c. Because the acceleration is small
 - d. Because the calculator shows two digits
3. How does the lecture estimate the height of a thirty-nine-floor building?
- a. By measuring its shadow
 - b. By guessing a hundred metres
 - c. By scaling up: about two metres per person, two people per floor, thirty-nine floors
 - d. By counting windows
4. How many hundred-dollar bank stacks make a trillion dollars?
- a. A thousand
 - b. A million
 - c. Ten thousand
 - d. A hundred million

Answers

1. **b)** Exact numbers, such as a count of bags, do not limit the digits of a result.

- 2. a)** When multiplying, keep as many significant figures as the least precise input: fifty-five has two.
- 3. c)** Two metres per person times two people per floor times thirty-nine floors gives about one hundred and fifty-six metres.
- 4. d)** A trillion has twelve zeros and a stack ten thousand has four; twelve minus four leaves eight zeros: a hundred million.

Worked examples

Height of a 39-floor building Estimate the height of a thirty-nine-floor building, taking an adult as about two metres tall and a floor as about two adults.

One floor is about two adults of two metres each, about four metres.

Thirty-nine floors times four metres per floor.

Thirty-nine times four is one hundred and fifty-six.

Answer: about one hundred and fifty-six metres

Practice

- 1.** Estimate the height of a twelve-floor building in the same way.

Hint: Take about four metres per floor.

Answer: About forty-eight metres: twelve times four.

- 2.** A woman has three bags: two weigh thirteen point five pounds each and one weighs ten point two. What is the total, to the right digits?

Hint: The count of two bags is exact; keep the tenths of the measured weights.

Answer: Thirty-seven point two pounds: two times thirteen point five is twenty-seven, plus ten point two.

- 3.** One bank stack is six by three by half an inch. What is its volume?

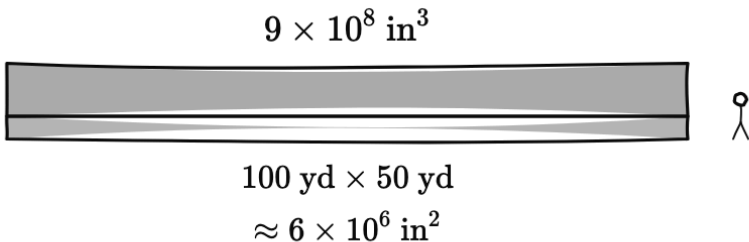
Hint: Multiply length, width and height.

Answer: Nine cubic inches: six times three is eighteen, times one half is nine.

How High Is a Trillion?

We have a hundred million bank stacks, each about nine cubic inches.
Now spread them over the football field.

$$h = \frac{V}{A} \quad \frac{9 \times 10^8}{6 \times 10^6} = 150 \text{ in}$$
$$\approx 12 \text{ ft}$$



Volume over area gives height

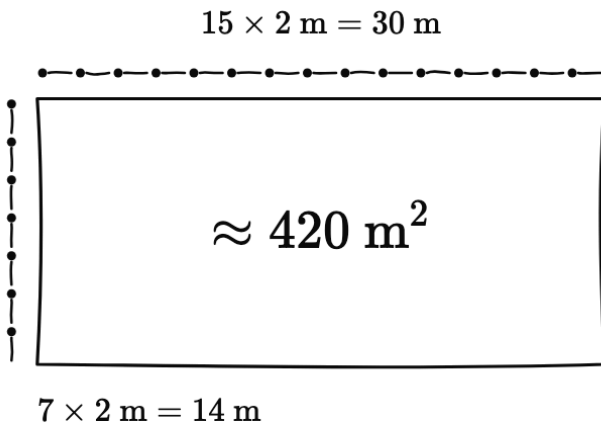
The field is a hundred yards long and fifty wide, which is five thousand square yards. To match the stacks, measured in inches, convert: a yard is three feet and a foot is twelve inches, so one square yard holds three times three times twelve times twelve, which is one thousand two hundred and ninety-six square inches. Five thousand square yards is therefore about six million four hundred and eighty

thousand square inches, rounded to six million, keeping just one significant figure for this rough estimate.

The total volume of money is nine cubic inches per stack times a hundred million stacks: nine hundred million cubic inches. Volume equals area times height, so the height is the volume divided by the area. Nine hundred million divided by six million is one hundred and fifty, so the pile stands about a hundred and fifty inches high. Divide by twelve inches per foot, and that is about twelve feet. That is half again as high as the ceiling of an ordinary room, spread across an entire football field.

The friend who guessed three inches was far too low; the one who said ten feet was roughly right. The book asks you to compare your own first guess, and to think about the difference between a quick guess and a careful, step-by-step approximation.

One last exercise uses your own body as a ruler. How big is a regulation basketball court? A grown man is about two metres tall. About fifteen men laid end to end would cover its length, about thirty metres, and about seven would cover its width, about fourteen metres. Thirty times fourteen gives roughly four hundred and twenty square metres.



A court measured in people

That closes the chapter: from a few simple laws, through agreed units and honest uncertainty, to estimates you can make in your head.

Remember

A football field of five thousand square yards is about six million square inches. A hundred million stacks of nine cubic inches make nine hundred million cubic inches of money. Dividing volume by area gives a height of about a hundred and fifty inches, roughly twelve feet. A basketball court, measured in two-metre people, is about fifteen by seven of them, roughly four hundred and twenty square metres.

Questions

1. Why must square yards be converted using the factors for yards-to-feet and feet-to-inches twice each?
 - a. Because area has two dimensions, length and width, each needing conversion
 - b. Because the field is twice as long as it is wide
 - c. Because square yards are larger than yards
 - d. Because inches are smaller than feet
2. How is the height of the money pile found?
 - a. By adding the volume and the area
 - b. By dividing the volume of money by the area of the field
 - c. By multiplying the area by the number of stacks
 - d. By dividing the area by the volume
3. Which friend's guess was closer, and why does the exercise matter?
 - a. Three inches, showing guesses are usually right
 - b. Neither, showing estimates are useless
 - c. Ten feet, showing that careful step-by-step estimates can test quick guesses
 - d. Both equally
4. How is the basketball court's area estimated?
 - a. By looking it up
 - b. By pacing it with a phone

- c. By assuming it is square
- d. By counting two-metre people along the length and width and multiplying

Answers

- 1. **a)** One square yard is three times three times twelve times twelve square inches, since both length and width are converted.
- 2. **b)** Volume equals area times height, so height equals volume divided by area.
- 3. **c)** The pile is about twelve feet; ten feet was roughly right and three inches far too low.
- 4. **d)** About fifteen men cover the length, thirty metres, and seven the width, fourteen metres; thirty times fourteen is about four hundred and twenty.

Worked examples

The height of a trillion dollars A hundred million bank stacks of nine cubic inches each are spread evenly over a football field of about six million square inches. How high is the pile?

Total volume: nine cubic inches times a hundred million is nine hundred million cubic inches.

Height is volume divided by area.

Nine hundred million divided by six million is one hundred and fifty, about a hundred and fifty inches.

A hundred and fifty inches divided by twelve inches per foot is about twelve feet.

Answer: about a hundred and fifty inches, roughly twelve feet

Practice

1. How many square inches are in one square yard?

Hint: A yard is three feet and a foot is twelve inches; convert both length and width.

Answer: One thousand two hundred and ninety-six: three times three times twelve times twelve.

2. Estimate the area of a room that is about three adult heights long and two wide, with an adult about two metres tall.

Hint: Turn people into metres, then multiply length by width.

Answer: About twenty-four square metres: six metres by four metres.