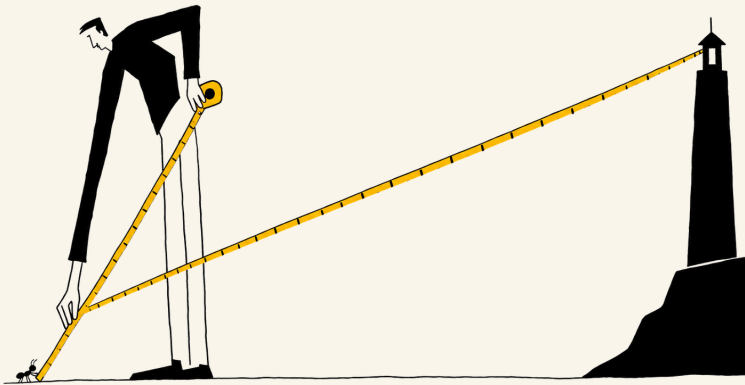


College Physics



Paul Peter Urone, Roger Hinrichs · 2022

Illustrated audio lectures

Lecturized

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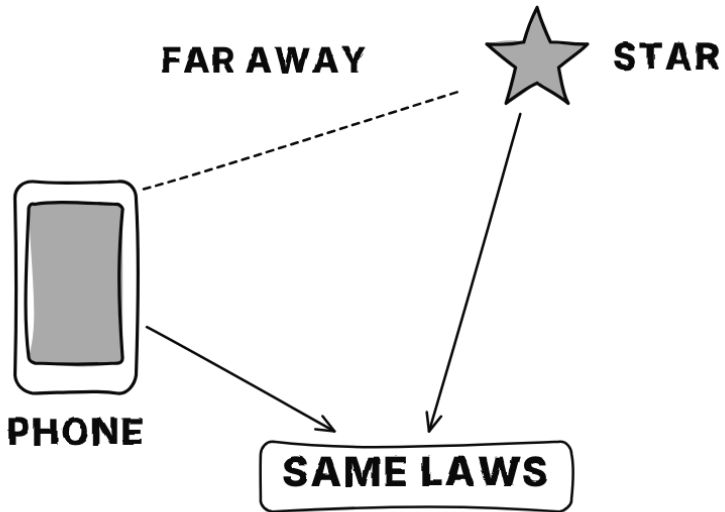
Give or take

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Honest digits and bold guesses

The same rules here and out there

Tonight, if the sky is clear, go to a window and pick out one bright star. Now hold your phone up next to it, so the lit screen and the tiny point of light sit side by side. You would think they could not be more different: one is a flat piece of glass in your hand, the other a ball of burning gas unimaginably far away. Yet the light from both is governed by the same rules of nature, the very ones that warm a pan on the stove and pull a dropped key to the floor. That idea, that the near and the far obey one set of rules, is the promise this book opens with.



One set of rules

Many people meet the word physics with a little dread. They picture pages of hard equations and formulas that seem to have nothing to do with ordinary life. The book answers that fear head on: physics turns out to play a far bigger part in your life than you might think, whatever you do for a living.

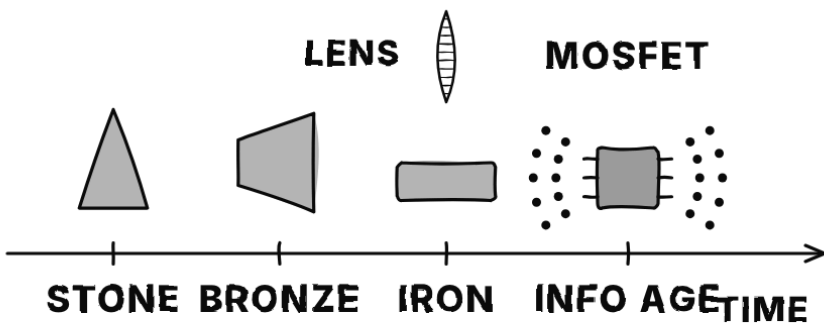
Its first picture is a cloud in space called the Veil Nebula. It lies about two thousand four hundred light-years away, and a light-year is the distance light travels in a whole year, nearly ten trillion kilometers. Imagine a giant glowing veil of thin threads and sheets, like smoke caught in a flash of light. It is what remains of a star that blew up about eight thousand years ago, and the blast is still rushing outward, slamming into gas and dust and making them glow. Scientists compare new photographs with ones the Hubble telescope took in 1997, and from how the cloud has changed they learn how such clouds grow. And the forces shaping that cloud are the same forces we deal with on Earth, whether we are sending a rocket into space or simply heating a new home. Even its colours are the glow of hydrogen, sulfur and oxygen, the same stuff that everything around us is made of.

Try it yourself

On a clear night, look at the stars for a minute, then look at a lit lamp in the room. The book's point is that the same handful of rules explains both lights; the difference is only scale and distance.

The story then moves from the sky to our own past. We name the great ages of history after materials, the Stone Age, the Bronze Age, the Iron Age, because each was built on knowing how some material

behaves and how to work it. Knowing just where to strike a stone so it flakes into a sharp point, learning in the furnaces of South India and Sri Lanka that adding carbon to iron makes the earliest fine steel, grinding and polishing glass until it forms a lens: all of that is physics, learned by hand. Our own time is called the Information Age, and it too rests on a single tiny invention. In 1959, in a laboratory in New Jersey, Mohamed Atalla, born in Egypt, and Dawon Kahng, born in Korea, invented a microscopic electronic switch, the MOSFET. Hardly anyone has heard of it, yet more of these switches have been made than any other object in human history. Your phone contains billions of them, and others work inside a car's controls, a microwave oven and the instruments at a doctor's office.



Ages built on materials

The book then points to the newest marvels in the news: trains that float above their rails, cloaks that bend light around themselves, tiny robots that attack cancer cells inside the body. However amazing or however ordinary, all of them rest on physics. And physics is part of many jobs that never mention it. A pilot has to understand how the wind pushes the plane off its course, and a physical therapist has to understand the forces on a muscle as the body bends. Even the satellite map on your phone, which shows where you are within a few meters, is a quiet piece of physics at work.

Finally, the book lays out the road ahead. First comes a short history of how people began to study nature, from ancient thinkers of the Middle East and the Mediterranean up to Isaac Newton and the laws named after him. Then the shared system of units, the agreed meter and second and kilogram, which lets scientists everywhere speak one mathematical language. It is the reason a kitchen scale in one country and a pharmacy's scale in another can agree on what a gram is. Last comes the honest part: how close any measurement can come to the truth, and why scientists take such pains to say exactly how sure, or unsure, they are.

Remember

Physics is not a pile of formulas cut off from life; it is part of almost everything you use and do. The forces that shape a distant exploding star are the same ones we meet on Earth. Every age of technology, from sharpened stone to the billions of tiny switches in a phone, grew out of understanding how materials behave. The book will go from the history of the subject to shared units of measurement, and then to how sure we can be of any measurement.

Questions

1. Why does the book open with a cloud in space that is thousands of light-years away?
 - a. To show that the forces shaping it are the same ones we deal with on Earth
 - b. To show that space is governed by rules quite unlike ours
 - c. To show that only astronomers really need physics
 - d. To show that the cloud will soon reach the Earth
2. What did the Stone, Bronze and Iron Ages have in common?
 - a. Each began when a new star was seen in the sky
 - b. Each was built on knowing how a material behaves and how to work it
 - c. Each ended when people forgot how to make tools
 - d. Each was named after the people who invented its tools

3. What makes the MOSFET a fitting symbol of our Information Age?
 - a. It is the largest machine ever built
 - b. It is famous and known to almost everyone
 - c. More of them have been made than any other object, and they sit in nearly every device
 - d. It was the first device ever to use electricity
4. Why does the book say scientists take great pains to state how sure they are of a measurement?
 - a. Because measurements are always exactly right
 - b. Because units differ from country to country
 - c. Because only rough guesses are allowed in physics
 - d. Because no measurement is perfect, and honesty about its limits is part of doing science

Answers

1. **a)** The forces shaping that cloud are the same ones at work when we send a rocket up or heat a home.
2. **b)** Every one of those ages rested on understanding a material, and that understanding is physics learned by hand.
3. **c)** Hardly anyone knows its name, yet more of these tiny switches exist than any other object, inside phones, cars and ovens.
4. **d)** The last part of the road ahead is about how close any measurement can come to the truth and saying exactly how sure one is.

Practice

1. Name one job you know that is not called physics, and explain what part of physics the person in that job has to understand, the way the book does for a pilot and a physical therapist.

Hint: Think about what pushes, pulls, heats or moves in that job.

Answer: For example, a delivery driver has to judge how long a loaded van takes to stop, which is about forces and motion. Like the pilot who must understand how wind pushes a plane off course, the driver uses physics every day without calling it that.

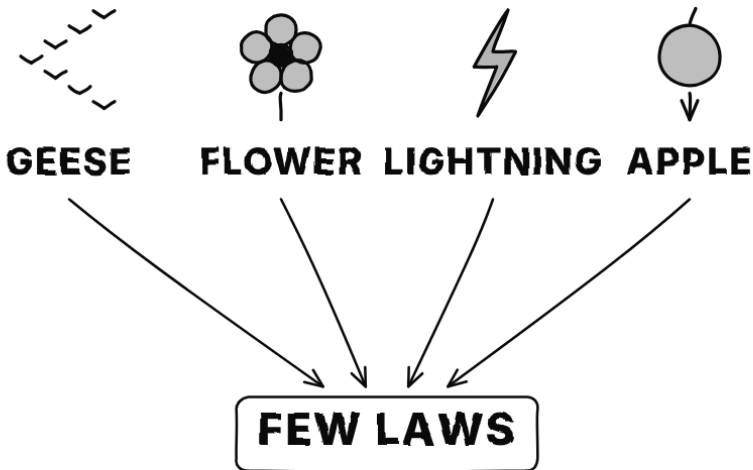
2. Why does it matter that scientists everywhere use the same meter, second and kilogram? Explain with an everyday example.

Hint: Imagine two people in different countries trying to compare a measurement.

Answer: A shared system of units lets everyone speak one language of measurement. A gram on a kitchen scale in one country is the same gram on a pharmacy's scale in another, so a result measured in one place means the same thing everywhere.

One law for chips and batteries

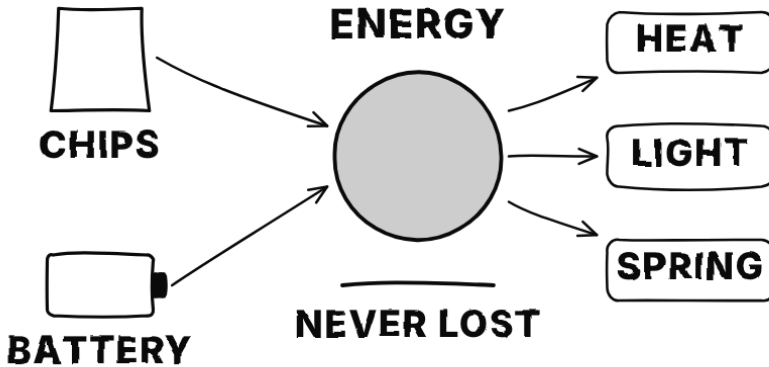
Picture a flock of geese flying south in autumn, strung out in a long V across the sky. Now picture a flower, a flash of lightning, an apple dropping from a branch. You would expect each of these to need its own explanation, a separate rulebook for birds, for flowers, for storms. The surprise the book opens this chapter with is the opposite: even the geese in their V formation are following the laws of physics, the same laws that govern everything else.



Many things, few laws

The world is endlessly detailed. Every day we see a huge variety of things and events, and over the centuries people have gathered mountains of facts about them, from the flight of birds and the colours of flowers to the tiniest particles, the largest groups of galaxies, the flow of time and the beginning of the universe. And yet, faced with all that detail, we have found that a surprisingly small set of laws, which fit together, explains what we see. People naturally look for patterns and order, and nature turns out to cooperate: underneath, it is orderly and simple. That hidden order is exactly what makes physics a pleasure to study rather than a burden.

Here is the book's own example. What do a bag of chips and a car battery have in common? They look like they belong to different worlds, one to the kitchen and one to the garage. But both hold energy that can be turned into other forms: the chips into the warmth and movement of your body, the battery into the spark that starts an engine. One law ties them together, the law of conservation of energy: energy can change from one form to another, but it is never lost. The same law links the calories printed on a snack packet and the charge in your phone, and it also covers heat, light and the wound-up spring of an old watch. Once you know that one rule, all these forms of energy stop being a list to memorize and become a single story told in different settings.



Energy changes form

So the book makes its promise. Its theme is that physical laws unite things and that nature is basically simple. Along the way you will meet the most important topics of physics, but the real prize is a skill: the ability to take a few laws and apply them far beyond anything a single book could cover. That skill, careful thinking with a small set of rules, helps in study and in any line of work.

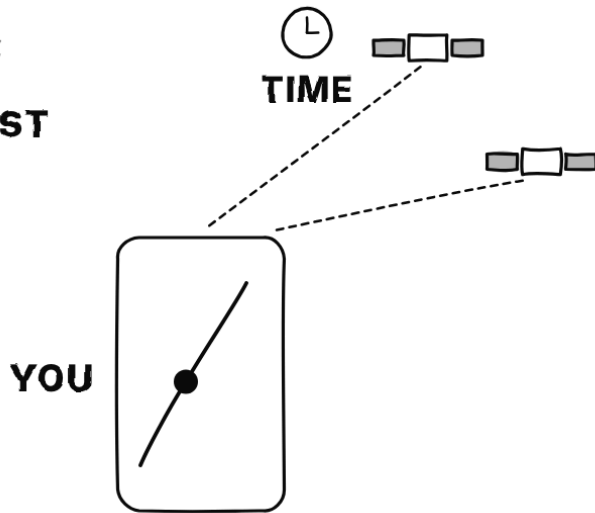
What exactly is physics, then? Science, in the book's words, is the general truths of nature, its theories and laws, together with all the knowledge they hold, and scientists keep adding to that knowledge and stating the laws more clearly. Physics is the part of science that describes how energy, matter, space and time act on one another, and above all asks what basic mechanism lies underneath every

happening. Its reach runs from tiny electrically charged particles to the motion of people, cars and spaceships, and almost everything around you can be described quite accurately by its laws.

Take a smartphone. Physics describes how electricity moves through its circuits, and engineers use that knowledge to choose the materials and lay out the circuits when they build one. Then open the map on the same phone: the blue dot that follows you down the street rests on a simple relation from physics, between how far something travels, how fast it goes, and how long it takes. To place you correctly, the system has to account for hills and valleys on the ground, the exact distances to the satellites overhead, and even a subtle effect in which time runs at a slightly different pace for the satellites than for us. Most of these calculations grew out of the work of Gladys West, a mathematician and computer scientist who programmed the first computers able to locate places on Earth very precisely. Every time your phone shows you how long a drive will take, it is using her methods and these laws of physics.

$$d = v \times t$$

GLADYS WEST



Physics behind the map

Remember

The world is enormously varied, yet a small set of laws that fit together explains most of what we see. Nature is orderly and simple underneath, and that is what makes physics enjoyable to study. The law of conservation of energy, that energy changes form but is never lost, links things as different as chips, batteries, heat and light. Physics describes how matter, energy, space and time act on one another, and looks for the basic mechanism behind each happening. Your phone's circuits and its map both run on these laws.

Questions

1. What did people discover after gathering mountains of facts about nature?
 - a. That every kind of thing needs its own separate set of rules
 - b. That a surprisingly small set of laws that fit together explains most of it
 - c. That nature has no order and can only be listed
 - d. That only the largest objects follow physical laws
2. What do a bag of chips and a car battery have in common?
 - a. Both hold energy that can be turned into other forms
 - b. Both are made of the same materials
 - c. Both lose their energy without a trace once used
 - d. Both work only when they are warm
3. What does physics ask about every happening in nature, above all?
 - a. How beautiful it is
 - b. Who first saw it
 - c. What basic mechanism lies underneath it
 - d. Whether it can be sold
4. Why does the map on a phone have to account for the satellites and even for time itself?
 - a. Because the satellites need to be repaired regularly
 - b. Because the phone has no clock of its own

- c. Because maps change their shape every day
- d. Because placing you rests on how far, how fast and how long, and time runs slightly differently up there

Answers

- 1. **b)** Faced with all that detail, we found that a small, unified set of laws explains what we see.
- 2. **a)** Each stores energy that changes into other forms, and the law of conservation of energy covers both.
- 3. **c)** Physics describes how energy, matter, space and time act on one another and seeks the basic mechanism behind each happening.
- 4. **d)** The blue dot rests on the relation between distance, speed and time, and the calculation must allow for exact satellite distances and time's slightly different pace for them.

Practice

- 1. The book links food calories, batteries, heat, light and watch springs with one law. Choose one of these and describe how its energy changes form, using the law of conservation of energy.

Hint: Start with where the energy is stored, then follow where it goes.

Answer: For example, a wound-up watch spring stores energy; as it unwinds, that energy turns the gears and becomes motion, and a little becomes heat from friction. The energy changes form along the way, but none of it is lost.

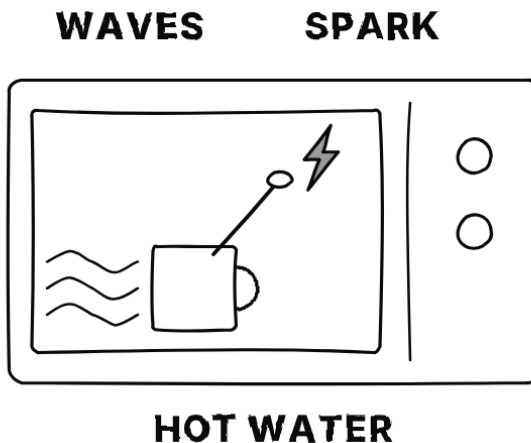
- 2. Why does the book say that learning a few laws is worth more than memorizing many facts? Explain in your own words.

Hint: Think about what you can do with a rule that you cannot do with a list.

Answer: A short set of laws lets you understand things you have never studied, because they connect many apparently unrelated topics. A list of facts only covers what is on it; a law can be applied far beyond any single book, in study and in any work.

From the microwave to the atom

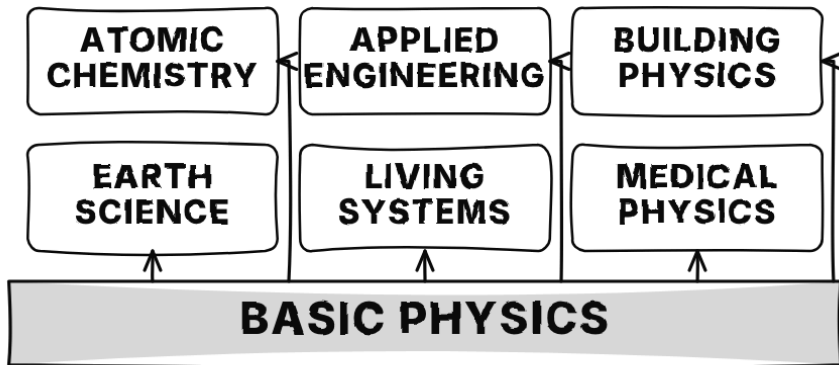
Put a cup of water in the microwave and run it for a minute. The cup comes out barely warm in places, but the water inside is hot. Now imagine doing the same with a metal spoon left in the cup: you would see sparks crackle against the oven wall, and every manual warns you never to do it. Why does the water heat and the spoon spark? You do not need to be a scientist to ask that, and the book's point is that physics gives the answer to ordinary people, not only to experts.



Water heats, spoon sparks

Physics helps you understand how a microwave oven heats food, why metal should stay out of it, and why such ovens might disturb a pacemaker. It helps you weigh the dangers of radiation calmly, instead of out of fear or rumour. It explains why a black radiator helps carry heat away from a car engine, and why a white roof keeps the inside of a house cooler on a hot day. Picture two cars parked in the summer sun, one black and one white: the white one stays more comfortable inside, and the reason is the same physics that makes a white roof cool a house. It even makes sense of a car's ignition and of the electrical signals racing along the nerves in your own body. None of these needs a laboratory, only a feel for the basic rules.

Physics is also the floor that many other subjects stand on. Chemistry is about how atoms and molecules behave together, so it grows straight out of the physics of atoms. Most kinds of engineering are simply physics put to work. In architecture, physics keeps buildings standing, and it shapes how they sound, how they are heated, lit and cooled. Geologists lean on it to date rocks by their natural radioactivity, to study earthquakes and to follow heat inside the Earth. Some fields even carry the word in their names, such as biophysics and geophysics, which are half physics by birth.



Physics under the sciences

Living things are full of physics too. On the smallest scale it helps describe the walls of plant cells, like the neat little bricks you can see in an onion skin under a microscope, and the thin membranes that wrap animal cells, where something as basic as a nerve firing takes place. On the scale of the whole body it explains the heat we give off, the work our muscles do and the power they deliver. Medicine uses it constantly: x-rays, magnetic resonance scans and ultrasound measurements of blood flow are all physics. Treatment can be physics as well, as when radiation is aimed at a tumour to fight cancer. And it explains our senses: how a musical instrument makes sound, how the eye tells colours apart, how a laser carries information.

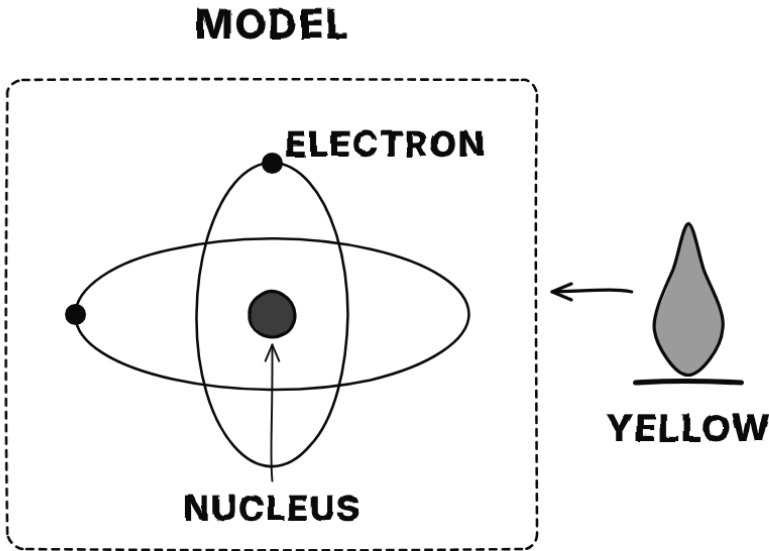
The book adds a pair of pictures that look unrelated at first. One is a microwave oven, the other a scan of a human brain. The link is that both work with the same kind of invisible waves; the oven uses them to heat food, the scanner to build an image of the brain precise enough to show where a tumour is. So the box in your kitchen and the machine in a hospital are cousins, built on the same piece of physics.

You cannot study every one of these uses, and the book says you need not. What is worth having is the basic laws and the skill to apply them, which also sharpens your problem solving in general. Because physics keeps the most basic parts of science, all the other sciences use it, and learning it makes them easier to understand.

Where do these laws come from? The book is clear that the laws of nature are not invented by people. They are built into the universe; we did not make them and cannot change them, we can only discover them and come to understand them. Discovering them is a deeply human adventure, full of mystery, imagination, struggle, triumph and disappointment. Isaac Newton had to be talked into publishing his revolutionary work, and late in life he ran the Royal Mint, where he put ridges on the edges of coins so that cheats could not shave silver off them. Marie Curie spent her own money on her early research and damaged her health through exposure to radiation; she remains the only person to win Nobel Prizes in both physics and chemistry. And the foundation of all such discovery is observation: science has to describe the world as it really is, not as we might like to imagine it.

It starts with ordinary curiosity. We look up at the sky and wonder whether a certain kind of cloud means a storm is coming. When we get serious about it, we become organized: we collect and sort our observations carefully, try to measure more exactly, run controlled

tests where we can, and write down ideas about how the facts fit together. From all that we build the tools science uses to sum up what it has found, and the first of them is the model.



The planetary model of the atom

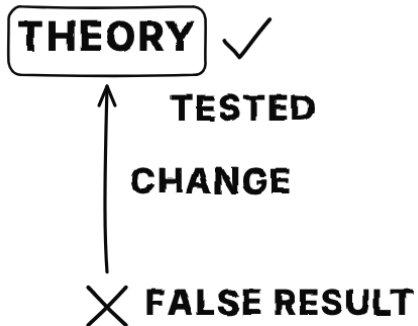
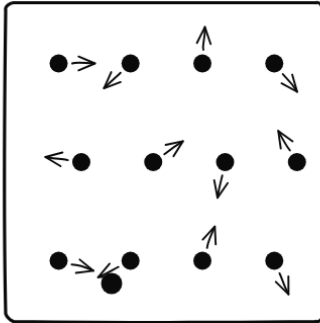
A model is a picture of something that is too hard, or impossible, to show directly. It is backed by evidence, but it only works well within certain limits. The book's example is the planetary model of the atom, in which tiny electrons circle the centre of the atom the way planets circle the Sun. No one can actually watch an electron going round, but this mental picture helps explain things we can see, such as the particular colours of light that glow from a hot gas.

Try it yourself

Look at a sodium street lamp, or sprinkle a pinch of table salt into a gas flame on the stove. You will see the flame flash a strong yellow-orange instead of blue. That fixed colour, the same every time, is the kind of observation the atom model was built to explain.

Physicists use models in many ways: to think a situation through and calculate an answer, or to recreate it as a simulation on a computer. A weather app showing a storm moving across a map is a model of this kind: nobody can see the whole atmosphere at once, so a computer pictures it to help forecast what comes next. A gas is another good case: its atoms and molecules are far too small to see, so we picture them as tiny bouncing particles in order to make sense of what our instruments tell us. To a scientist, a theory is not a guess but a testable explanation of patterns in nature, backed by evidence and confirmed many times by different groups of researchers. And if an experiment ever shows that something a theory predicts is false, the theory is either thrown out or changed, for example by narrowing the cases it claims to cover. Some scientific explanations need such a picture and others do not, and how a model differs from a theory and a law is the subject of the next part.

GAS MODEL



A gas as bouncing particles

Remember

Physics is useful to everyone: it explains the microwave oven, the white roof, the car radiator and the signals in our nerves. Chemistry, engineering, architecture, geology, biology and medicine all rest on it. What matters is not learning every use, but knowing the basic laws and how to apply them. The laws of nature are discovered, not invented, and observation of the world as it really is comes first. A model is a picture of something we cannot see directly, like the atom drawn as a tiny solar system; it helps explain what we can observe, but only within limits.

Questions

1. What links a microwave oven and a brain scanner?
 - a. Both are meant to be used only by doctors
 - b. Both work with the same kind of invisible waves, used for different jobs
 - c. Both heat whatever is placed inside them
 - d. Both were invented to fight cancer
2. Why does chemistry grow straight out of physics?
 - a. Because chemistry deals with atoms and molecules, which the physics of atoms describes
 - b. Because chemists and physicists use the same buildings
 - c. Because chemistry is older than physics
 - d. Because chemistry does not need any experiments
3. Why do we need a model of the atom at all?
 - a. Because the atom changes its shape all the time
 - b. Because the model is a proof that electrons are planets
 - c. Because the atom is too small to show directly, and the picture explains what we can observe
 - d. Because a model is always exactly right in every case
4. What happens to a theory if an experiment shows that one of its predictions is false?
 - a. It is kept unchanged, because theories are never wrong
 - b. It becomes a law

- c. It turns into a model
- d. It is thrown out or changed, for example by narrowing the cases it covers

Answers

- 1. **b)** The oven uses the waves to heat food, the scanner to build an image of the brain precise enough to locate a tumour.
- 2. **a)** Chemistry studies how atoms and molecules behave together, so it rests on the physics of atoms.
- 3. **c)** No one can watch an electron going round, but the picture helps explain things we can see, such as the colours glowing from a hot gas.
- 4. **d)** A theory is a tested explanation; when a prediction fails, the theory is discarded or adjusted.

Practice

- 1. The book lists many fields that rest on physics. Pick two of them and describe, for each, one thing physics does there.

Hint: Think of geology, architecture or medicine.

Answer: For example, in geology physics is used to date rocks by their natural radioactivity and to study earthquakes; in medicine it gives x-rays, magnetic resonance scans and ultrasound measurements of blood flow.

- 2. Explain in your own words what a model is, and why the planetary picture of the atom is useful even though no one can see an electron moving.

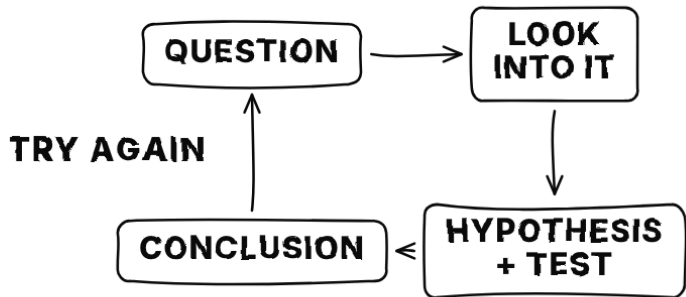
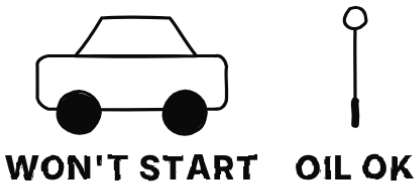
Hint: Ask what a model is for, and what it lets us explain that we can actually observe.

Answer: A model is a picture of something too hard or impossible to show directly; it is backed by evidence but works only within limits. The

planetary picture of the atom helps explain things we can see, such as the particular colours of light given off by a hot gas.

Laws, theories and a car that won't start

It is a cold morning, you turn the key, and the car will not start. Nobody stands there helpless for long; you start to wonder why, and to guess. Perhaps there is no oil in the engine. So you lift the bonnet, pull out the dipstick, and look: the oil is right where it should be. That guess is ruled out, and you move on to the next one, perhaps the battery. Without any lab coat, you have just done what scientists do.



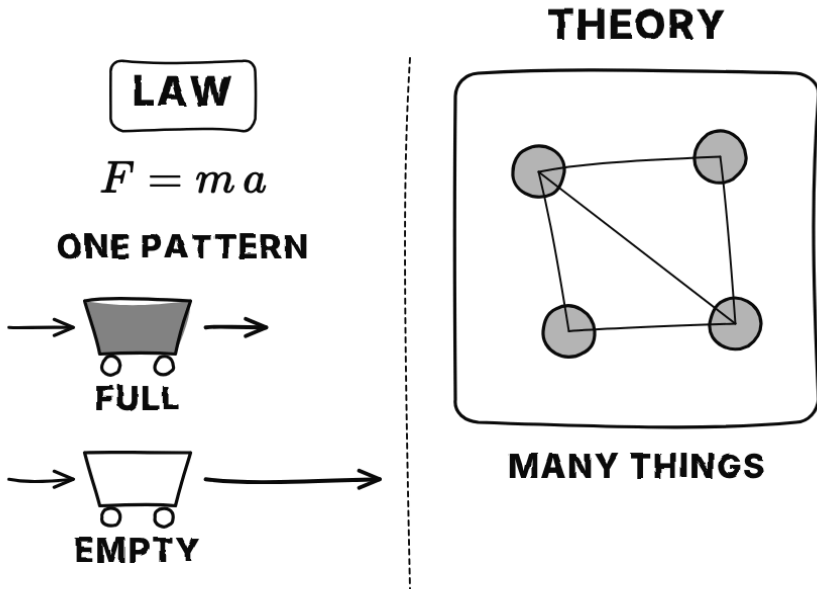
The method as a loop

The book calls this the scientific method, and it describes it in a few steps. You notice something and ask a question about it. You look into

the matter to learn what might be behind it. You propose a possible explanation, a hypothesis, and then you test it with an experiment. Finally you look at what the test showed and draw a conclusion; if the guess failed, as the oil guess did, you make a new one and go round again. The method is not only for science; it works for many everyday puzzles, and it can be adjusted to fit the problem.

This way of working has a long history. A thousand years ago, in Cairo, a scholar named Ibn al-Haytham made great advances in understanding light and how we see. He did something even more important: he told scientists to be ready to throw out what they had been taught, to be the enemy of everything they read, and to trust only evidence they could check. He insisted on testing things again and again, and he warned that our senses and our own expectations can fool us. His thinking was an early form of the method we use today.

Out of all that testing come two kinds of statements scientists rely on: laws and theories. Both grow from hypotheses that were tested, and both are backed by evidence, but they are different in size and shape. A law is short and very general: one brief statement of a pattern in nature, confirmed by repeated experiments, and often written as a single equation. One example is the law you already know, that energy is never lost in any process, only changed. Another is Newton's second law of motion, which ties together the push on an object, how much matter is in it, and how quickly its speed changes. Push a full supermarket trolley and an empty one with the same shove, and the empty one speeds up far more; Newton's law says exactly how push, mass and change of speed are linked.

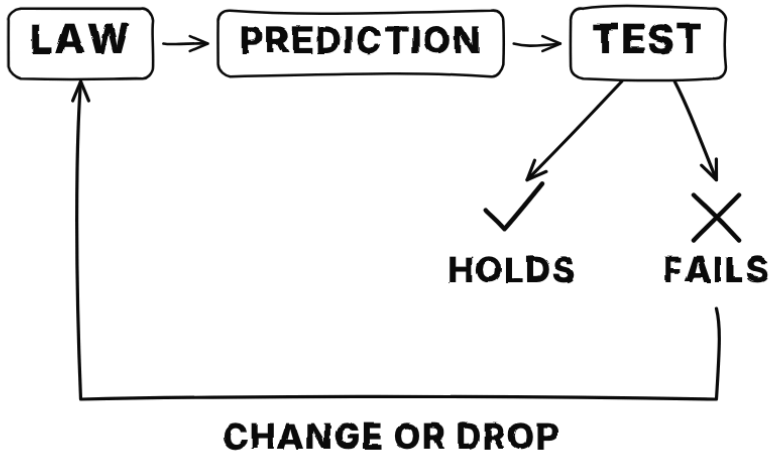


Law versus theory

A theory, as we saw, is a well-tested explanation, but it is much bigger than a law. It cannot be squeezed into a single line; the theory of evolution and the theory of relativity are examples that are far too rich to be called laws. The simplest way to keep them apart: a law describes one kind of action, while a theory explains a whole family of related happenings. And where a law is a starting point that the scientific method builds on, a theory is where that long process ends up. Rules that apply only in narrow settings are usually called principles, such as one that holds only for liquids and gases, though people are often loose about where a law ends and a principle begins.

Laws and theories are not just tidy summaries of old facts. Often, once one is in place, it points scientists to something nobody had found yet,

sometimes even an object or event that had never been seen. Such predictions are among the great triumphs of science, and they are possible only because the universe is orderly underneath. But if an experiment does not bear out the prediction, the law or theory is wrong, however elegant or convenient it is.



Valid until contradicted

That means no law can ever be known with total certainty, because no one can try every possible experiment in every possible situation. So physicists treat laws and theories as valid until something contradicts them. If a careful experiment that others can repeat goes against a well-established law, the law has to be changed or thrown out completely. It is like the car again: the oil guess looked reasonable until the dipstick said otherwise, and then it had to go. The book

compares science to sailing into an uncharted ocean: discoveries are made, laws and theories are formed, and the world looks more beautiful for what we have learned.

Remember

The scientific method is a loop: notice and ask, look into it, propose a hypothesis, test it, conclude, and try again if the guess fails. A law is a short, general statement of one pattern in nature, often a single equation; a theory is a large explanation of a whole family of happenings. Laws and theories can predict things never seen before, but a failed prediction proves them wrong. No law is ever certain; each holds only until a reliable experiment contradicts it.

Questions

1. In the car example, what should you do once the oil turns out to be fine?
 - a. Conclude that nothing is wrong with the car
 - b. Keep believing the oil is the cause
 - c. Propose a new explanation and test it in turn
 - d. Stop, because the method has failed
2. Which is the clearest difference between a law and a theory?
 - a. A law describes one kind of action; a theory explains a whole family of related happenings

- b. A law is a guess, while a theory has been proved
 - c. A theory is always shorter than a law
 - d. Only theories are backed by evidence
3. Why can no law of nature ever be known with total certainty?
- a. Because laws change their meaning over time
 - b. Because scientists disagree about every law
 - c. Because laws are only opinions
 - d. Because no one can try every possible experiment in every possible situation
4. What did Ibn al-Haytham add to the way science is done?
- a. He said scientists should trust what they read in old books
 - b. He said scientists should trust only checked evidence and test things again and again
 - c. He said our senses are never mistaken
 - d. He said experiments are unnecessary once a law is written

Answers

- 1. c)** When a hypothesis fails its test, you make a new one and go round the loop again.
- 2. a)** Both are tested and backed by evidence, but a law states one pattern briefly while a theory explains many related ones.
- 3. d)** A law can only be checked in the cases tried, so it holds until a reliable experiment contradicts it.
- 4. b)** He urged scientists to be the enemy of what they read, rely on evidence, repeat tests, and remember that senses and expectations can mislead.

Worked examples

The car that will not start You try to start your car and it will not start. Use the scientific method to test the idea that the engine has no oil.

Notice and ask: the car will not start, so why not?

Look into it: find out the different reasons a car can fail to start.

Propose a hypothesis: maybe the engine has no oil.

Test it: open the bonnet and check the oil level; it is at an acceptable level.

Conclude: the oil is not the problem, so propose a new hypothesis and repeat the process.

Answer: The test rules out the oil, and the search goes on with a new hypothesis.

Practice

1. Your phone suddenly will not charge. Describe how you would use the scientific method to find out why, with at least one hypothesis and the test for it.

Hint: Start by asking the question, then pick one possible cause you can check.

Answer: Question: why will the phone not charge? Hypothesis: the cable is faulty. Test: try a different cable that works with another device. If the phone now charges, the hypothesis is supported; if not, the cable is ruled out and you try a new hypothesis, such as the charger or the socket.

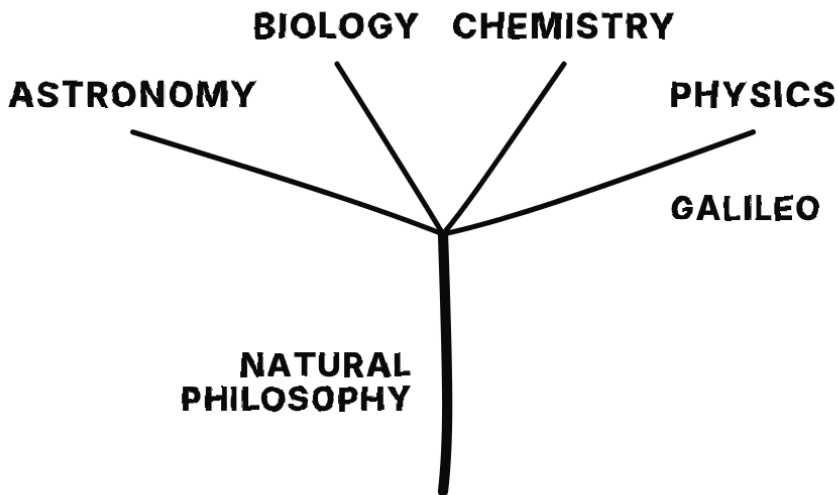
2. A scientist has a beautiful, convenient theory, but a careful experiment that others can repeat does not confirm what it predicts. What does the book say must happen, and why?

Hint: Ask what counts more in science: elegance or experiment.

Answer: The theory is wrong and must be changed or thrown out, however elegant or convenient it is. Experiment is the final judge, and laws and theories hold only until a reliable experiment contradicts them.

When the everyday rules run out

Toss a set of keys from one hand to the other. You know, without thinking, roughly where they will land; your brain has been predicting flights like that since childhood. The rules of physics written down centuries ago predict the same arc almost perfectly. But take those same rules to the inside of an atom, or to a probe sweeping close past the Sun, and they start to give the wrong answers. How can rules that work so well in your hand fail out there? The book answers by telling how physics grew up.



Natural philosophy branches out

For most of history, physics was not a subject of its own. The word itself comes from the Greek for nature, and the study of nature was called natural philosophy. From ancient times to the Renaissance, natural philosophy held everything together: astronomy, biology, chemistry, physics, mathematics and medicine. The Greek thinker Aristotle is the book's example; he wrote about physics, but also about animals, the soul, politics and poetry, all as parts of one great inquiry.

As knowledge piled up over the last few centuries, no single person could hold it all, and natural philosophy branched into separate fields. Physics kept the most basic part, which is why, as we saw, other sciences still lean on it today. The book honours two figures who pushed this change along: Ibn al-Haytham, whom we met as an early champion of testing, and Galileo, who were so important to mathematics, light, physics and astronomy that a later scientist put both of them on the cover of his major work. Think of a family business that grew into a large company: at first one person did everything, then separate departments formed, but the original core still holds them together.

The physics that grew from the Renaissance to the end of the nineteenth century is called classical physics. At the beginning of the twentieth century a series of revolutionary discoveries transformed it into what we call modern physics. Classical physics is not a perfect description of the universe, but it is an excellent approximation as long as three conditions hold. Things must move slower than about one percent of the speed of light; they must be big enough to see with a microscope; and gravity must be weak, like the Earth's. A car on the motorway, a plane, even a rocket heading for the Moon all move far slower than that limit, so the classical rules describe them very well.

CLASSICAL

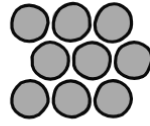
SPEED < 1 % OF C



SIZE > MICROSCOPE



MODERN



WEAK GRAVITY



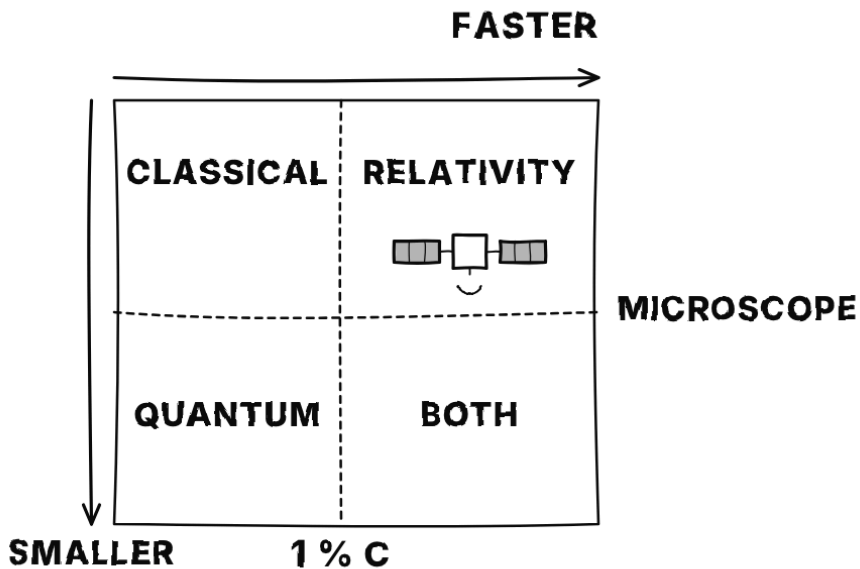
GOLD ATOMS

Where classical physics works

We humans live entirely inside those conditions, and that is why classical physics feels like common sense while much of modern physics feels bizarre. This is exactly where models earn their keep: they let us picture things we never experience, such as objects rushing along at enormous speeds or things far too small for our senses. We can understand an atom because we can imagine one, even though no one has ever seen an atom with the naked eye. New tools help too; with a special microscope that feels its way across a surface, scientists can now actually see the separate atoms that make up a sheet of gold, lined up like oranges packed in a crate.

Modern physics has brought some of the most spectacular advances in all of science. Many classical laws had to be changed or dropped,

and the result has transformed technology, society and our picture of the universe. Like science fiction, modern physics is full of strange objects far from daily life, but unlike science fiction it is real. It rests on two revolutionary theories. Relativity is needed whenever something moves faster than about one percent of the speed of light, or sits in strong gravity, like that near the Sun. Quantum mechanics is needed for anything too small to be seen with a microscope. The satellites that put the blue dot on your phone's map are a real case where everyday rules are not quite enough: as we heard, time runs at a slightly different pace for them, and the system has to allow for it.



Which theory where

Put the two theories together and you get relativistic quantum mechanics, which handles tiny things that move very fast or feel

strong gravity. It is the best all-purpose theory we have, but it is so demanding mathematically that scientists use it only when they must, and use the simpler theories whenever those are accurate enough. So why does the book spend most of its time on classical physics? For two reasons: it describes everyday life with great accuracy, and you need it before you can understand modern physics at all. The book adds an encouraging note: a surprising amount of modern physics can be done with ordinary school mathematics.

It closes with a little check on what came before. If a friend tells you they have learned a new law of nature, you already know something before they say another word: it will be a short statement of a rule that nature always follows. If instead they say they have learned a theory, you can expect something much broader, an explanation that covers a whole range of related happenings.

Remember

Physics grew out of natural philosophy, one great study of nature that branched into separate sciences, with physics keeping the most basic part. Classical physics works superbly as long as things are slower than about one percent of the speed of light, big enough to see with a microscope, and in weak gravity.

Beyond those limits we need modern physics: relativity for the very fast and strong gravity, quantum mechanics for the very small. Models let us picture this unfamiliar world, and new microscopes can even show single atoms. Classical physics stays at the heart of the book because it describes daily life so well and modern physics is built on it.

Questions

1. Why does classical physics feel like common sense to us?
 - a. Because it was the first physics ever written
 - b. Because it is a perfect description of the whole universe
 - c. Because we live where things are slow, large enough to see and in weak gravity, which is where it works
 - d. Because modern physics has been proved wrong
2. Which theory is needed to describe an electron inside an atom?
 - a. Quantum mechanics, because the electron is too small to see with a microscope
 - b. Relativity, because the atom has strong gravity
 - c. Classical physics, because atoms move slowly
 - d. No theory, because electrons cannot be studied
3. Why does the book spend most of its time on classical physics?
 - a. Because modern physics is only science fiction
 - b. Because classical physics is more exact than relativity
 - c. Because relativistic quantum mechanics has been abandoned
 - d. Because it describes everyday life very accurately and is needed to understand modern physics
4. What happened to natural philosophy as knowledge grew?
 - a. It stayed one subject taught by one teacher
 - b. It branched into separate fields, and physics kept the most basic part

- c. It was replaced entirely by medicine
- d. It turned into science fiction

Answers

1. **c)** We live entirely inside the conditions where classical physics is an excellent approximation, so it matches our experience.
2. **a)** Quantum mechanics is needed for anything too small to be seen with a microscope.
3. **d)** Classical physics fits daily life with great accuracy, and modern physics cannot be understood without it.
4. **b)** Over the last few centuries natural philosophy split into separate sciences, with physics keeping the most basic part.

Practice

1. A friend tells you they have learned about a new law of nature. What can you know about it before they describe it, and what would be different if they had learned a theory instead?

Hint: Recall what every law is, and what every theory is.

Answer: Any law will be a short description of a rule that all natural processes follow. A theory would instead be a broad, widely applicable explanation that covers a whole range of related happenings.

2. For each case, say whether the everyday classical rules are enough or which modern theory is needed: a car on a motorway; a probe flying close past the Sun; the inside of an atom.

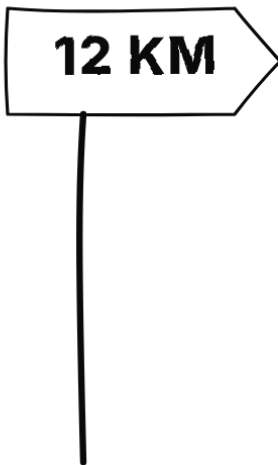
Hint: Check the three conditions: slow compared with light, big enough to see with a microscope, weak gravity.

Answer: The car: classical physics, since it is slow, visible and in weak gravity. The probe near the Sun: relativity, because of the Sun's strong

gravity. The inside of an atom: quantum mechanics, because it is far too small to see with a microscope.

Twelve what?

Imagine a friend phones you with directions: "Go straight for twelve, then turn left." Twelve what? Twelve steps, twelve blocks, twelve miles? The number alone is worse than useless; it sounds helpful and tells you nothing. The book makes the same point with a cartoon of a signpost whose distances are in unknown units, and calls such numbers maddeningly useless. That is where measurement in physics begins: a number only means something together with its unit.



12 STEPS

-

12 BLOCKS

—

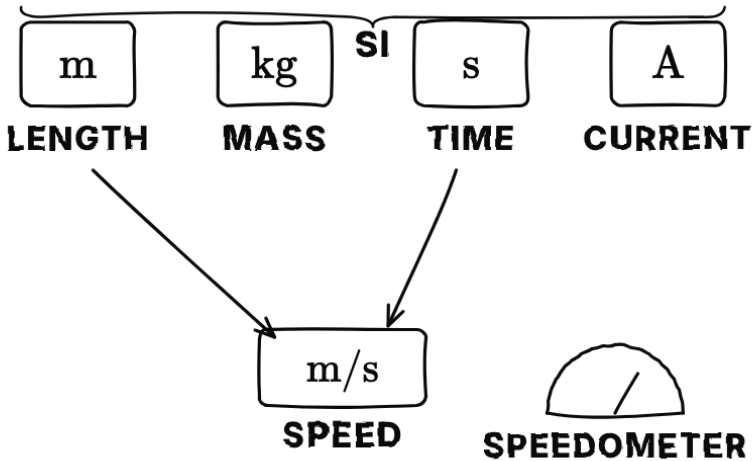
12 MILES

—————

Twelve what?

And physics needs a lot of numbers. It deals with things that last an unimaginably short moment, like the life of an atomic nucleus, and things as old as the Earth. It deals with particles far smaller than an atom and with distances out to the edge of the known universe; with the tiny push of a jumping flea and the enormous pull between the Earth and the Sun. Even the distance to the Moon, which feels huge, is only a tiny step compared with the distances to other bodies in the sky. Putting numbers on such things, instead of only describing them in words, lets us understand nature far more deeply, but only if everyone agrees on the units.

Here nature shows its simplicity once more. Almost everything physics measures can be built out of just four basic quantities: length, mass, time and electric current. Some quantities are defined by saying how to measure them, the way we measure a distance with a ruler or a time with a clock. Others are defined by saying how to work them out from those measurements; average speed, for instance, is the distance travelled divided by the time it took. The speedometer in your car does that sum for you continuously, turning distance and time into one number on the dial.

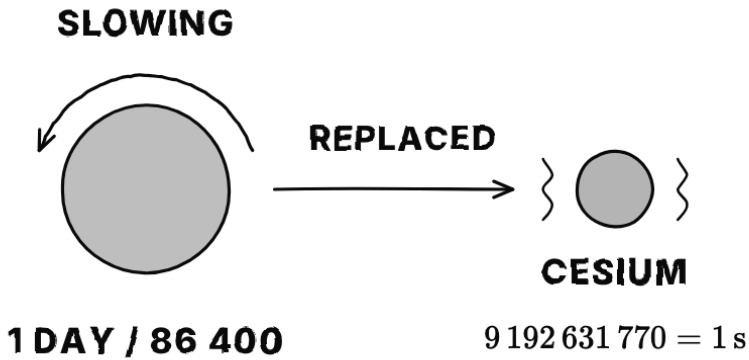


Four base units, many derived

A unit is an agreed, standard amount. The length of a race can be given in meters for sprinters or in kilometers for long-distance runners; it is the same kind of quantity, just a handier size of unit. Two systems of units are used in the world. One is the metric system, known as SI from its French name; virtually every country uses it, and it is the system scientists everywhere agree on. The other is the older English or imperial system of inches, feet and miles, inherited from the British Empire and still widely used in the United States. The book sticks to SI, and where it uses a familiar outside unit, such as the millimeters of mercury in which blood pressure is measured, it always ties it back to SI.

The four fundamental units are the meter for length, the kilogram for mass, the second for time and the ampere for electric current, which the book will reach much later. They are called fundamental because they can only be defined by describing how they are measured. Every other unit is built from them and is called a derived unit: a unit of speed, for example, is a length divided by a time. When your weather app shows wind in kilometers per hour, it is using a derived unit made of a length and a time.

Take the second, the unit of time. For many years it was defined as a fixed fraction of an average day, one part in eighty-six thousand four hundred. But the Earth's spin is very slowly getting slower, so the day is gradually growing longer, and a unit tied to it would quietly change. So scientists found something that never changes: cesium atoms can be made to vibrate in an extremely steady way, and those vibrations can be watched and counted. Since 1967, one second has been the time it takes for just over nine billion of these vibrations, an exact, fixed count. Clocks built on this, called atomic clocks, keep time to better than a millionth of a second over a whole year; one of them, seen from above, is a tower almost ten meters tall. The clock on your phone is regularly set from networks that rely on atomic time, which is why it agrees with everyone else's to the second.



From the day to cesium

Why all this fuss over one unit? Because every measurement in physics is in the end expressed in terms of the fundamental units, and no measurement can be more accurate than the units it rests on. A shaky second would make every timing shaky, and a shaky meter every distance.

Remember

A number without an agreed unit tells you nothing. Almost everything in physics can be built from four basic quantities: length, mass, time and electric current, measured in meters, kilograms, seconds and amperes. Every other unit, such as a unit of speed, is a derived unit made from these. Most of the world and all of science use the metric system, SI. The second is now defined by counting the steady vibrations of cesium atoms in atomic clocks, because the day itself is slowly getting longer.

Questions

1. Why is average speed called a derived quantity rather than a fundamental one?
 - a. Because it is worked out from two measurements, a distance and a time
 - b. Because it can only be measured with a speedometer
 - c. Because it was invented after the meter
 - d. Because it has no unit at all
2. Why was the second tied to cesium atoms instead of to the length of the day?
 - a. Because cesium is easier to buy than a calendar
 - b. Because days are different lengths in summer and winter only

- c. Because the Earth's spin is slowly getting slower, so the day is gradually growing longer
 - d. Because the day was too short to divide
- 3. Why does it matter so much that the fundamental units are very accurate?
 - a. Because derived units do not depend on them
 - b. Because every measurement is in the end expressed through them and can be no more accurate than they are
 - c. Because only scientists use them
 - d. Because they change every year
- 4. Which of these is one of the four fundamental quantities?
 - a. Speed
 - b. Force
 - c. Electric charge
 - d. Mass

Answers

- 1. **a)** Average speed is defined by how to calculate it, distance travelled divided by time taken, so its unit is built from length and time.
- 2. **c)** A unit tied to a lengthening day would quietly change; cesium vibrations stay perfectly steady.
- 3. **b)** All measurements rest on the fundamental units, so a shaky unit would make everything built on it shaky.
- 4. **d)** The four fundamental quantities are length, mass, time and electric current; speed, force and charge are built from them.

Practice

1. A signpost says only "Station 3". Explain why this is not a useful measurement, and what would make it one.

Hint: Ask what the number is counting.

Answer: A number without a unit tells you nothing: 3 could be meters, kilometers or miles. Adding an agreed unit, such as "3 kilometers", turns it into a measurement anyone can understand and compare.

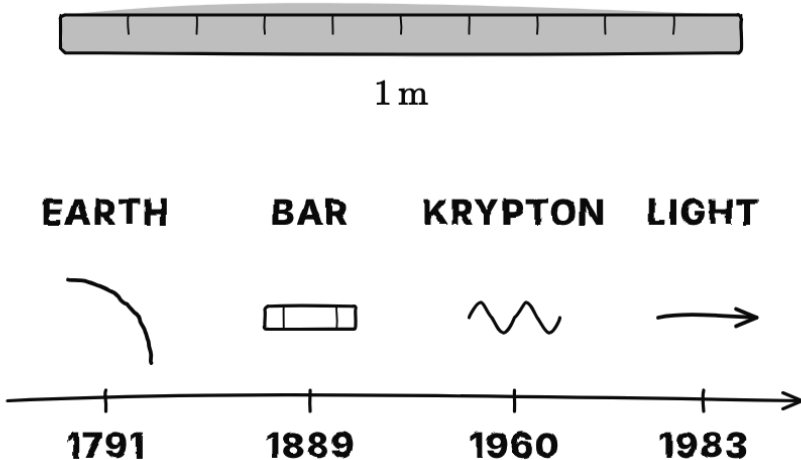
2. Sort these into fundamental or derived units, and explain each: the second, a unit of speed such as kilometers per hour, the kilogram, the meter.

Hint: A fundamental unit is defined by how it is measured; a derived unit is built from others.

Answer: The second, the kilogram and the meter are fundamental: each is defined by how it is measured. Kilometers per hour is derived: it is a length divided by a time.

A meter of light, a kilogram of constants

Take a tape measure and pull out exactly one meter. How do you know the factory printed the marks in the right place? Because its tape was checked against a better one, which was checked against a better one still, all the way back up a chain. At the very top of that chain there has to be something that defines what a meter is. The story of what sits at the top is the story of this part.

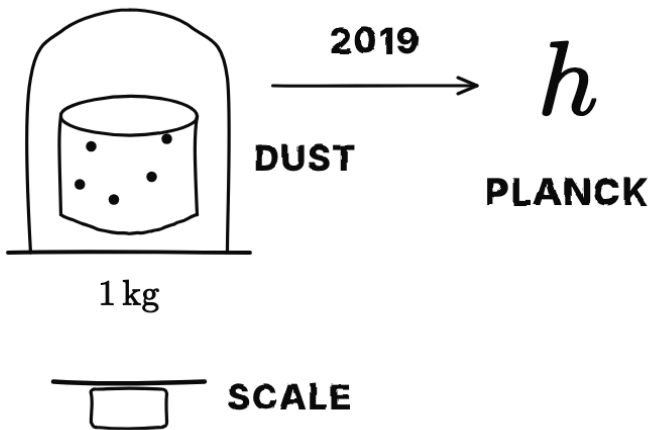


Four meters through history

The first meter, in 1791, was a slice of the Earth itself: one ten-millionth of the distance from the equator to the North Pole. In 1889

it became something you could hold: the distance between two fine lines engraved on a bar of platinum and iridium, kept near Paris. By 1960 scientists could do better, and the meter was redefined as a fixed number of wavelengths of the orange light given off by krypton atoms, just over one and a half million of them. Since 1983 the meter has been defined by light itself: it is the distance light travels in empty space in a tiny, exact fraction of a second, one part in just under three hundred million. That choice fixes the speed of light as an exact number of meters per second. Each step made the meter more accurate and more precise, and moved it away from objects that can wear, bend or be lost.

The kilogram followed a similar road, only later. For more than a century it was a small cylinder of platinum and iridium, kept with the old meter bar at the international bureau of weights and measures near Paris. Exact copies were kept elsewhere, for instance at the national standards institute just outside Washington, and every mass on Earth could, in the end, be traced back to a comparison with that cylinder. The kilogram printed on a bag of flour or shown on your kitchen scale was, ultimately, a promise that it matched that one lump of metal. The cylinder did not rust, but tiny specks of dirt from the air still clung to its surface and very slowly changed its mass. A standard that drifts is a poor standard, so in May 2019 scientists adopted a steadier definition: the kilogram is now defined through the second, the meter and Planck's constant, a fixed number from quantum physics that links the energy of light to its frequency.



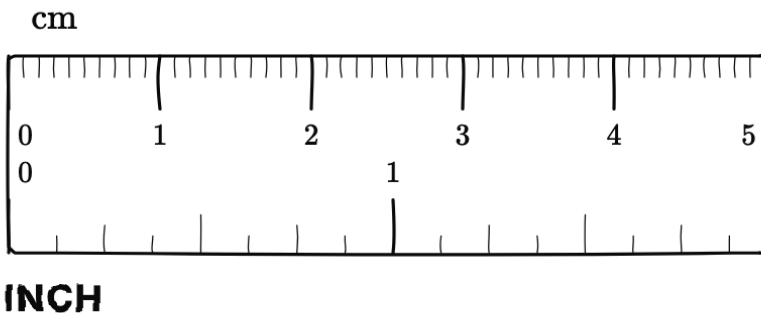
The kilogram leaves its cylinder

The fourth fundamental unit, the ampere for electric current, will wait for the chapters on electricity. The early chapters, on motion, fluids, heat and waves, need nothing beyond length, mass and time.

Behind all these changes is a wish physicists share. Since everything is made of tiny parts, it would be most satisfying to build every standard on those tiny parts and on fixed features of nature, like the speed of light, rather than on objects in a vault. Time already rests on the vibrations of cesium atoms, and length, once tied to the light of krypton atoms, now rests on the speed of light. The book mentions that mass might one day be tied to atoms too, for instance by measuring an extremely perfect sphere of silicon atoms, and that

electric charge might one day rest on the charge of single electrons and protons.

Now the second big idea of this part: the metric system counts in tens. There are a hundred centimeters in a meter and a thousand meters in a kilometer, so switching units means only moving the decimal point. The older American system is far less tidy, with twelve inches to a foot and five thousand two hundred and eighty feet to a mile.

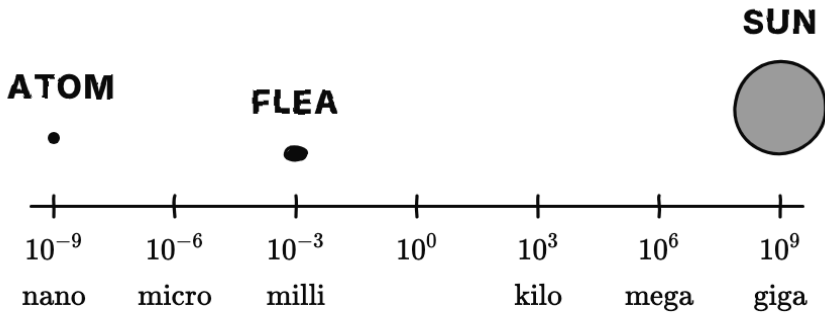


Tens against twelves

Try it yourself

Look at a ruler marked in both centimeters and inches. On the metric side, every tenth small mark is a bigger mark, so you can read millimeters, centimeters and meters at a glance. On the inch side, the inch is cut into halves, quarters, eighths and sixteenths, and turning inches into feet or miles needs real arithmetic.

The second advantage is that one unit can cover any size, simply by putting a short word in front of it, called a prefix. Kilo means a thousand times, centi a hundredth, milli a thousandth, micro a millionth, nano a billionth; at the large end come mega, giga and tera for a million, a billion and a trillion. The book's table pairs each prefix with a picture: a millimeter is about the height of a flea at its shoulders, a centimeter the thickness of a fingertip, a kilometer about six tenths of a mile, a nanogram a small speck of dust, a femtometer the size of a proton. So builders work in meters, pilots in kilometers and lens designers in nanometers, and no one has to invent a new unit for their own trade. Your phone's storage counted in gigabytes and a medicine dose in milligrams are the same trick, one unit stretched by a prefix.



Prefixes along powers of ten

Thinking in tens also gives a quick sense of scale, called the order of magnitude: the ballpark size of a number. Eight hundred and four hundred and fifty are different, but both are "hundreds", so they share an order of magnitude. An atom is about a ten-billionth of a meter across, and the Sun about a billion meters; that difference in order of magnitude tells you at once how far apart those worlds are.

Remember

The meter began as a slice of the Earth, then a metal bar, then a count of light waves, and is now the distance light travels in a tiny, exact fraction of a second. The kilogram was a metal cylinder near Paris until 2019; now it is defined through the second, the meter and Planck's constant. Physicists want every standard to rest on nature's smallest parts and unchanging constants. The metric system counts in tens, and a prefix such as kilo, milli or nano stretches one unit over any size. The order of magnitude is a quick, ballpark sense of how big something is.

Questions

1. Why was the old metal kilogram replaced in 2019?
 - a. Because it rusted badly
 - b. Because tiny specks of dirt from the air clung to it and slowly changed its mass
 - c. Because it was too small to see
 - d. Because it was moved from Paris to Washington
2. What does the meter rest on today?
 - a. The distance from the equator to the North Pole
 - b. Two lines on a metal bar near Paris
 - c. The distance light travels in empty space in a tiny, exact fraction of a second
 - d. The size of a krypton atom

3. Why is switching between metric units so easy?
 - a. Because metric units step in factors of ten, so you only move the decimal point
 - b. Because metric units never need converting
 - c. Because there is only one metric unit
 - d. Because metric numbers are always round
4. Why would physicists like every standard to rest on atoms and constants of nature?
 - a. Because atoms are cheaper than metal bars
 - b. Because objects in a vault are easier to measure
 - c. Because constants of nature change slowly over time
 - d. Because nature's smallest parts and fixed constants do not wear, drift or get lost

Answers

1. **b)** A standard that drifts is a poor standard, so the kilogram was redefined through fixed constants.
2. **c)** Since 1983 the meter has been defined by light, which also fixes the speed of light exactly.
3. **a)** A hundred centimeters make a meter and a thousand meters a kilometer, unlike twelve inches to a foot.
4. **d)** Building standards on tiny parts and fixed features like the speed of light avoids the drift of physical objects.

Practice

1. Using the prefixes, say how many milligrams are in a gram and how many meters are in a kilometer, and explain why the metric system

makes such questions easy.

Hint: Milli means a thousandth and kilo means a thousand times.

Answer: There are a thousand milligrams in a gram and a thousand meters in a kilometer. Because every prefix is a power of ten, converting means only moving the decimal point, unlike inches, feet and miles.

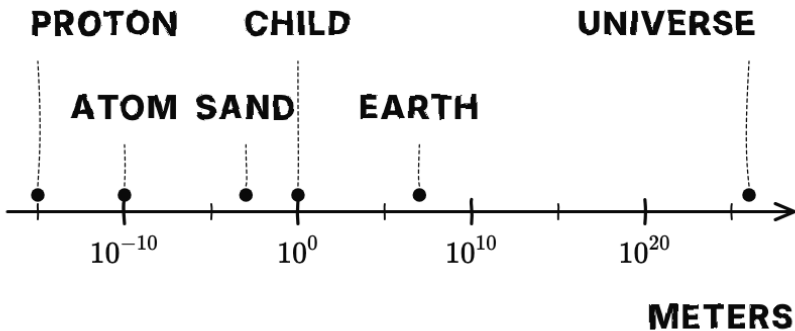
2. Put these in order from smallest to largest by order of magnitude, using the book's examples: a centimeter, a femtometer, a kilometer, a millimeter.

Hint: Femto is the smallest prefix here, kilo the largest.

Answer: Femtometer (about the size of a proton), millimeter (a flea at its shoulders), centimeter (a fingertip's thickness), kilometer (about six tenths of a mile).

From a proton to the edge of the universe

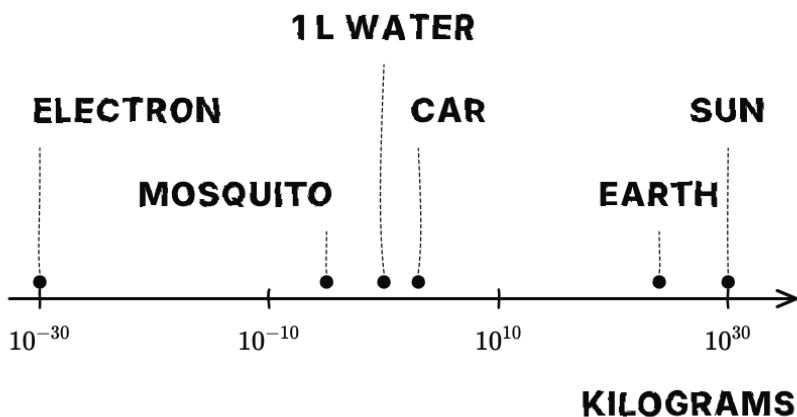
Pick up a single grain of sand and hold it on your fingertip. It is about a millimeter across, a thousandth of a meter. Now try to imagine something a trillion times smaller: that is roughly the size of a proton, the core of a hydrogen atom. And now something so large that light, the fastest thing there is, needs billions of years to cross it: the known universe. Physics talks about all of these, and the book's first job in this part is simply to let that range sink in.



A ladder of lengths

It does so with a long table of known lengths, masses and times, each given only roughly, as an order of magnitude. Read the lengths from top to bottom and you climb a ladder. At the bottom is the smallest detail any experiment can make out today, then a proton, the nucleus of a uranium atom, a hydrogen atom, the thin skin around a living cell, and the wavelength of visible light. Then come things you can see: a grain of sand, a four-year-old child about a meter tall, a football field about a hundred meters long, and the deepest point of the ocean, around ten kilometers down. Then the Earth itself, the distance to the Sun, the distance light travels in a year, the width of our galaxy, the distance to the nearest large galaxy, Andromeda, and finally the edges of the known universe.

The masses climb in the same way. At the bottom sits the electron, then a hydrogen atom, a bacterium, a mosquito and a hummingbird. In the middle, a liter of water weighs about a kilogram, a person about a hundred kilograms, a car about a thousand. The liter bottle of water in your fridge is a handy one-kilogram weight, the same rung of the ladder the book uses. Above that come a large ship, a large iceberg, the core of a comet, the Moon, the Earth, the Sun, our whole galaxy and, at the very top, all the known matter in the universe.



A ladder of masses

And the times: the moment it takes light to cross a proton; the life of an extremely unstable nucleus; one swing of a light wave; one vibration of an atom in a solid; one swing of an FM radio wave. Then human-sized times: a nerve impulse lasting about a thousandth of a second, one heartbeat of about a second, a day, a year. Then the long ones: half a human life, all of recorded history, the age of the Earth and the age of the universe. Two photographs frame the table: tiny plankton drifting among ice crystals in the Antarctic sea, from a few thousandths of a millimeter up to two millimeters long, and two galaxies smashing into each other more than two billion light-years away. The range of nature, the book says, challenges the imagination.

With so many sizes and so many units in use, we keep needing to change from one unit to another. A European recipe asks for milk in liters and your measuring jug shows cups; walking directions give a distance in feet and you want to know how many miles that is. The tool for this is the conversion factor: a ratio saying how many of one unit make one of another. There are twelve inches in a foot, a hundred centimeters in a meter, sixty seconds in a minute, a thousand meters in a kilometer.

$$\frac{1 \text{ km}}{1000 \text{ m}}$$

CONVERSION FACTOR

- 1. HAVE AND WANT**
- 2. FIND THE FACTOR**
- 3. MULTIPLY, CANCEL**

$$80 \cancel{\text{ m}} \times \frac{1 \text{ km}}{1000 \cancel{\text{ m}}} = 0.080 \text{ km}$$

A conversion factor cancels units

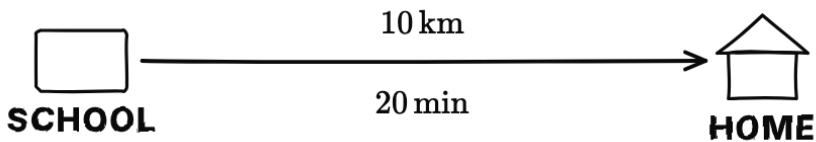
The method has three steps. First, write down which unit you have and which you want. Second, find the conversion factor that links them. Third, multiply what you have by that factor, arranged so that the unit you do not want cancels out and the one you want is left

behind. The book's first example turns eighty meters into kilometers: since a thousand meters make a kilometer, the meters cancel and you are left with eight hundredths of a kilometer. The same recipe works for any kind of unit at all.

Try it yourself

Take a kitchen measuring jug with both liters and cups printed on it. Fill it to the one-liter line and read off the cups, about four. That reading is your own conversion factor between the two units, and you could use it to rewrite any recipe.

Then the book works through a real trip. You drive ten kilometers from school to home, and it takes twenty minutes. How fast were you going on average, first in kilometers per hour and then in meters per second? Average speed is simply the distance divided by the time, which gives half a kilometer every minute. To turn minutes into hours you use the fact that an hour holds sixty minutes, and the answer comes out at thirty kilometers per hour. That is a typical city speed on your car's speedometer, which already tells you the answer is sensible.



$$\frac{10 \text{ km}}{20 \text{ min}} = 0.500 \text{ km/min}$$

$$\times \frac{60 \text{ min}}{1 \text{ h}} = 30.0 \text{ km/h}$$

$$1 \text{ h} = 3600 \text{ s}$$



Ten kilometres in twenty minutes

The book insists on checking such answers, and this is what it calls dimensional analysis: letting the units themselves keep watch over the calculation. If you put the conversion factor in upside down, the units will not cancel; instead you get a strange mix, something like kilometer-hours per minute squared, which is plainly not a speed. So you check three things: that the units cancelled properly, that the answer is in the units you were asked for, and that it makes sense. Here it does: ten kilometers in a third of an hour means thirty kilometers in a whole hour. The book also points out that the answer was written with exactly three digits, as were the distance and the time, and that the sixty minutes in an hour do not limit this at all, because an hour is defined as exactly sixty minutes.

For the second half of the question, meters per second, two conversion factors are needed at once. One swaps hours for seconds, since an hour holds three thousand six hundred seconds, and the other swaps kilometers for meters. Setting them up so that both unwanted units cancel is the next step, and it is where the next part picks up.

Remember

Physics stretches from the size of a proton to the edges of the known universe, from the mass of an electron to the mass of all known matter, and from the time light takes to cross a proton to the age of the universe. A conversion factor is a ratio saying how many of one unit make another, such as a thousand meters in a kilometer. To convert, multiply by the factor arranged so the unwanted unit cancels and the wanted one is left. Dimensional analysis means letting the units check the work: if they do not cancel to the units you want, something is upside down. Ten kilometers in twenty minutes is thirty kilometers per hour, a sensible city speed.

Questions

1. When converting units, how must the conversion factor be set up?
 - a. So that the unit you do not want cancels and the one you want is left

- b. So that the larger number is always on top
 - c. So that both units remain in the answer
 - d. It does not matter which way round it is
2. What would it mean if a speed calculation ended in units of kilometer-hours per minute squared?
- a. That the car was going very fast
 - b. That the answer must be in miles
 - c. That the conversion factor was put in upside down
 - d. That the units are correct but rare
3. Why does the sixty minutes in an hour not limit how many digits the speed answer can have?
- a. Because sixty is a round number
 - b. Because an hour is defined as exactly sixty minutes, so the factor is perfectly exact
 - c. Because conversion factors are never used in calculations
 - d. Because minutes are not real units
4. Why is thirty kilometers per hour a sensible answer for ten kilometers in twenty minutes?
- a. Because the speedometer always reads thirty in a city
 - b. Because thirty is three times twenty
 - c. Because ten plus twenty makes thirty
 - d. Because twenty minutes is a third of an hour, so in a whole hour you would go three times as far

Answers

1. **a)** You multiply by the factor arranged so the unwanted unit cancels, leaving the wanted one behind.
2. **c)** An upside-down factor stops the units from cancelling and leaves a mix that is plainly not a speed.
3. **b)** An hour is defined to be sixty minutes, so the conversion factor carries no uncertainty.
4. **d)** Ten kilometers in a third of an hour means thirty kilometers in a full hour.

Worked examples

Eighty meters in kilometers Convert 80 meters into kilometers.

Write down the unit you have, meters, and the unit you want, kilometers.

Find the conversion factor: 1,000 meters make 1 kilometer.

Multiply 80 meters by 1 kilometer per 1,000 meters, so the meters cancel.

80 divided by 1,000 is 0.080.

Answer: 0.080 km

Practice

1. You drive the 10.0 km from school to home in 20.0 min. What is your average speed in kilometers per hour?

Hint: First divide distance by time to get kilometers per minute, then use 60 minutes in an hour.

Answer: 30.0 km/h: 10.0 km divided by 20.0 min is 0.500 km per minute, and $0.500 \times 60 = 30.0$ km/h.

2. Using the book's table, put these in order from the shortest time to the longest: one heartbeat, the duration of a nerve impulse, one day, the age of the Earth.

Hint: A nerve impulse lasts about a thousandth of a second, a heartbeat about a second.

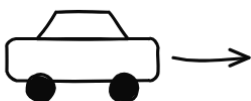
Answer: Nerve impulse (about a thousandth of a second), heartbeat (about one second), one day, the age of the Earth.

Close to the truth, or close to each other

We left our drive home half finished: ten kilometers in twenty minutes, thirty kilometers per hour, and now the same speed in meters per second. Two swaps finish the job. An hour is three thousand six hundred seconds, so the hours give way to seconds; a kilometer is a thousand meters, so the kilometers give way to meters. Once both unwanted units have cancelled, the answer is about eight and a third meters every second. Picture it as roughly the length of a car covered every half second, a calm pace through town. The book adds that you could have started from half a kilometer per minute instead; you would need different conversion factors, but you would land on exactly the same eight and a third.

30.0 km/h

$$30.0 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1 \cancel{\text{h}}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} = 8.33 \frac{\text{m}}{\text{s}}$$



car length per $\frac{1}{2}$ s

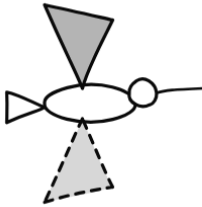
8.333333

8.33

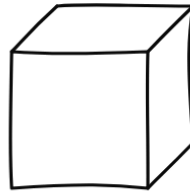
30 km/h in metres per second

Then it asks a sharp question. Why were all these answers written with just three digits, thirty point zero and eight point three three? A calculator would happily show eight point three three three three three and more. Why not write them all down? The book promises the answer in the coming pages, and it has to do with how good the measurements were in the first place.

Before that, a few side notes. Not every unit is as tidy as the meter. There are curious old ones, such as the firkin, which was used for beer and holds about thirty-four liters, or the barleycorn, a measure of length. The book invites you to look up such units and work out how each one relates to SI.



$\approx 20 \text{ ms}$



1 cm

1 cm

$$1 \text{ cm}^3 = 1 \text{ mL}$$



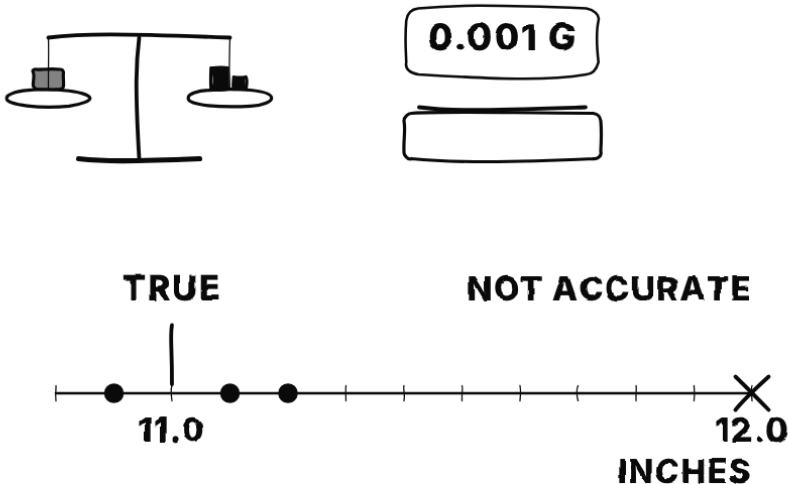
A cube of one millilitre

Here is a puzzle the book sets. Some hummingbirds beat their wings more than fifty times every second. If a scientist wants to time a single wing beat, what unit should they use? The fundamental unit is still the second, but one beat lasts only a tiny slice of a second, about twenty thousandths of it. So the natural choice is the millisecond, a thousandth of a second, and the beat is about twenty milliseconds long.

And another: one cubic centimeter, a tiny cube one centimeter on each side, holds exactly one milliliter. That tells you how neatly SI fits together. The unit of volume is not invented separately; it is built straight out of the unit of length. The little plastic cup that comes with

children's medicine, marked in milliliters, is really marked in tiny cubes of one centimeter each.

Now the book turns to the heart of measurement: how good is a measurement? It starts with an old double-pan balance. You put the object in one pan and standard metal weights, one gram, ten grams, a hundred grams, in the other, and when the bar between them sits level, the two sides weigh the same. Many such balances have been replaced by digital scales, which are more precise: an old balance may read only to the nearest tenth of a gram, a good digital scale to the nearest thousandth. Your kitchen scale sits somewhere between the two, usually reading to the nearest gram.



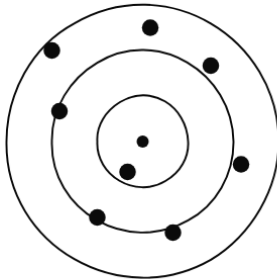
Three readings of 11 inches

All of science rests on measurements, and to judge them the book uses two words that sound alike but mean different things. The first is accuracy: how close a measurement comes to the true value. Say a packet of printer paper states that each sheet is eleven inches long. You measure one sheet three times and get eleven point one, eleven point two and ten point nine. These are accurate, because they all sit close to the true eleven inches; a reading of twelve would not be.

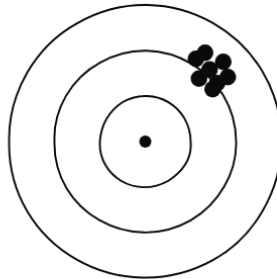
Try it yourself

Take a ruler and measure the long side of a sheet of paper three times, lifting the ruler between tries. You will probably not get exactly the same number each time, nor exactly what the packet says. How close your readings come to the stated size is their accuracy; how close they come to each other is the next idea.

The second word is precision: how closely repeated measurements, taken the same way, agree with one another. Your three readings range from ten point nine up to eleven point two, a spread of only three tenths of an inch, so they are fairly precise. If you had read ten point nine, eleven point one and eleven point nine, the spread would be much larger, and the measurements would not be precise.



ACCURATE



PRECISE

Accurate or precise

The paper readings happen to be both accurate and precise, but the two can come apart. The book pictures a GPS trying to pinpoint a restaurant at the centre of a dartboard, each attempt drawn as a black dot. In the first picture the dots are scattered widely, far from one another, yet they are grouped around the centre: low precision, but high accuracy. In the second the dots huddle tightly together, but well off to one side, far from the restaurant: high precision, but low accuracy. That second case is like a phone map that always puts you, very steadily, on the wrong side of the street. Being consistent is not the same as being right, and being right on average is not the same as being consistent.

Remember

Ten kilometers in twenty minutes is about eight and a third meters per second, whichever way you convert. A hummingbird's single wing beat is best timed in milliseconds, and a cubic centimeter is exactly a milliliter. Accuracy is how close a measurement is to the true value. Precision is how closely repeated measurements agree with one another. A measurement can be accurate without being precise, or precise without being accurate, like dots scattered around a target or bunched off to one side.

Questions

1. A GPS gives positions scattered widely but grouped around the true spot. What is true of it?
 - a. It is precise but not accurate
 - b. It is accurate but not precise
 - c. It is both accurate and precise
 - d. It is neither accurate nor precise
2. Why is a hummingbird's single wing beat best described in milliseconds?
 - a. Because one beat lasts only about twenty thousandths of a second
 - b. Because seconds are not a fundamental unit
 - c. Because hummingbirds beat their wings once a minute

- d. Because milliseconds are larger than seconds
3. What does it tell you that one cubic centimeter is exactly one milliliter?
- a. That volume is a fundamental quantity
 - b. That centimeters and milliliters are the same kind of quantity
 - c. That the liter was invented before the meter
 - d. That SI builds its unit of volume straight out of its unit of length
4. Your three paper readings of 11.1, 11.2 and 10.9 inches are called precise. Why?
- a. Because one of them is exactly eleven inches
 - b. Because they were measured with a digital scale
 - c. Because they spread over only three tenths of an inch
 - d. Because they are all larger than ten

Answers

- 1. b)** The dots are far from one another, so precision is low, but they centre on the restaurant, so accuracy is high.
- 2. a)** Fifty beats a second leaves about twenty milliseconds for each beat, so the millisecond fits the size of the measurement.
- 3. d)** The milliliter depends on the centimeter, so the unit of volume is derived from length.
- 4. c)** Precision is how closely repeated readings agree, and these differ by at most three tenths of an inch.

Worked examples

The drive home in meters per second Your average speed was 30.0 km/h. Express it in meters per second.

Two units must change: hours into seconds and kilometers into meters.

An hour is 3,600 seconds, so divide by 3,600 to make the hours cancel.

A kilometer is 1,000 meters, so multiply by 1,000 to make the kilometers cancel.

$$30.0 \times 1,000 \div 3,600 = 8.33.$$

Answer: 8.33 m/s

Practice

1. Some hummingbirds beat their wings 50 times per second. How long does one beat take, in seconds and in milliseconds?

Hint: Divide one second into 50 equal parts.

Answer: One fiftieth of a second, which is 0.02 s, or 20 milliseconds.

2. Measured three times, a sheet of paper that should be 11.0 inches long gives 10.9, 11.1 and 11.9 inches. Are these measurements accurate, precise, both or neither? Explain.

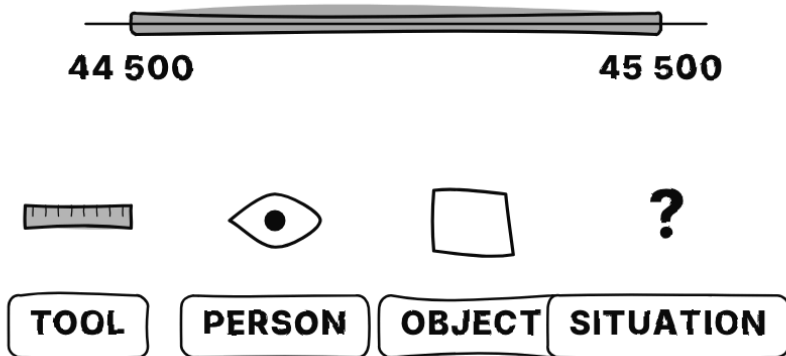
Hint: Look first at how far apart the readings are, then at how close they come to 11.0.

Answer: They are not precise, because they spread over a full inch, from 10.9 to 11.9. Two readings are close to 11.0, but 11.9 is well off, so as a set they are not very accurate either; the book uses this set as its example of measurements that are not precise.

Give or take

Someone asks how far your car has gone, and you glance at the dashboard and say: about forty-five thousand miles, give or take five hundred. That small "give or take" is honest and useful. It says the true number could be as low as forty-four thousand five hundred or as high as forty-five thousand five hundred, or anywhere in between. The book gives that give or take a name: the uncertainty, a kind of disclaimer attached to every measured value.

$45\,000 \pm 500$ miles

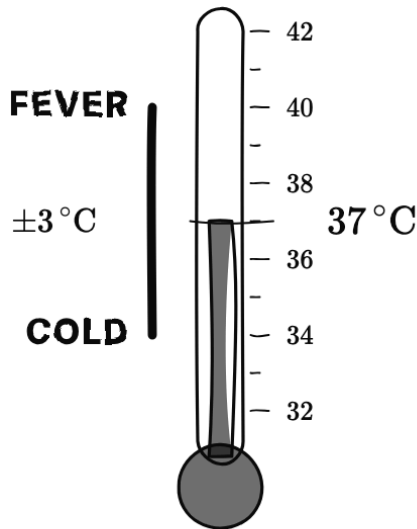


Where the doubt comes from

Uncertainty is a number that says how far your measurement may stray from the true or expected value. It is tied to the accuracy and precision we just met: if your measurements are not very accurate or not very precise, the uncertainty is large. And every measurement has some. The sheet of printer paper from before could honestly be reported as eleven inches, give or take two tenths of an inch.

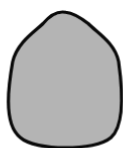
Where does the uncertainty come from? The book names four sources. The measuring tool has limits: if the smallest mark on the ruler is a tenth of an inch, you cannot read much finer than that. The person measuring has limits too, perhaps poor eyesight or an unsteady hand. The thing being measured may be uneven, like a sheet of paper with one side slightly longer than the other. And there may be other influences, depending on the situation. A sound uncertainty comes from thinking carefully about all of them.

Why does this matter so much? The book's answer is a sick child. You suspect a fever and reach for a thermometer, but imagine that it could be off by three degrees Celsius either way. It reads thirty-seven, a normal temperature, yet the truth could be anything from a dangerously cold thirty-four to a dangerous fever of forty. Such a thermometer would be useless. That is why a medical thermometer has to be far more trustworthy than that: a few degrees of doubt would hide exactly the difference you need to see.



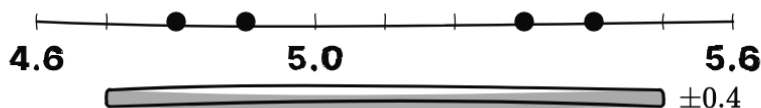
A thermometer three degrees off

There is a handy way to judge how big an uncertainty really is: compare it with the measurement itself, as a percentage. This is called the percent uncertainty: how many hundredths of the measured value the give or take amounts to. The book shows it with a bag of apples. A shop sells bags marked five pounds, and over a month you buy four and weigh them: four point eight, five point three, four point nine and five point four pounds. Their average is five point one pounds, and from earlier weighings you know the bag's weight is uncertain by about four tenths of a pound.



5 lb

AVERAGE 5.1



$$\frac{0.4}{5.1} \approx 8\%$$

Five pounds, give or take

Try it yourself

Weigh the same packet of pasta or rice on your kitchen scale a few times, taking it off and putting it back each time. Compare the readings with the weight printed on the packet. The spread of your readings, and how far they sit from the label, give you a feel for the give or take of your own scale.

Four tenths of a pound out of about five is roughly eight percent. So the bag is five pounds, give or take eight percent, and the average you measured sits comfortably within that range of the label. The shop is not cheating you; the swings from week to week are within the honest

give or take. The book adds a question to think about: if the bag were only half as heavy but had the same give or take, the uncertainty would be a much larger share of it, twice as large in percent. And a practical warning: a percent uncertainty is a fraction turned into hundredths, so remember to turn it into a percentage rather than leave it as a small decimal.

Remember

Uncertainty is the give or take on a measurement, how far it may be from the true value, and every measurement has some. It comes from the limits of the tool, the skill of the person, unevenness in the object, and the circumstances. A measurement with too much uncertainty, like a thermometer that may be three degrees off, is useless. Percent uncertainty compares the give or take with the measured value itself. A five-pound bag of apples known to within about four tenths of a pound is five pounds give or take eight percent.

Questions

1. You say your car has done 45,000 miles, give or take 500. What are you claiming?
 - a. That the true mileage is exactly 45,000
 - b. That the true mileage lies somewhere between 44,500 and 45,500
 - c. That the true mileage is at least 45,500

- d. That the odometer is broken
2. Why would a thermometer that may be three degrees off be useless for a sick child?
- a. Because it would always show a fever
 - b. Because three degrees is too small to matter
 - c. Because a normal reading could hide anything from dangerously cold to a dangerous fever
 - d. Because thermometers cannot measure children
3. If a bag were half as heavy but had the same give or take, what would happen to its percent uncertainty?
- a. It would be twice as large
 - b. It would be half as large
 - c. It would not change
 - d. It would vanish
4. Which of these is one of the book's sources of uncertainty?
- a. The colour of the object
 - b. The time of day the measurement is written down
 - c. The brand name of the ruler
 - d. Unevenness in the object being measured

Answers

1. **b)** The give or take is the uncertainty, so the true value could be anywhere 500 miles either side.
2. **c)** A reading of 37 could really mean anything from 34 to 40 degrees, so it cannot tell you what you need to know.

3. a) The same give or take is a bigger share of a smaller value, twice as large in percent.

4. d) The book lists the tool's limits, the person's skill, unevenness in the object, and other influences of the situation.

Practice

1. A bag marked 5 pounds weighs 4.8, 5.3, 4.9 and 5.4 pounds in four weeks, and its weight is known to within about four tenths of a pound. Is the bag's label honest, and roughly how big is the uncertainty as a share of the weight?

Hint: Find the average, then compare four tenths of a pound with about five pounds.

Answer: The average is 5.1 pounds, within four tenths of a pound of the label, so the label is honest. Four tenths out of about five is roughly eight percent.

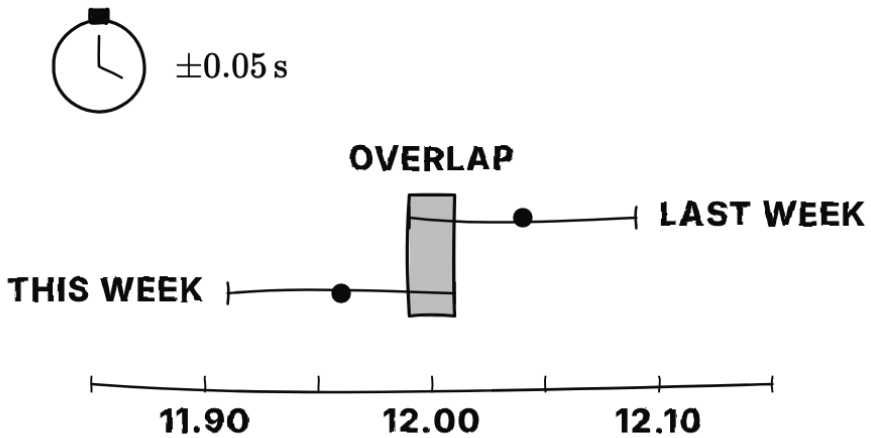
2. You measure a sheet of paper with a ruler whose smallest mark is a tenth of an inch. Name two things, besides the ruler, that could add to the uncertainty of your result.

Hint: Think of the person measuring and of the sheet itself.

Answer: For example, poor eyesight or an unsteady hand of the person measuring, and a sheet cut slightly crooked so that one side is longer than the other.

How many digits you have earned

A new stopwatch arrives for a school running team, and the manual says it may be off by five hundredths of a second either way. Last week the best sprinter ran the hundred meters in twelve point zero four seconds; this week the watch says eleven point nine six. Did the sprinter get faster? It looks like it, but the book's answer is no: the difference is so small that the stopwatch's own give or take could account for it. You simply cannot tell the two runs apart with that watch.



Two runs the watch cannot split

That is the first lesson of this part: anything you work out from measurements carries their doubt along with it. Suppose you measure a room's floor to find its area. The length is a little uncertain and so is the width, so the area you calculate from them must be uncertain too. For multiplying or dividing measurements whose uncertainties are small, a few percent or less, the book gives a handy rule: just add up their percent uncertainties.



$$4 \text{ m} \pm 2 \%$$

$$3 \text{ m} \pm 1 \%$$

$$2 \% + 1 \% = 3 \%$$

$$A = 12 \text{ m}^2 \pm 3 \%$$

$$\approx \pm 0.4 \text{ m}^2$$

Percent uncertainties add

Its example is a floor four meters long and three meters wide. The length is uncertain by two percent and the width by one percent. The area comes to twelve square meters, and its uncertainty is the two and the one added together, three percent. Three percent of twelve square meters is a little over a third of a square meter, which the book rounds

to about four tenths of a square meter. If you are buying flooring for that room, that is why it is wise to order a bit extra: your own measurements carry their give or take into the area.

The second lesson is about the tool. A precise measuring tool is one that can show very small steps. An ordinary ruler measures to the nearest millimeter, while a caliper, a sliding gauge with fine jaws, measures to a hundredth of a millimeter. The caliper is the more precise tool, and the more precise the tool, the more precise and accurate your measurements can be.

Try it yourself

Lay a ruler beside a pencil and read its length as carefully as you can. You will find the end falls between two millimeter marks, so you have to judge the last digit by eye. That last, judged digit is exactly what the book means by the first digit with some doubt in it.

This leads to the idea of significant figures: you write down only as many digits as your tool truly measured. If a ruler tells you a stick is thirty-six point seven centimeters, you may not write thirty-six point seven one, because the ruler cannot see a hundredth of a centimeter. And even the last digit you do write, the seven, was partly a guess: the end of the stick seemed to fall somewhere between thirty-six point six and thirty-six point seven, and you had to judge. To count significant figures, start at the first measured digit on the left and count through the last one written on the right; thirty-six point seven has three. The number of significant figures tells you how precise the tool was. A

kitchen scale that shows whole grams cannot honestly give a spice weight to a tenth of a gram, however carefully you read it.

Zeros need special care, because some are measured and some are only placeholders. In zero point zero five three, the zeros in front just show where the decimal point sits, so they do not count; the number has two significant figures. In ten point zero five three, the zeros sit between other digits and were really measured, so all five figures count. A round number like thirteen hundred is ambiguous: its zeros might have been measured, or might just fill space, so it could have two, three or four significant figures. To avoid that confusion, the book suggests writing such numbers in powers of ten. The rule of thumb: zeros count, except when they are only holding a place.

$0.053 \rightarrow 2$	$0.0009 \rightarrow 1$
$10.053 \rightarrow 5$	$15\,450.0 \rightarrow 6$
$1300? \rightarrow 2-4$	$87.990 \rightarrow 5$
$1.3 \times 10^3 \rightarrow 2$	$30.42 \rightarrow 4$

Which zeros count

The book closes with a quick check. Zero point zero zero zero nine has one significant figure, since all the zeros are placeholders. Fifteen thousand four hundred fifty point zero has six, because the zeros, including the one after the decimal point, show a measurement made to a tenth. Eighty-seven point nine nine zero has five, the final zero recording a measurement made to a thousandth. And thirty point four two has four, because a zero sitting between other digits always counts.

Remember

Anything calculated from measurements is uncertain too; for multiplying or dividing, add the small percent uncertainties. If a tool's give or take is as large as the difference you want to see, as with the sprinter's stopwatch, you cannot tell the results apart. A more precise tool, like a caliper compared with a ruler, measures in smaller steps. Write only as many digits as the tool really measured; the last one is already partly estimated. Zeros count when they were measured, not when they only hold the decimal point in place.

Questions

1. Why can't the coach be sure the sprinter got faster, going from 12.04 s to 11.96 s?
 - a. Because the stopwatch's give or take is large enough to explain the small difference

- b. Because sprint times are never compared week to week
 - c. Because 100 meters is too short to time
 - d. Because the second run was slower
2. A floor's length is uncertain by 2% and its width by 1%. How uncertain is its area?
- a. 1%
 - b. 2%
 - c. 3%
 - d. 6%
3. Why may you not report a stick's length as 36.71 cm when you measured it with an ordinary ruler?
- a. Because 36.71 is too long for a stick
 - b. Because a ruler cannot measure to a hundredth of a centimeter
 - c. Because lengths must always end in a zero
 - d. Because rulers only measure in inches
4. How many significant figures does 30.42 have?
- a. Two
 - b. Three
 - c. Five
 - d. Four

Answers

1. a) When the tool's uncertainty is as big as the difference you want to see, you cannot tell the results apart.

- 2. c)** For multiplying measurements with small uncertainties, the percent uncertainties add: $2\% + 1\% = 3\%$.
- 3. b)** You write only as many digits as the tool truly measured, and a ruler reads to millimeters, not hundredths of a centimeter.
- 4. d)** A zero sitting between other digits was measured and always counts, so all four digits are significant.

Practice

- 1.** Decide how many significant figures each number has, and say why: 0.053; 10.053; 0.0009; 87.990.

Hint: Zeros in front only place the decimal point; zeros between digits or at the end after a decimal point were measured.

Answer: 0.053 has two, since the leading zeros are placeholders. 10.053 has five, since its zeros sit between other digits. 0.0009 has one. 87.990 has five, the final zero showing a measurement to a thousandth.

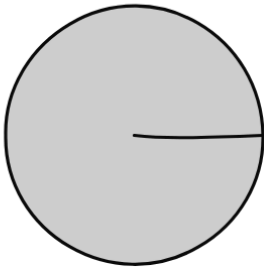
- 2.** A floor is 4.00 m long and 3.00 m wide, with uncertainties of 2% and 1%. What is the area, and roughly how many square meters could it be off by?

Hint: Multiply for the area and add the percentages for its uncertainty.

Answer: The area is 12.0 square meters with a 3% uncertainty, which is about a third of a square meter, rounded by the book to 0.4 square meters.

Honest digits and bold guesses

Measure a round tabletop with a tape and you find its radius, from the centre to the edge, is about one point two meters. Ask a calculator for the area and it proudly shows eight digits, four point five two three eight nine and so on. Every digit looks certain. But your tape only gave you two digits to begin with, so how can the answer have eight? It cannot, and this last part is about how many digits an answer has truly earned, and then about the opposite art: making a good guess with hardly any digits at all.



$$r = 1.2 \text{ m}$$

$$4.5238934 \text{ m}^2$$

$$A = \pi r^2 \approx 4.5 \text{ m}^2$$

2 SIG FIGS

Digits the radius allows

The book's ground rule is simple. When you combine measurements of different quality, the answer can be no more precise than the least precise of them. A chain is only as strong as its weakest link, and a calculation is only as good as its roughest measurement. There are two versions of the rule, one for multiplying and dividing, and one for adding and subtracting.

For multiplying and dividing, the answer keeps as many significant figures as the measurement that has the fewest. The table's radius had two, so the area is honestly four point five square meters, two figures, no more. It does not matter that the number pi, which you also use to find the area, is known to many more digits; the radius is the weakest link. It is the same with a medicine dose worked out from a weight read on a bathroom scale: the dose can be no more exact than that scale reading.

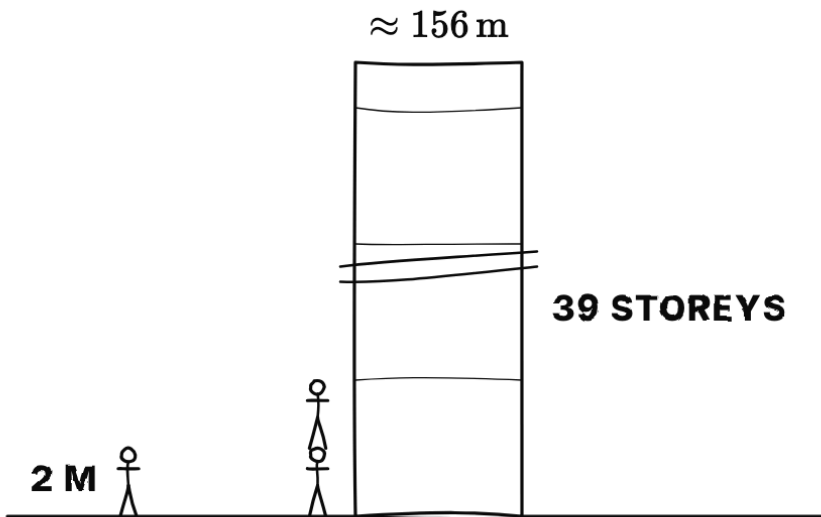
For adding and subtracting, the rule is about decimal places instead. The book follows a load of potatoes. You buy seven point five six kilograms at a shop, on a scale that reads to hundredths of a kilogram. You drop off six point zero five two kilograms at a laboratory, on a finer scale that reads to thousandths. Then you add thirteen point seven kilograms at home, weighed on a bathroom scale that reads only to tenths. Do the sums and you get fifteen point two zero eight kilograms, but the bathroom scale only knew tenths, so the honest answer is fifteen point two.

$$\begin{array}{r}
 7.56 \text{ kg} \\
 - 6.052 \text{ kg} \\
 + 13.7 \text{ kg} \quad \textbf{TENTHS ONLY} \\
 \hline
 15.208 \text{ kg} \longrightarrow 15.2 \text{ kg}
 \end{array}$$

Adding: keep the fewest decimals

Some numbers never limit the digits at all, because they are exact. If a woman carries two bags of thirteen point five pounds and one of ten point two, the "two" is a count, not a measurement, so it carries no doubt; the total is thirty-seven point two pounds. Likewise the two in the rule for the distance round a circle is exact. The book also explains its own habit: most numbers in it carry three significant figures, more only where a subject such as optics needs extra care. And it sets a quick test: a wagon of fifty-five kilograms speeding up gently feels a push you find by multiplying its mass by how quickly its speed grows; because fifty-five has only two figures, the answer, one point four in the unit of force called the newton, has only two as well.

Now the book turns the whole question around. Very often, scientists and engineers do not need many digits at all. They need a quick, sensible estimate, which the book cheerfully calls a guesstimate. How far away is that town? Roughly how dense is this object? About how much current flows in this circuit? Such estimates rule out impossible answers, let you challenge other people's claims, and guide your next step. The skill grows with practice, with a feel for units, and with a willingness to take a risk and commit to a number.



A building in people

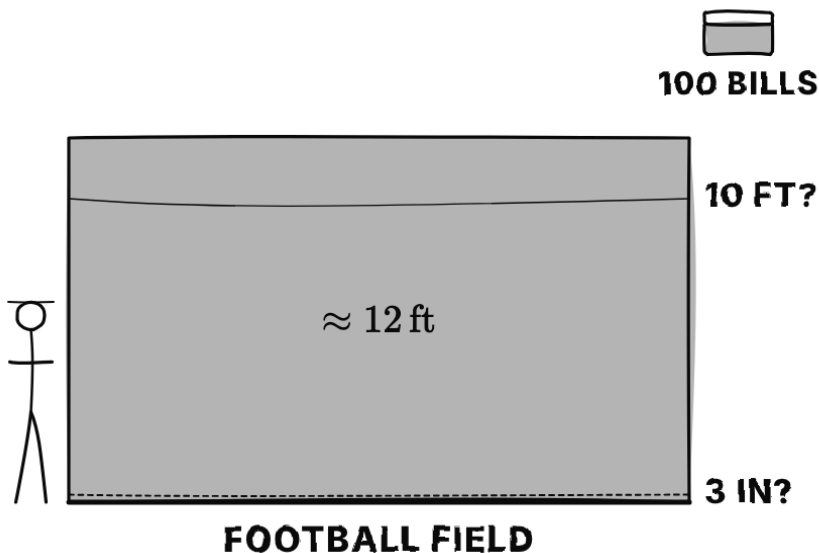
The first example is a tall building of thirty-nine storeys. How high is it, with no tape measure? Scale up from a person. The book takes a grown man as about two meters tall, and one storey as about two people standing on each other's heads. So each storey is about four

meters, and thirty-nine of them come to roughly a hundred and fifty-six meters.

Try it yourself

Measure your own hand across the palm; for many people it is about ten centimeters. Now walk your hand across the width of a table, counting hand-widths. Ten hand-widths means about a meter, and you have measured the table without any ruler, just as the book suggests.

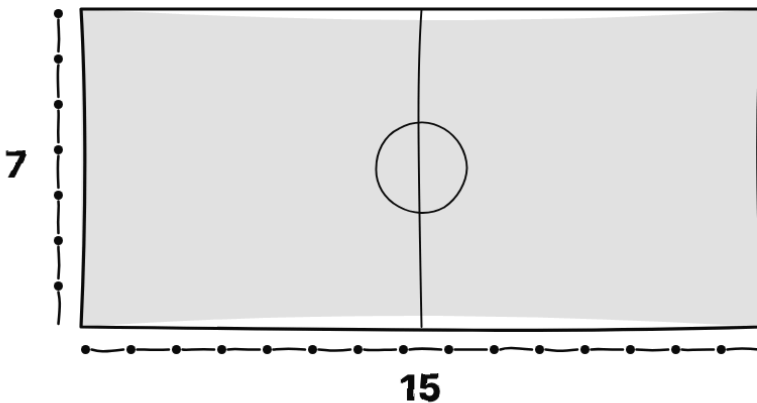
The second example takes on a number almost nobody can picture: a trillion dollars. The book notes that the United States owed a little more than twenty-eight trillion dollars in 2021, yet few people can imagine even one trillion. So picture it in hundred-dollar bills, bundled into bank stacks of a hundred bills each, the kind seen in films, each stack worth ten thousand dollars. Now spread all of those stacks evenly over an American football field, end zone to end zone. How high would the pile reach? One friend guesses three inches; another guesses ten feet.



A trillion dollars on a field

The book works it through with deliberately rough numbers, keeping just one significant figure. A single bill is about three inches by six, and a stack of a hundred is about half an inch thick, so each stack is a small brick of paper. A trillion dollars in stacks of ten thousand makes about a hundred million stacks. The field, a hundred yards by fifty, works out at roughly six million square inches. Spread the bricks over that area and the pile comes out at about a hundred and fifty inches, around twelve feet deep. So the three-inch guess was far too small, and the ten-foot guess was not bad at all. The lesson is not the exact height but the method: a careful rough estimate, built from things you can picture, beats a gut feeling.

The book's last challenge is to estimate the floor of a basketball court with nothing but mental arithmetic. Picture people about two meters tall lying end to end: about fifteen of them along the court, about seven across. That gives roughly thirty meters by fourteen, and an area of about four hundred and twenty square meters. You could do the same with your living room, counting how many of your own body lengths fit along each wall.



$$30 \text{ m} \times 14 \text{ m} \approx 420 \text{ m}^2$$

A court in body lengths

That brings the first chapter, and this course, to its end. Physics, as we have heard, looks for a few simple laws beneath an endlessly varied world, measures that world in agreed units, says honestly how sure it is, and knows when to count every digit and when a bold, sensible guess will do.

Remember

An answer can never be more precise than the least precise measurement behind it. When multiplying or dividing, keep as many significant figures as the measurement with the fewest; when adding or subtracting, keep as many decimal places as the roughest measurement. Exact numbers, such as a count of bags, never limit the digits. A good approximation scales up from familiar things, such as a person's height or a hand's width, and quickly rules out impossible answers. A trillion dollars in hundred-dollar bills would cover a football field roughly twelve feet deep.

Questions

1. Why is the area of a table with a radius of 1.2 m given as 4.5 square meters, not 4.5238934?
 - a. Because pi is only known to two digits
 - b. Because the radius was measured to only two significant figures
 - c. Because areas are always rounded to tenths
 - d. Because calculators are unreliable after two digits
2. Why is the total weight of potatoes given as 15.2 kg and not 15.208 kg?
 - a. Because the bathroom scale, the roughest one, read only to tenths

- b. Because the laboratory scale was broken
 - c. Because potatoes lose weight over time
 - d. Because adding always removes digits
3. Why does the 'two' in 'two bags of 13.5 pounds' not limit the answer's digits?
- a. Because two is a small number
 - b. Because bags are not weighed
 - c. Because two has only one digit
 - d. Because it is a count, an exact number with no doubt in it
4. In the trillion-dollar example, what is the main lesson?
- a. That the money pile would be exactly twelve feet
 - b. That gut feelings are usually right
 - c. That a rough estimate built from things you can picture beats a gut feeling
 - d. That only exact calculations are useful

Answers

1. **b)** When multiplying, the answer keeps as many significant figures as the measurement with the fewest, here the radius.
2. **a)** In adding and subtracting, the answer keeps no more decimal places than the least precise measurement.
3. **d)** Counted or defined numbers carry no uncertainty, so they never limit the significant figures.
4. **c)** The point is the method: careful rough reckoning from familiar pieces rules out wild guesses like three inches.

Worked examples

How tall is a 39-storey building? Estimate the height of a 39-storey building, taking a person as about 2 m tall and one storey as about two people high.

One storey is about two people, each about 2 m, so a storey is about 4 m.

There are 39 storeys.

39 storeys of about 4 m each make about 156 m.

Answer: About 156 m.

Practice

1. A woman has three bags: two weigh 13.5 pounds each and one weighs 10.2 pounds. What is their total weight?

Hint: The number of bags is exact; add the weights and look at their decimal places.

Answer: 37.2 pounds: $13.5 + 13.5 + 10.2 = 37.2$, and the count of two bags does not limit the digits.

2. Using mental arithmetic, estimate the floor area of a basketball court by imagining people about two meters tall lying end to end.

Hint: Guess how many people fit along the length and across the width.

Answer: About fifteen people along it and seven across give roughly 30 m by 14 m, an area of about 420 square meters.